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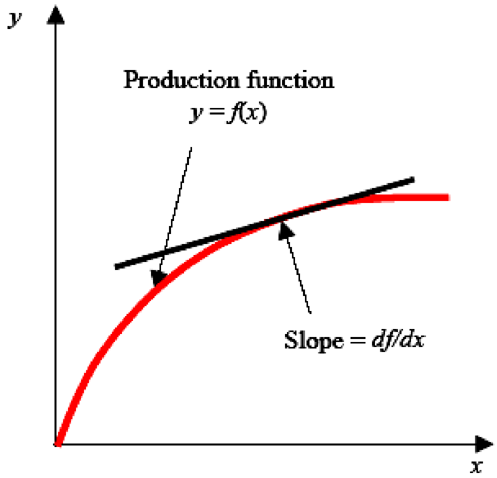
Firms

- ***Firms produce outputs from inputs (like water)***
- ***Firm objective: maximize profit***

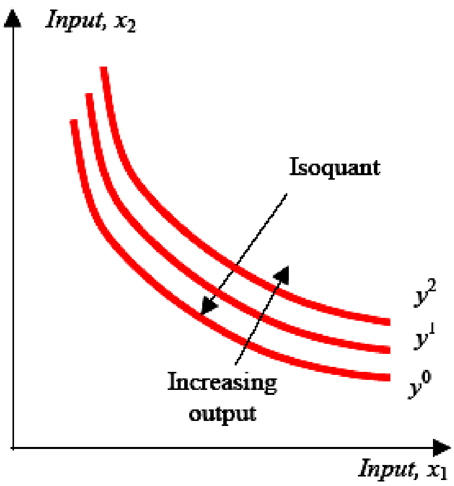
Production function

$$y = f(x)$$

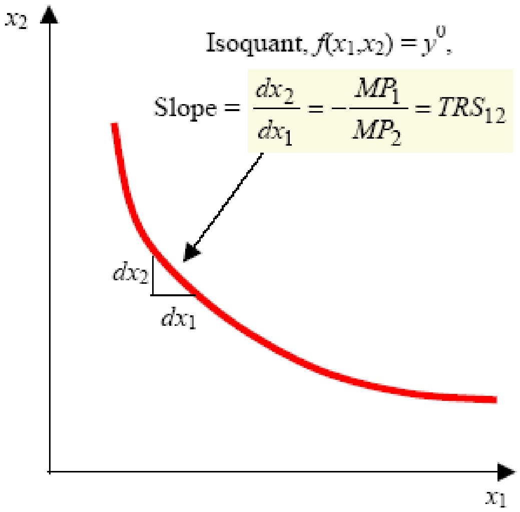


Isoquant

$$f(x) = y^0$$



- ***Suppose firm wants to increase one input and decrease another maintaining constant output***



$$y = f(x_1, x_2) = y^0 = const.$$

$$dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x2} dx_2 = 0$$

Marginal productivity

$$MP_i = \frac{\partial y}{\partial x_i} = \frac{\partial f(\vec{x})}{\partial x_i}, \quad i = 1,2$$

Technical Rate of Substitution

$$\frac{dx_2}{dx_1} = -\frac{MP_1}{MP_2} = TRS_{12}$$

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Profit

- Output $y = f(\boldsymbol{x})$
- Input $\boldsymbol{x}$
- Revenue $R = py$
- Cost $C = \sum_{n=1}^{N} w_n x_n$
- Profit

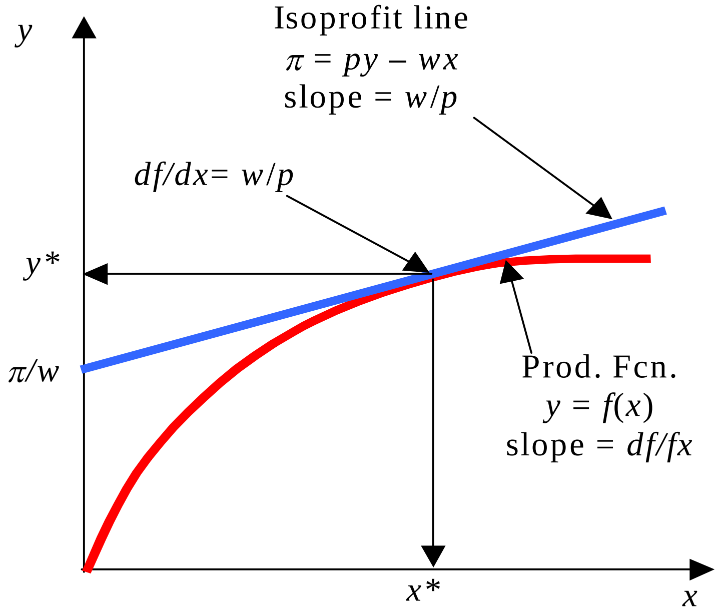
$$\pi = R - C \qquad \pi = pf(\boldsymbol{x}) - \sum_{n=1}^{N} w_n x_n$$

The Firm's Problem

- Maximize profit
- Optimality conditions $\frac{\partial \pi}{\partial x_n} = 0 \Rightarrow p\frac{\partial f}{\partial x_n} = w_n, \quad n = 1,\ldots,N$

$$\Rightarrow \frac{\partial f}{\partial x_n} = \frac{w_n}{p} \qquad n = 1,\ldots,N$$

- Value of marginal product (price times marginal product) for input n must equal price of that inp



Revenue

- Revenue $R = py$
  - receipt for selling y at price p
- Marginal Revenue
  - derivative WRT y
- Increase in output has two effects

- Adds revenue from sale of more units, and
- Causes value of each unit to decrease

$$\frac{dR}{dy} = \frac{\partial R}{\partial y} + \frac{\partial R}{\partial p}\frac{dp}{dy}$$
$$= p + y\frac{dp}{dy}$$

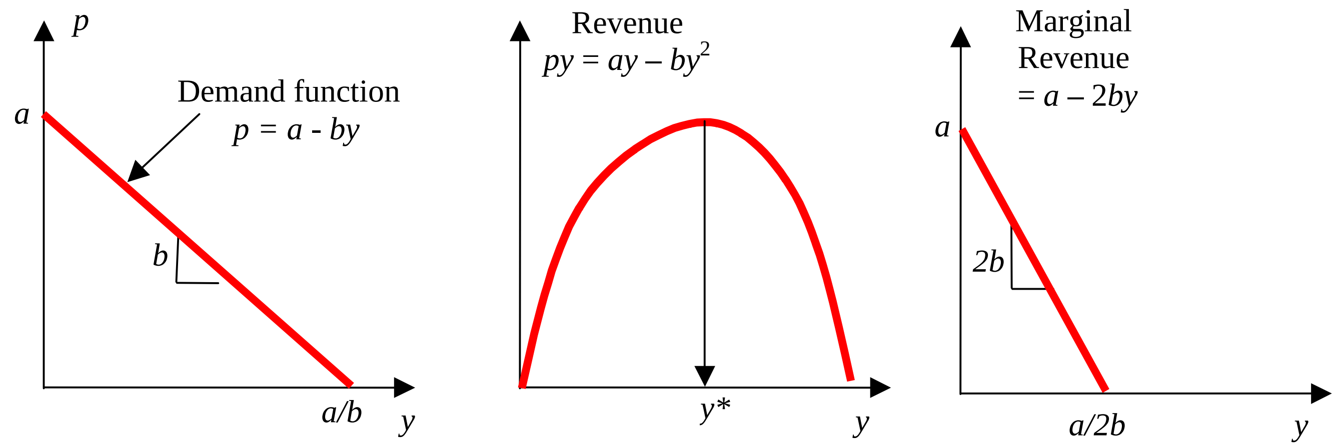
- Competitive firm: p is constant $p' = {dp}/{dy} = 0$

- Monopolistic firm: p is not constant $p' = {dp}/{dy} \neq 0$

Example

Linear demand function $p(y) = a - by$

- Revenue $R = py = ay - by^2$
- Marginal revenue $\frac{dR}{dy} = a - 2by$
- marginal revenue slope is twice demand curve



The Firm's Problem – 2nd Way

- Minimize cost

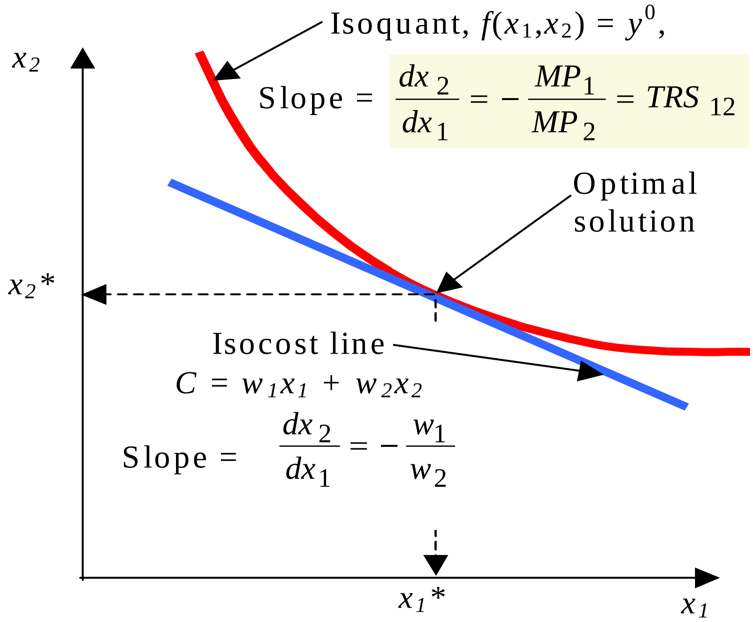
$$\text{minimize} \sum_{n=1}^{N} w_n x_n$$
$$\text{subject to}$$
$$f(x_1,\dots,x_N) = y_0$$

- Optimality conditions $\frac{\partial L}{\partial x_n} = w_n - \lambda\frac{\partial f}{\partial x_n} = 0 \quad n = 1,\dots,N$

$$\frac{\partial L}{\partial \lambda} = f - y_0 = 0$$

Technical rate of substitution equals price ratio

$$\frac{w_i}{w_j} = \frac{MP_i}{MP_j} = -TRS_{ij}$$



Cost Functions

Cost to producing level, y_0

$$TC(y) = \min\{\mathbf{w}\cdot\mathbf{x} : y = f(\mathbf{x})\}$$

- ***Cost comprised of fixed and variable*** costs $\quad TC(y) = FC + VC(y)$
- ***Average cost* is** cost per unit to produce *y* units $\quad AC = \frac{TC(y)}{y}$
- ***Marginal cost*** is cost of producing an additional unit $\quad MC = \frac{dTC}{dy} = \frac{dVC}{dy}$

Example – Competitive Firm

- How much water should a water industry firm sell (produce) and at what price?
- Firm's problem

$$\text{Maximize } \pi(y) = py - TC(y)$$

Optimality conditions

$$\frac{d\pi}{dy} = 0 = \frac{dp}{dy}y + p - \frac{dTC}{dy}$$

$$MR(y) = \frac{dp}{dy}y + p = MC(y)$$

Competitive firm

$$p' = {dp}/{dy} = 0$$

$$p = MC$$

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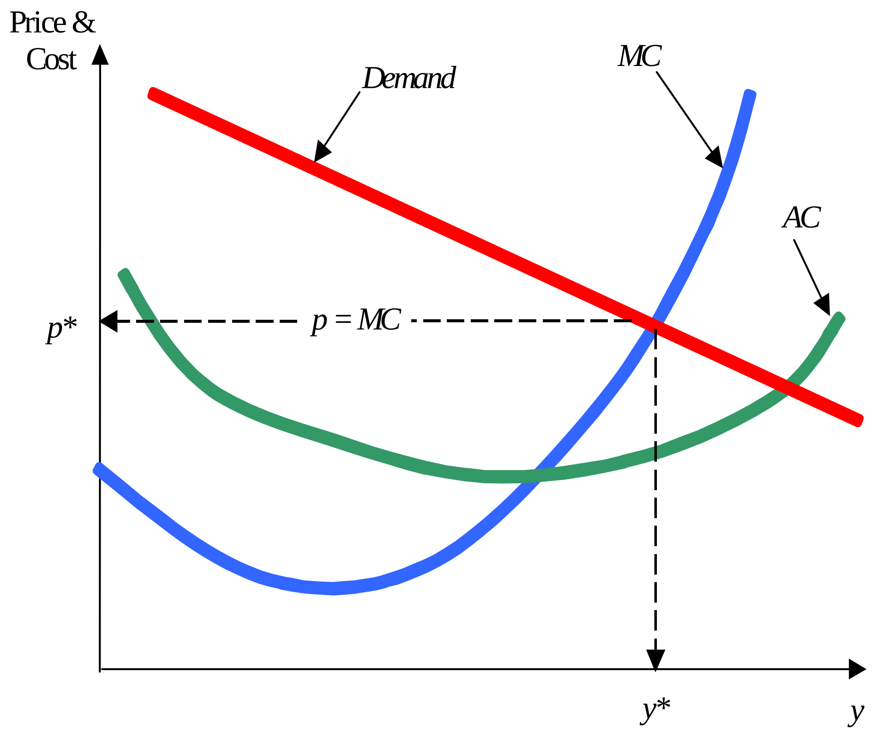
Example

Competitive firm



Example – Monopolistic Firm

- Recognizes its influence over market price
- Free to choose price and output to maximize profit

Optimality conditions

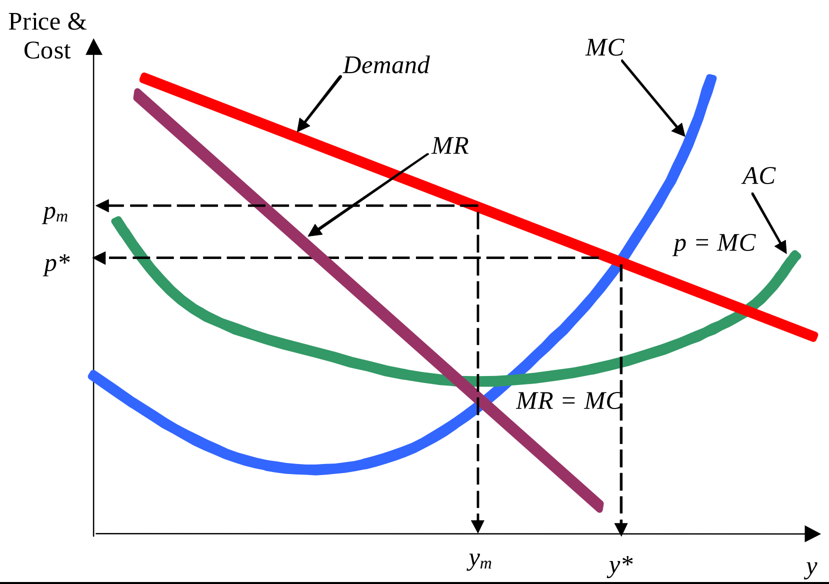
$$\text{Maximize } \pi(y) = py - TC(y)$$

$$\frac{d\pi}{dy} = 0 = \frac{dp}{dy}y + p - \frac{dTC}{dy}$$

$$MR(y) = \frac{dp}{dy}y + p = MC(y)$$

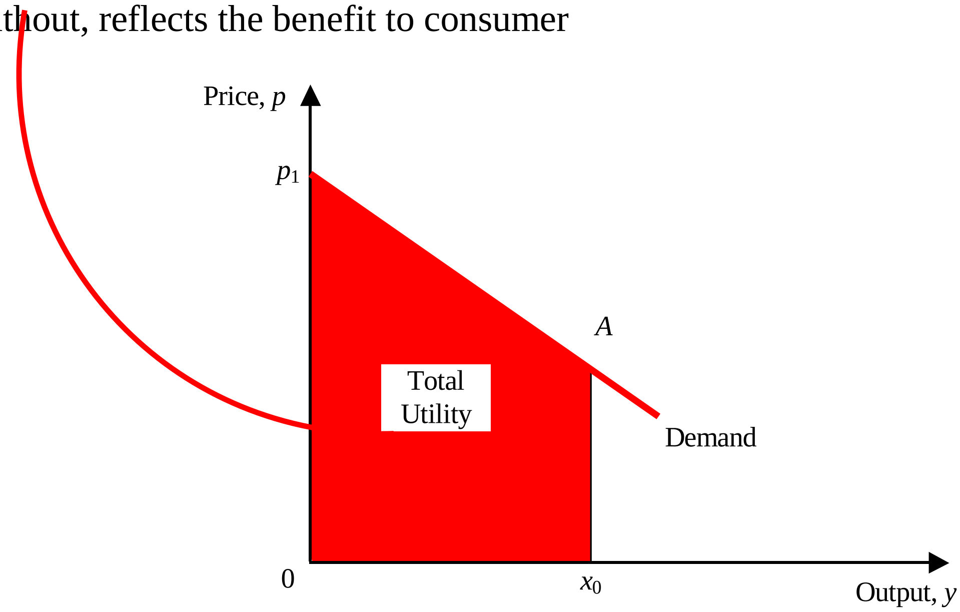
- Monopolistic firm
- So, Marginal Revenue = Marginal Cost

$$p' = {dp}/{dy} \neq 0$$



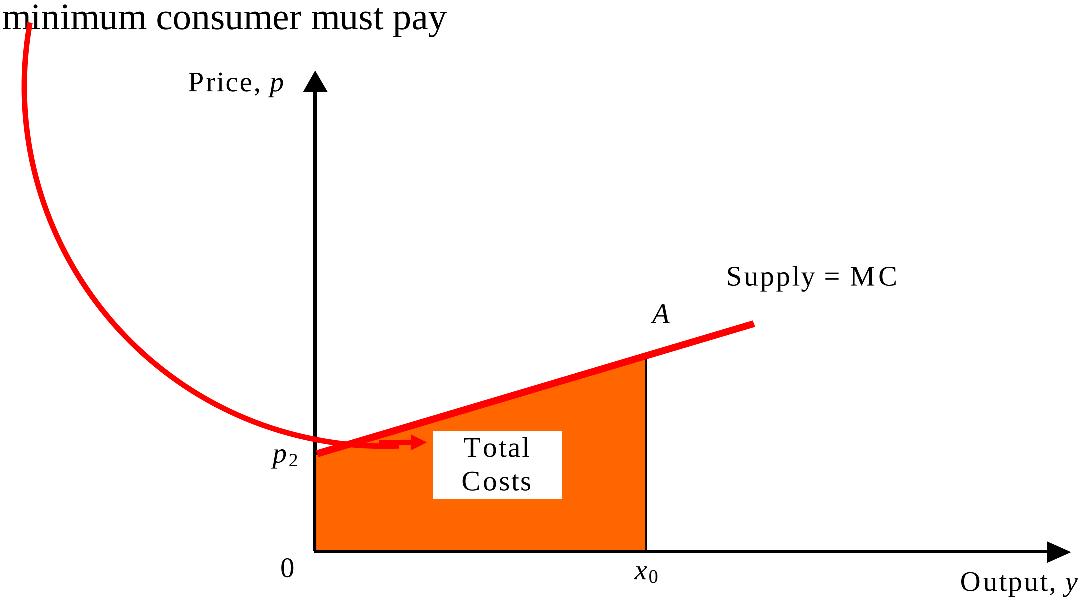
Consumers' WTP

- Maximum paid for *x*0 units rather than go without, reflects the benefit to consumer



Producers' Cost

Minimum producer will accept for x_0 units and the minimum consumer must pay



Pricing

- Demand affected by price of water
  - price elasticity of demand:
    - %change in demand for %change in price
- Conservation
  - Non-Price methods (education, etc.)
  - Price methods
    - "declining block rates" - the more water used, the lower the price for the last units of use (discourages conservation)
    - "alternative rate structures" - encourage users to reduce their consumption
      - *Increasing (or inverted) block: Rates increase at set usage level intervals • Seasonal block: Two different rate structures are set (one in the summer and one in the winter) • Baseline block: A baseline usage water usage amount is set based on a customer's winter use and a surcharge is then imposed for any use over the baseline during the summertime*

Price

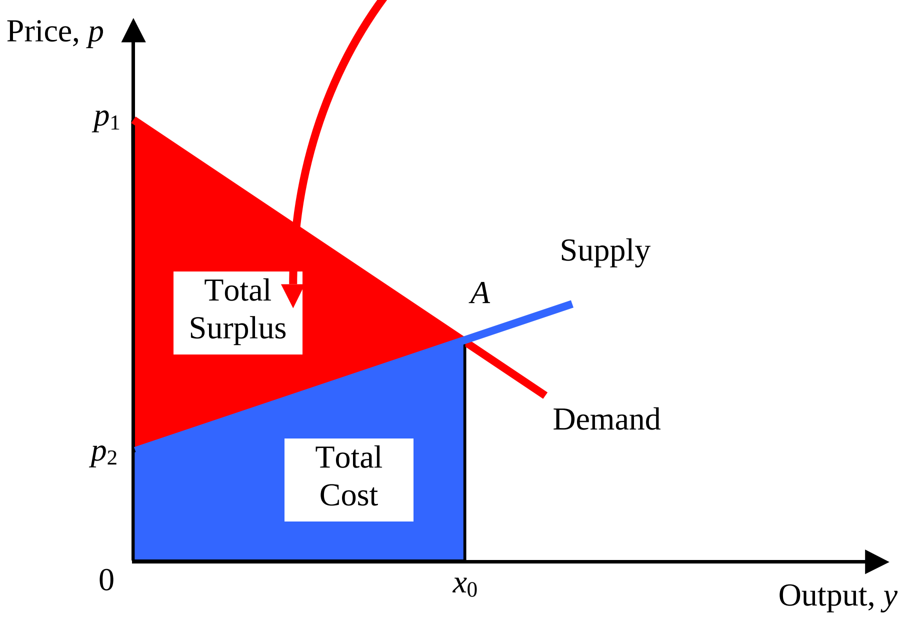
$3/1,000 gallons
= 174 gal/day/person
= 870 million gal/day
Dallas-Fort Worth area (5 million)

Elasticity

In Texas elasticity = -0.32
Consumption will decline 3.2% for every 10% rise in price

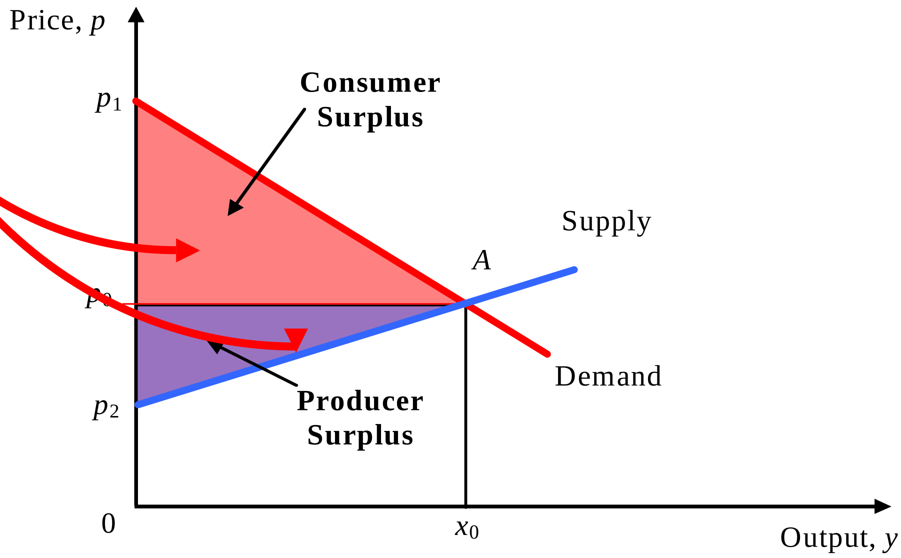
Consumers' & Producers' Surpluses

- total surplus net of resource costs = the difference between the maximum the consumer would be willing to pay rather than go without and the minimum he must pay in order to cover costs of production (total net benefit to society)



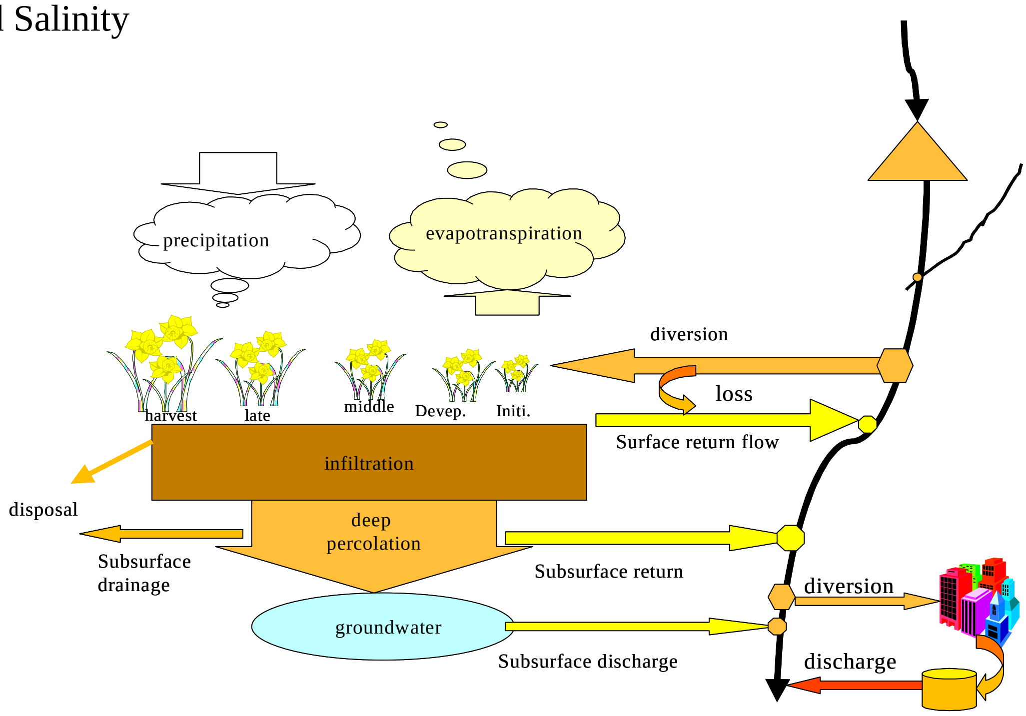
Surpluses – What they mean

- **Consumers' Surplus** = amount consumer would have been willing to pay, but didn't have to
- **Producers' Surplus** = amount producer would have been willing to accept, but was able to realize more



Example – Maipo basin, Chile

- **Water allocation to crops depends on**
  - Water requirements,
  - Economic profitability
- **Production functions incorporate**
  - Water, Technology, and Salinity



Production Functions

- Four broad approaches to production functions can be identified
  - Evapotranspiration models;
  - Simulation models;
  - Estimated models; and
  - Hybrid models

ET models

$$Y = Y_{\max}\left[1 - kc * \left(1 - \frac{E}{E_{\max}}\right)\right]$$

where

Y	**= yield**
Y_{max}	**= maximum yield**
kc	**= crop coefficient**
E	**= actual evapotranspiration (mm)**
E_{max}	**= maximum evapotranspiration (mm)**

Estimated models

$$\begin{aligned} Y / Y_{\max} = {} & a_0 + a_1 x + a_2 s + a_3 u \\ & + a_4 x \cdot s + a_5 x \cdot u + a_6 s \cdot u \\ & + a_7 x^2 + a_8 s^2 + a_9 u^2 \end{aligned}$$

where

Y	**= yield**
Y_{max}	**= maximum yield**
x	**= irrigation water applied,**
s	**= irrigation water salinity,**
u	**= irrigation uniformity,**
a_i	**= estimated coefficients (i=1,..9)**

Production Function - Wheat

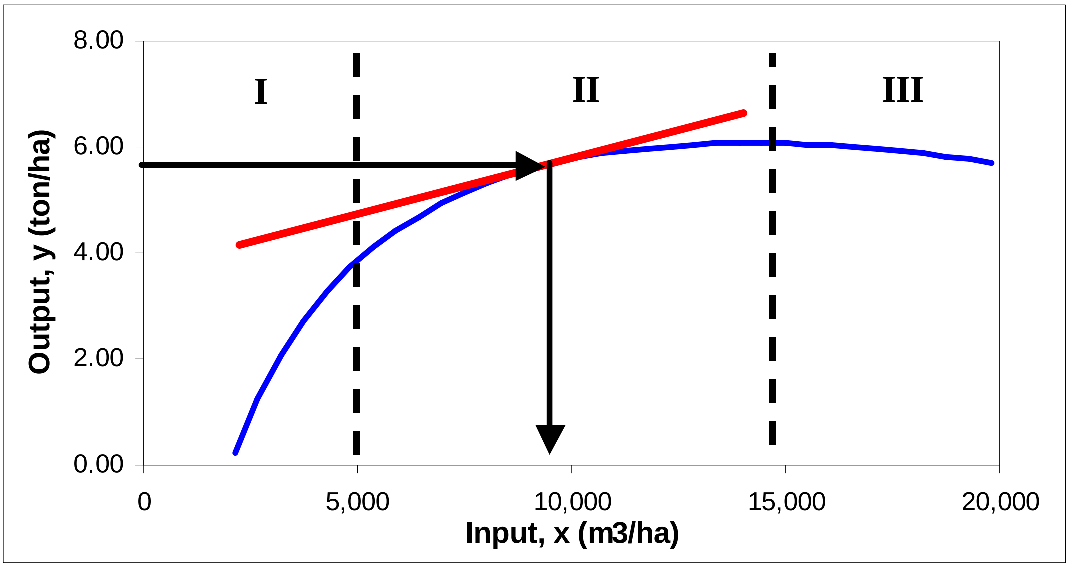
$$Y = Y_{\max}[a_0 + a_1(x / E_{\max}) + a_2 \ln(x / E_{\max})]$$

$$a_0 = b_0 + b_1 u + b_2 s$$

$$a_1 = b_3 + b_4 u + b_5 s$$

$$a_2 = b_6 + b_7 u + b_8 s$$

y_{max}	**= maximum yield (mt/ha)**
$b_0 - b_8$	**= coefficients,**
x	**= irrigation water applied (mm)**
E_{max}	**= Max ET (mm)**
s	**= irrigation water salinity (dS/m)**
u	**= irrigation uniformity**



Stages of Production

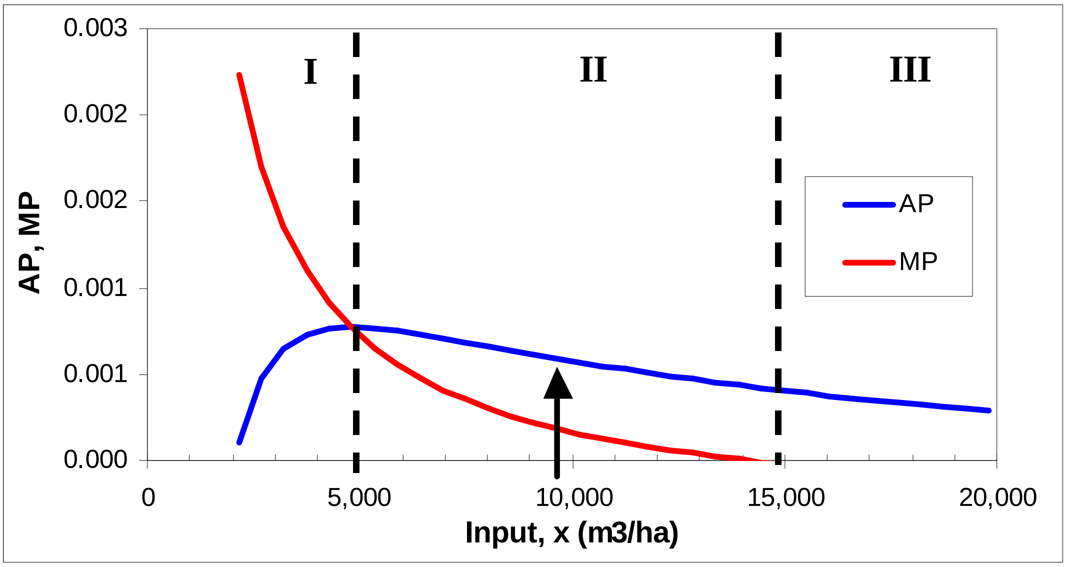
Region I ***MP*** **>** ***AP*****, not enough input is being used**

Physical efficiency of input increases throughout Region I

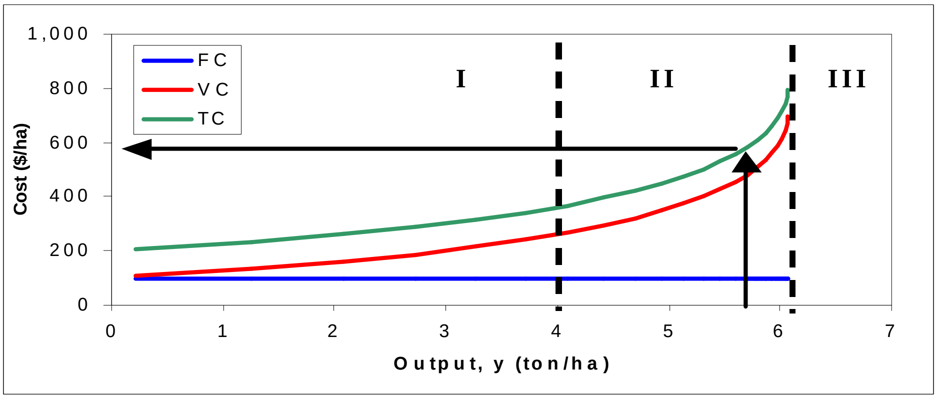
Region II ***MP*** **is decreasing and** ***MP*** **<** ***AP*****, just enough input is being used**

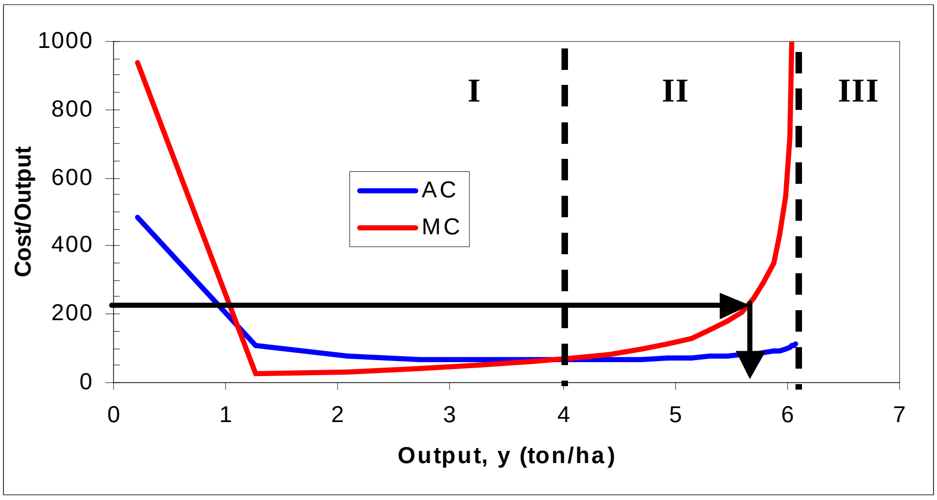
Optimal input use is in this Region, exact level depends on prices.

Region III ***MP*** **< 0, too much input is being used**



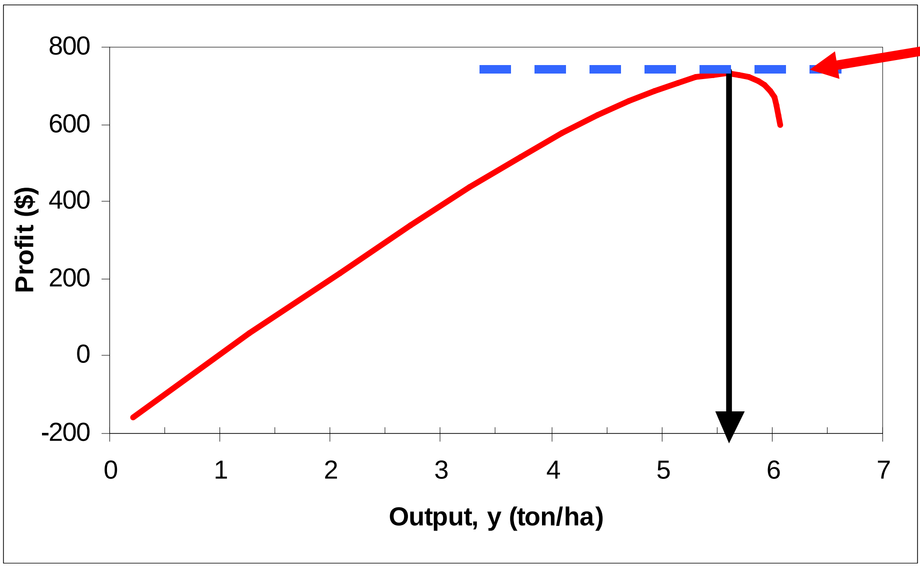
Cost Curves





Optimal Production - Wheat

$$\pi = py - wx$$



$$\frac{d\pi}{dx} = p\frac{dy}{dx} - w = 0$$

$$\frac{dy}{dx} = \frac{w}{p}$$

$$p = \frac{w}{\frac{dy}{dx}}$$

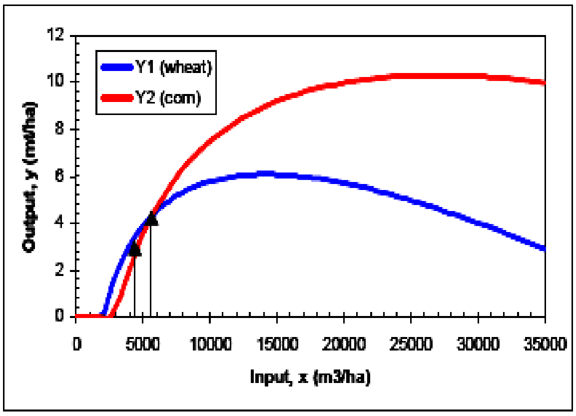
$$p = MC$$

Production – Wheat & Corn

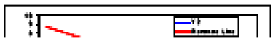
Production functions for wheat and corn

Water x (m3/ha)	Wheat y_1 (mt/ha)	Water x (m3/ha)	Corn y_2 (mt/ha)
0	0.00	0	0.00
536	0.00	883	0.00
1071	0.00	1765	0.00
1607	0.00	2648	0.00
2142	0.22	3530	1.21
3213	2.08	5295	3.87
4284	3.27	7060	5.60
5355	4.10	8825	6.83
6426	4.70	10590	7.74
7497	5.14	12355	8.43
8568	5.46	14120	8.96
9639	5.70	15885	9.37
10710	5.87	17650	9.68

Production Possibilities



0.2	6.3
0.0	6.9
0.0	7.5



Optimality Conditions

maximize $p_1 y_1 + p_2 y_2$
subject to
$f(x, y_1, y_2) = 0$

$$L(x, y_1, y_2, \lambda) = p_1 y_1 + p_2 y_2 - \lambda f(x, y_1, y_2)$$

$$\frac{\partial L}{\partial x} = p_1 \frac{\partial y_1}{\partial x} + p_2 \frac{\partial y_2}{\partial x} - \lambda \frac{\partial f}{\partial x} = 0$$

$$\frac{\partial L}{\partial y_1} = p_1 - \lambda \frac{\partial f}{\partial y_1} = 0$$

$$\frac{\partial L}{\partial y_2} = p_2 - \lambda \frac{\partial f}{\partial y_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = f(x, y_1, y_2) = 0$$

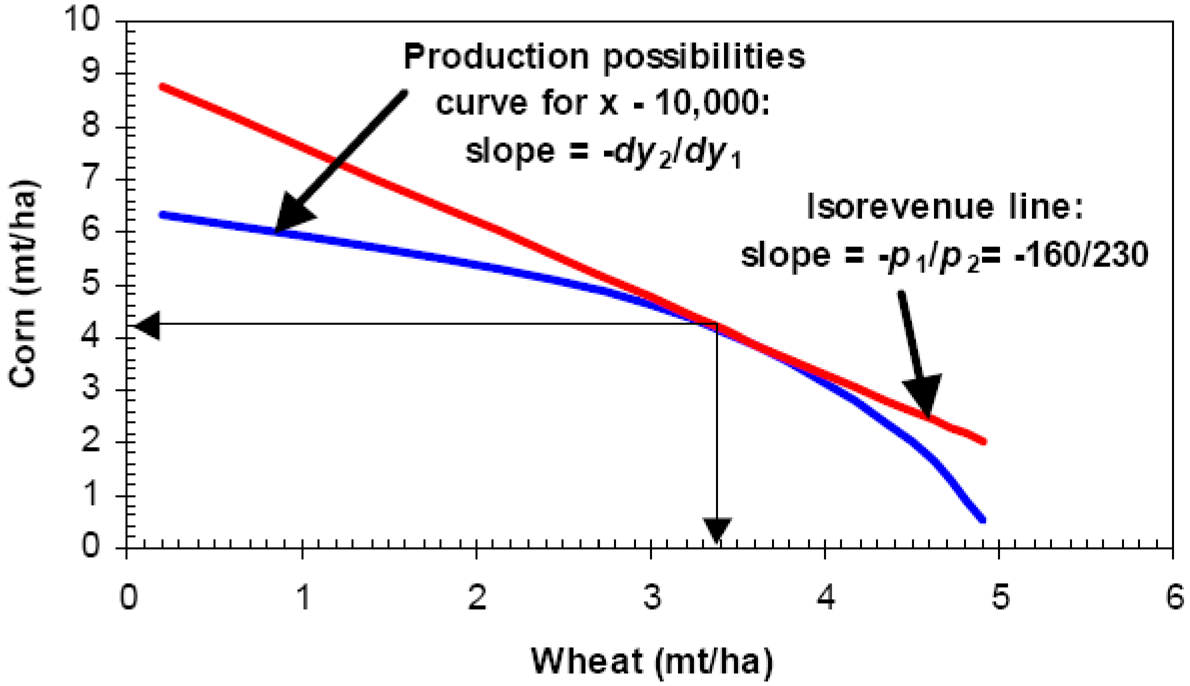
$$\frac{p_1}{p_2} = \frac{\partial f / \partial y_1}{\partial f / \partial y_2}$$

$$\frac{\partial f}{\partial y_1} dy_1 + \frac{\partial f}{\partial y_2} dy_2 = 0 \;\Rightarrow\; \frac{\partial f / \partial y_1}{\partial f / \partial y_2} = -\frac{dy_2}{dy_1} \;\Rightarrow\; \frac{p_1}{p_2} = -\frac{dy_2}{dy_1} = MRS_{1,2}$$

Optimal production

slope of isorevenue line (p_1/p_2) equals
slope of production possibilities curve ($-dy_2/dy_1$)

Multiple Outputs



Price of Corn = $160/mt
Price of Wheat = $230/mt