Fluid Flow in Soils

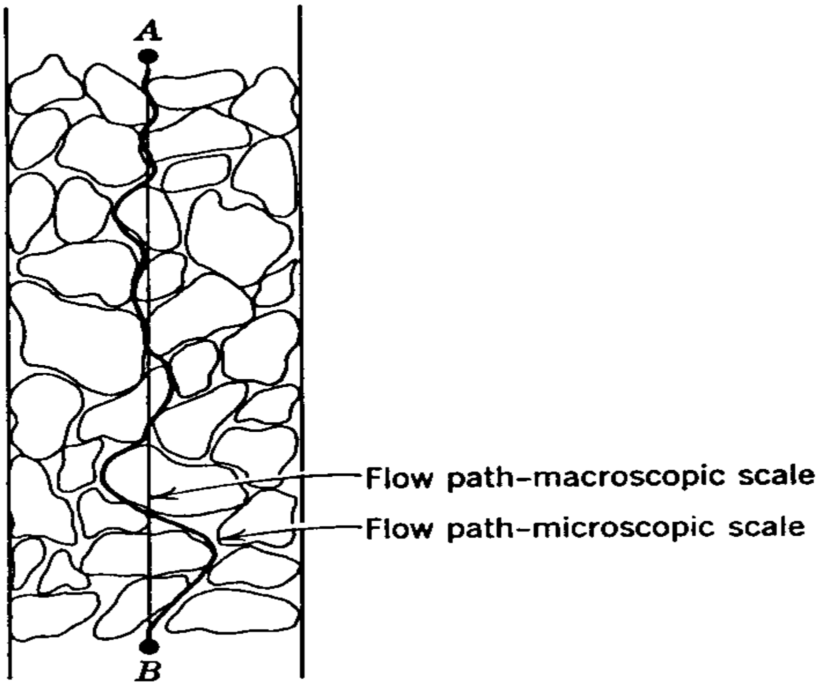
Problems of fluid flow in Soils

1. rate of flow of fluid through an earth dam (e.g. determination of rate of leakage through an earth dam).
2. problems involving compression (e.g. determination of the rate of settlement of a foundation).
3. problems involving strength (e.g. the evaluation of factor of safety of a given soil under a given loading).

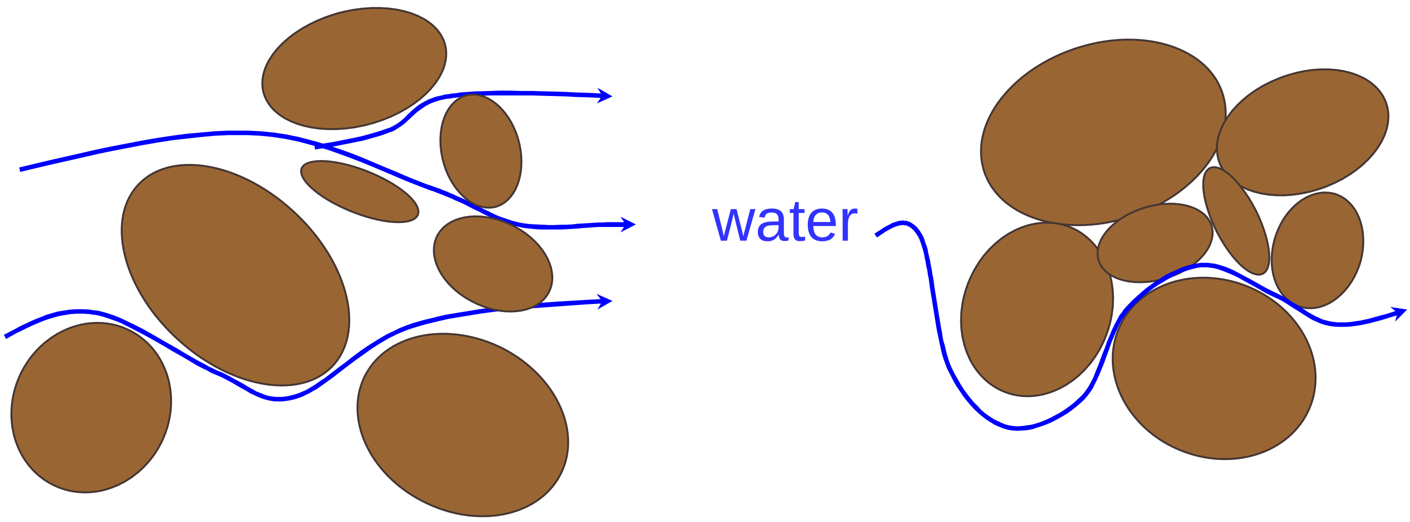
One dimensional flow

Flow Path in Soils

A measure of how easily a fluid (e.g., water) can pass through a porous medium (e.g., soils)



Macroscopic scale : Strait path (in soil engineering problems).
Microscopic scale : winding path (actual path)



Loose soil
- easy to flow
- **high** permeability

Dense Soil
- difficult to flow
- **Low** permeability

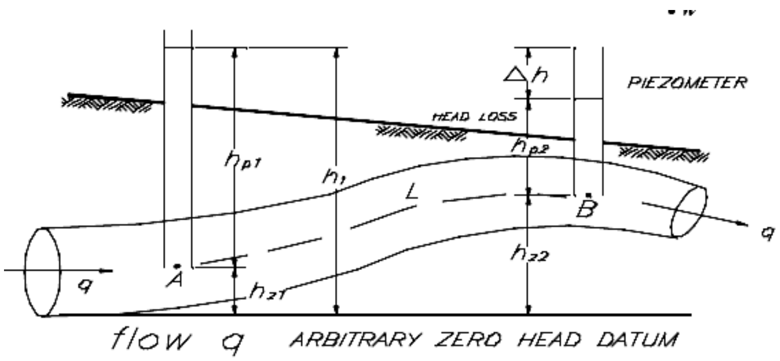
Hydraulic Gradient

According to Bernoulli's equation, the total head of a point under motion can be given by :

$$H = h_e + \frac{P}{\gamma_w} + \frac{v^2}{2g}$$

total head = position head + pressure head + velocity head

where h = total head,
P = Pressure ,
V = Velocity , and g = Acceleration due to gravity.



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Note: Velocity head $\frac{v^2}{2g}$ in soils is too small and can be neglected.

Hence:

$H = h_e + \frac{P}{g_w}$ and this is defined as the <u>piezometric head</u>

Now A→B $\Delta h = h_A - h_B$

And $\frac{\Delta h}{L}$ = i = hydraulic gradient

L = length of flow over which loss of head (Δh) is measured.

Therefore : *Flow of Water in Soils*

1- Hydraulic Head in Soil

Total Head = Pressure head + Elevation Head

$$h_t = h_p + h_e$$

- Elevation head at a point = Extent of that point from the datum

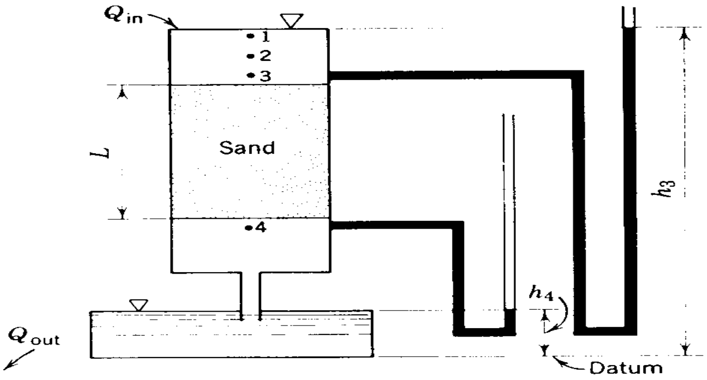
- Pressure head at a point = Height of which the water rises in the piezometer above the point.

- Pore Water pressure at a point = P.W.P. = $g_{water} \cdot h_p$

<u>Darcy's Law:</u> the velocity or discharge through a soil :

V ¥ i V= k i $\frac{Q}{A} = q = k\,i$

Where k = Coefficient of permeability



$i = \frac{\Delta h}{L} = \frac{h_3 - h_4}{L}$ = Hydraulic gradient between pts 3→ 4

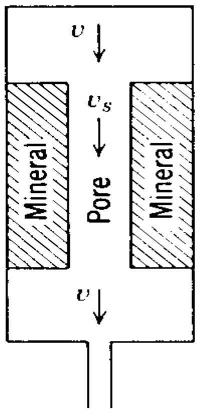
V = Velocity of water between pts.1→2→3

Now $Q_{in} = Q_{out}$

$VA = V_s A_v$

V_s = Velocity of water through the soil between pts. 3→ 4 = Seepage velocity

Then



$$V_s = V\frac{A}{A_v} = V\frac{AL}{A_v L} = V\frac{\text{total vol.}}{\text{voids vol.}}$$

$$\therefore V_s = \frac{V}{n}$$

(since n < 1 →Vs >V)

***How to measure the Pressure Head or the Piezometric Head**

1- Assume that you do not have seepage in the system (Before Seepage)

2- Assume that you have piezometer at the point under consideration

3- Get the measurement of the piezometric head (Water column in the Piezometer before seepage) = $h_{p(\text{Before Seepage})}$

4- Now consider the problem during seepage

5- Measure the amount of the head loss in the piezometer (D_h) or the drop in the piezometric head.

6- The piezometric head during seepage = $h_{p(\text{during seepage})} = h_{p(\text{Before Seepage})} - D_h$

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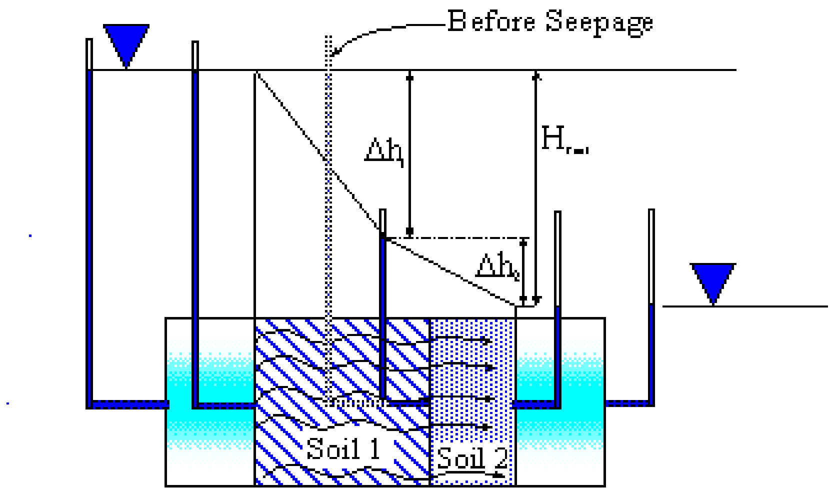
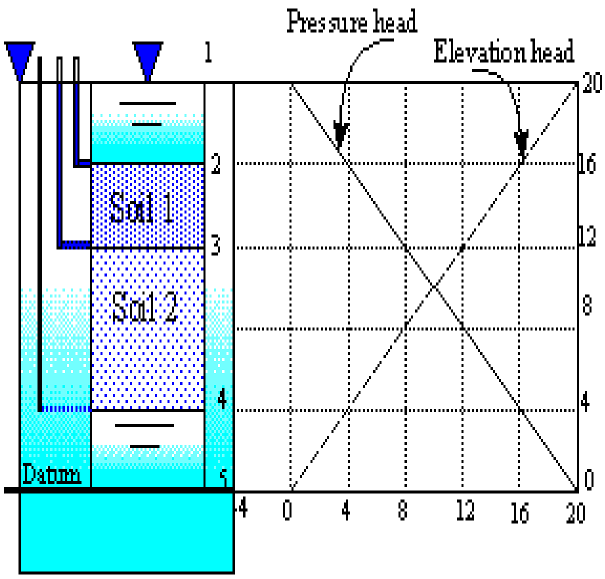
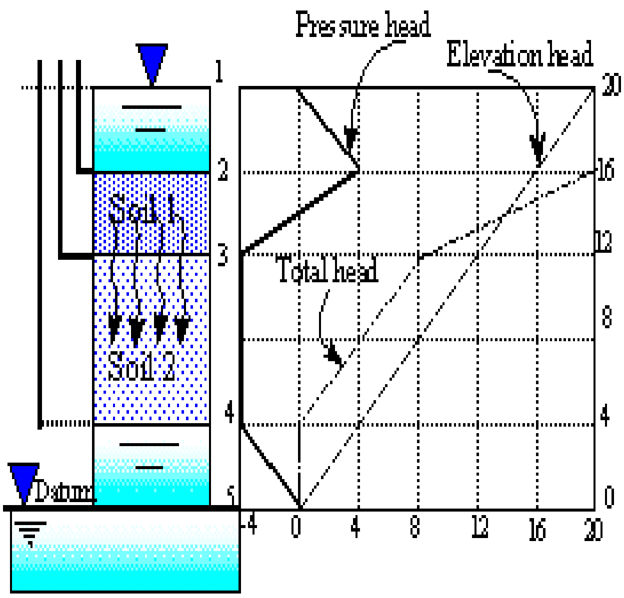


$$H_{total} = \Delta h_1 + \Delta h_2$$
$$q_1 = q_2 = v_1 A_1 = v_2 A_2 = k_1 \Delta h_1 / L_1 = k_2 \Delta h_2 / L_2$$



Before Seepage



During Seepage

Heads in Static Water

Head Distribution; Below Ground Water Table

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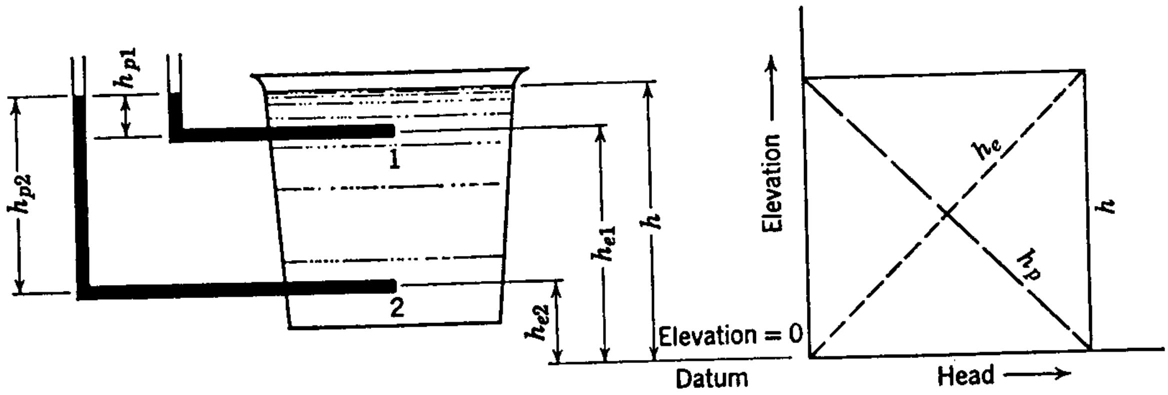
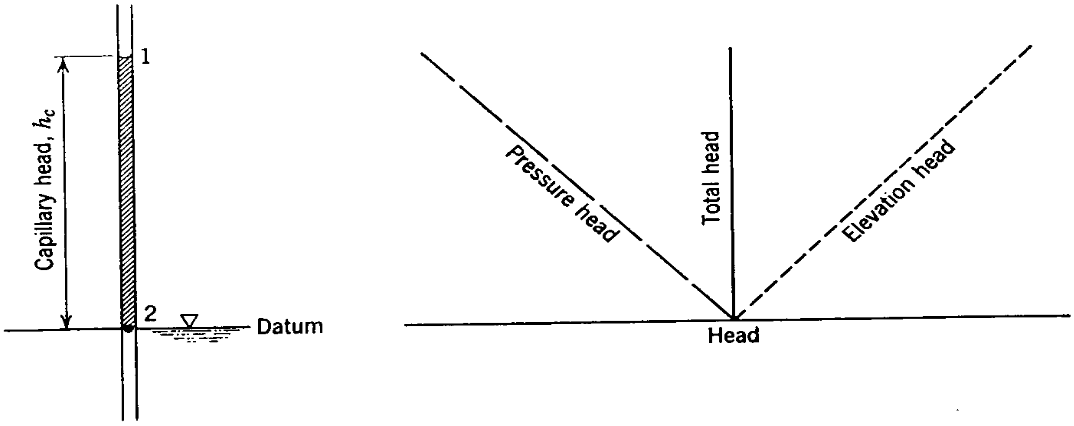
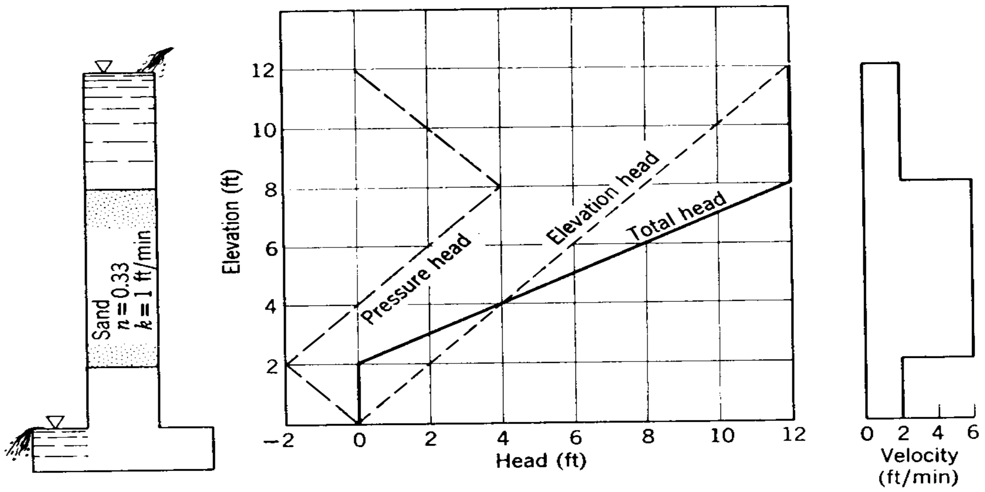


Head Distribution; Above Ground Water Table



Calculation of Pressure Head in Seepage Conditions (Downward Flow)



Calculation of Pressure Head in Seepage Conditions (Upward Flow)

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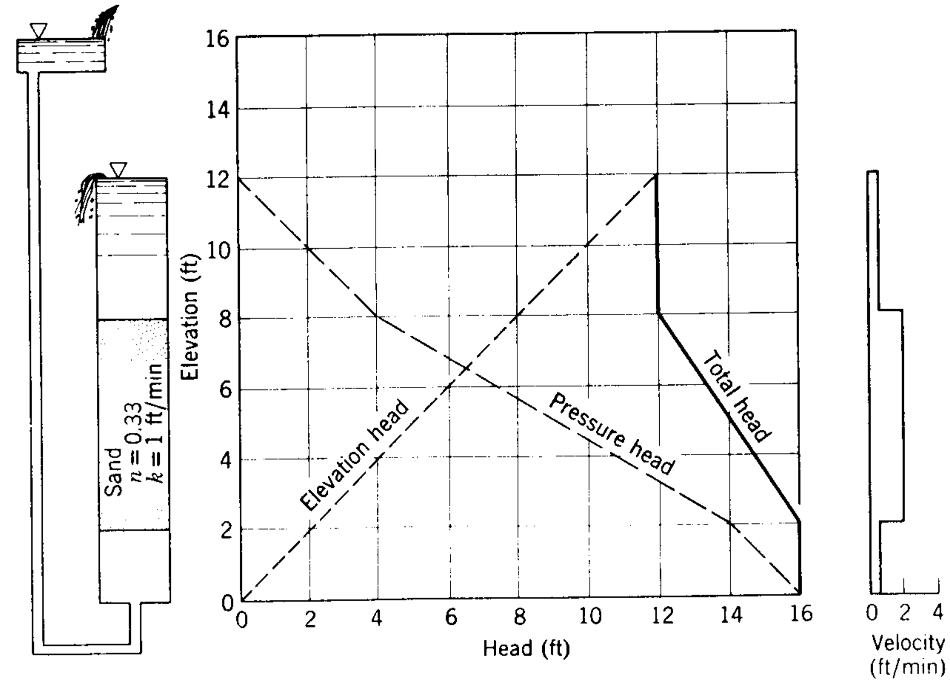
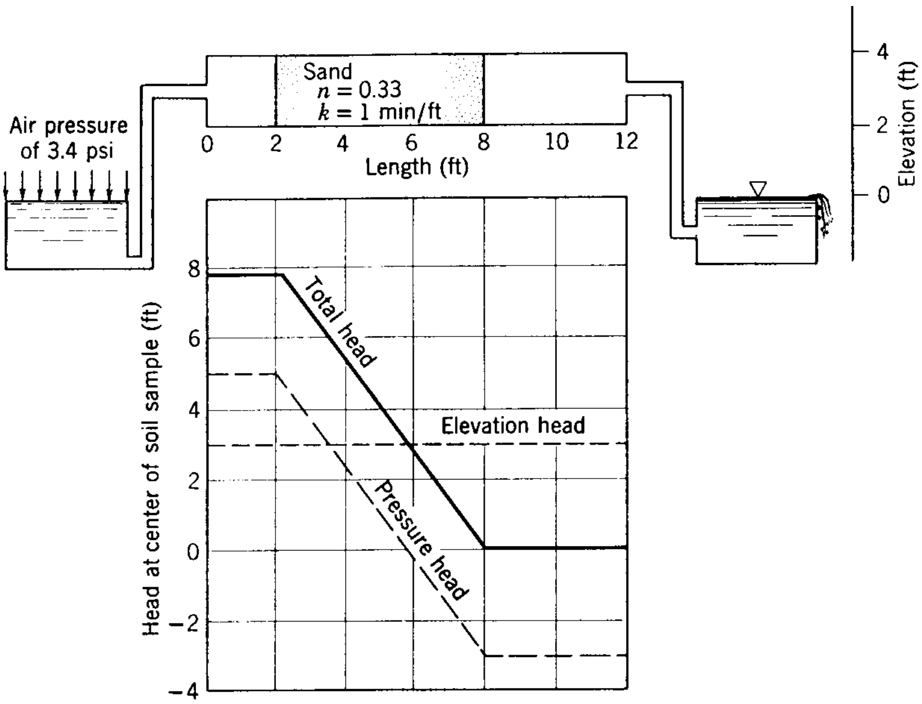
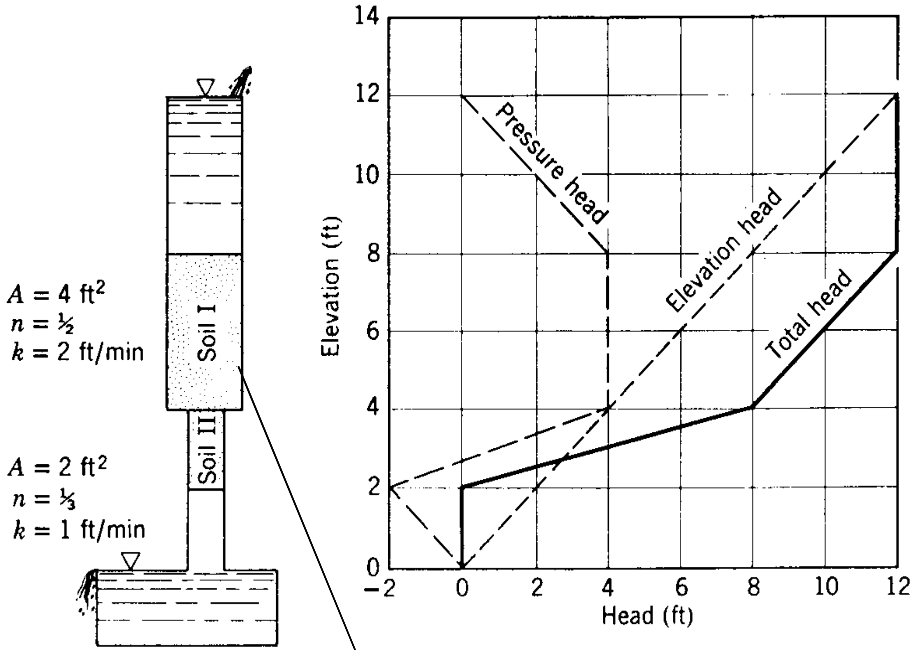


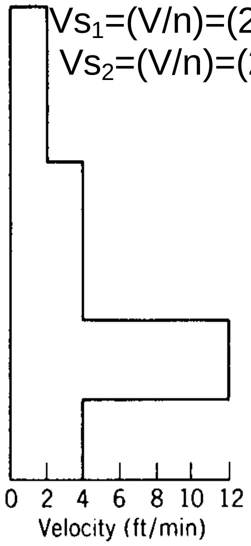
Calculation of Pressure Head in Seepage Conditions (Horizontal flow)



Calculation of Pressure Head in Seepage Conditions (two soils)

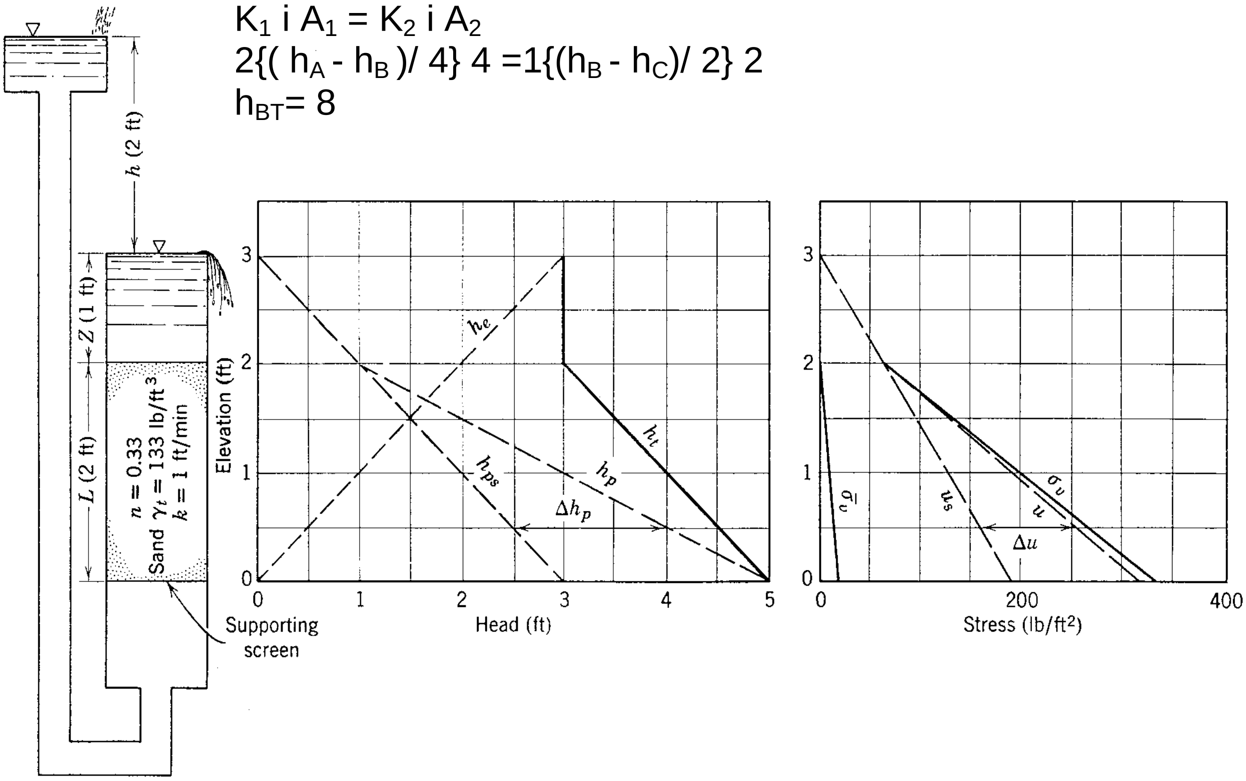


V_1= k i =2{(12-8)/2} = 2
V_2= k i =1{(8-0)/2} = 4
Vs_1=(V/n)=(2/1/2)=4
Vs_2=(V/n)=(2/1/3)=12



Seepage Force : is applied by moving water to the soil Skelton and acts in the flow direction

$Q_1 = Q_2$
K_1 i A_1 = K_2 i A_2
2{(h_A - h_B)/ 4} 4 =1{(h_B - h_C)/ 2} 2
h_{BT}= 8



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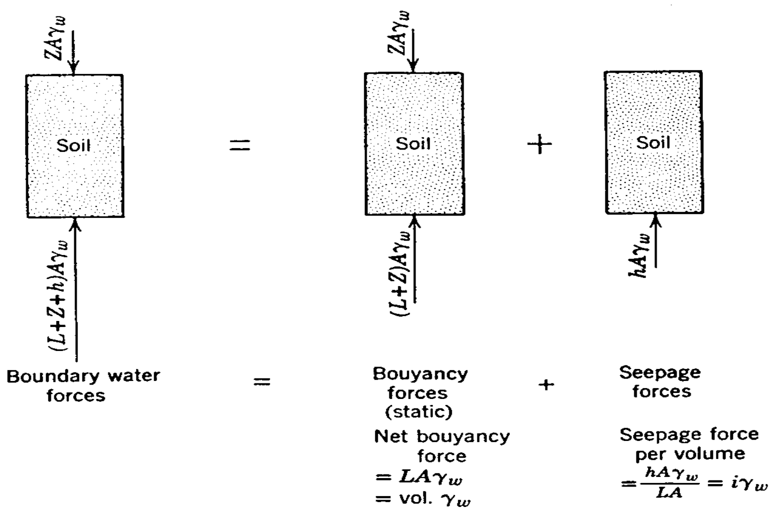


Hence :we can work either with :

- Boundary water forces + Total weights
- OR seepage forces + submerged weights

Seepage force/volume = $i\gamma_w$

$$j = \frac{\text{Seepage Force}}{\text{Vol. of Soil}} = \frac{hAg_w}{LA} = \frac{h}{L}g_w = ig_w$$

Note:

- seepage force exerted by the flowing water acts as an external force on the soil Skelton
- J is a constant value for a given soil mass.

Ex.: For the figure above find J if h=0.6 m and L=0.6 m.

$J = i\, g_w = \frac{h}{L} g_w = \frac{0.6}{0.6} x9.81 = 9.81$ kN/m^3

Force Equilibrium :for the setup shown in page above:

a-Total weight plus boundary water force :

$\sigma_A - u_A = \sigma`_A$

$F = Z\, g_w A + L\, A\, g_t - (h + Z + L)\, g_w A$

$= L\, A(g_t - g_w) - h\, g_w A$

$= L\, A\, g_b - h\, g_w A = A\,(L\, g_b - h\, g_w)$

$F = \sigma` A$

b-Submerged weight +seepage force

$F = L\, A\, g_b - h\, g_w A$

$F = \sigma' A$

Hence :total wt. + boundary water force =submerged weight + seepage force

Quick Condition (Boiling)

$$S = C + \sigma' \tan \phi$$ (Coulomb eq.)

Where S = Shear Strength
C = soil cohesion
Φ = internal friction

For cohesionless soils : C = 0 → $S = \sigma' \tan \phi$ Then S = 0 (boiling) when $\sigma' = 0$

Now $\sigma' = L\,A\,\gamma_b - h\,\gamma_w\,A$ and for $\sigma' = 0 \rightarrow \frac{h}{L} = \frac{\gamma_b}{\gamma_w} = i$

Hence: the gradient required to cause quick condition = critical gradient $= i_c$:

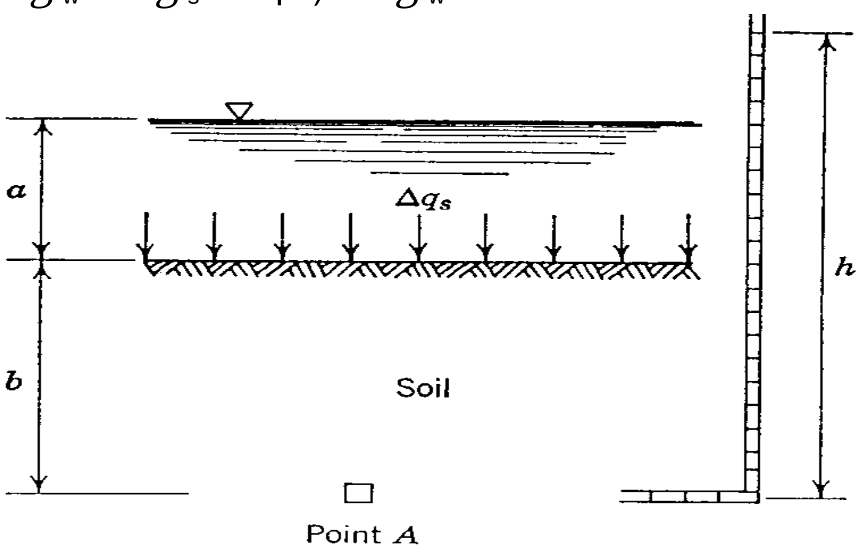
$$i_c = \frac{\gamma_b}{\gamma_w}$$

Show that $i_c = \frac{G_s - 1}{1 + e}$

Note : if γ_b not given ,take $\gamma_b \approx 1$

The effective stress at point A is

$$\sigma_v' = \sigma_v - u$$
$$= (a\,\gamma_w + b\,\gamma_s + \Delta q_s) - h\,\gamma_w$$



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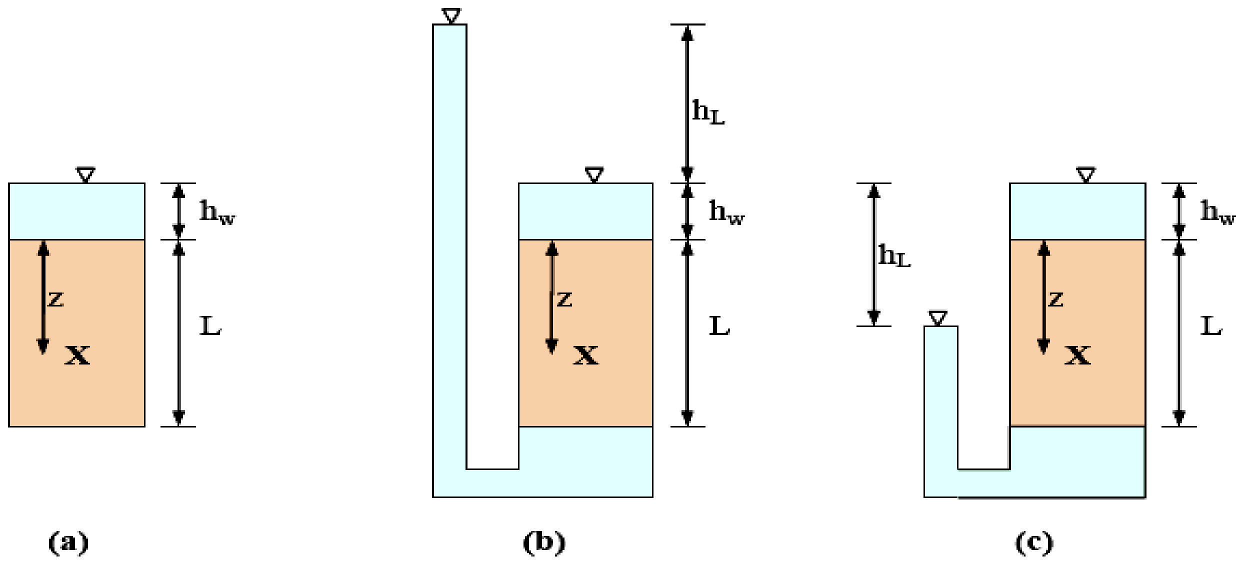
STRESSES IN SOILS DUE TO FLOW



Three different scenarios (a) Static (b) Flow-up (c) Flow-down

For all three situations, the total vertical stress is the same. The pore water pressures and effective stresses are summarized below.

$u = (h_L + h_w + z)\gamma_w$ $u = (+h_w + z - h_L)\gamma_w$

(a) Static situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z)$
$\sigma_v' = \gamma' z$

(b) Flow-Up Situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) + \mathbf{i\, z\, \gamma_w}$
$\sigma_v' = \gamma' z - \mathbf{i\, z\, \gamma_w}$

$i = h_L/z$, $h_L = i\, z$

(c) Flow-Down Situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) - \mathbf{i\, z\, \gamma_w}$
$\sigma_v' = \gamma' z + \mathbf{i\, z\, \gamma_w}$

When the flow is upwards in the soil, pore water pressure increases and effective stress decreases. When the flow is downward, the pore water pressure decreases and the effective stress increases. Higher the hydraulic gradient, higher the increase or decrease in the values of pore pressure and effective stress.

Now let's have a closer look at the flow-up situation, in a *granular soil*. The effective stress is positive as long as $\gamma' z$ is greater than $iz\gamma_w$. If the hydraulic gradient is too large, $iz\gamma_w$ can exceed $\gamma' z$, and the effective vertical stress can become negative. This implies that there is no inter-particle contact stress, and the grains are no longer in contact. When this occurs, the granular soil is said to have reached *quick condition*.

Determination of k in the Laboratory

Permeability in Soils

- Permeability is the measure of the soil's ability to permit water to flow through its pores or voids
- It is one of the most important soil properties of interest to geotechnical engineers

The Constant head test

- The constant head test is used primarily for coarse-grained soils
- This test is based on the assumption of laminar flow where *k* is independent of *i* *(low* values of i)
- ASTM D 2434
- This test applies a constant head of water to each end of a soil in a "permeameter"

Procedure (Constant head)

1. Setup screens on the permeameter
2. Measurements for permeameter, (D), (L), H_1
3. Take 1000 g passing No.4 soil (M_1)
4. Take a sample for M.C.
5. Assemble the permeameter – *make sure seals are air-tight*
6. Fill the mold in several layers and compact it as prescribed.
7. Put top porous stone and measure H_2
8. Weigh remainder of soil (M_2)
9. Complete assembling the permeameter. (keep outlet valve closed)
10. Connect Manometer tubes, but keep the valves closed.
11. Apply vacuum to remove air for 15 minutes (through inlet tube at top)
12. Run the Test (follow instructions in the lab manual) …..
13. Take readings
    - Manometer heads h_1 & h_2
    - Collect water at the outlet, Q ml at time t $\approx$ 60 sec.

- Determine the unit weight
- Calculate the void ratio of the compacted specimen

Calculate *k* as

- $$k = \frac{QL}{Aht}$$
- Calculate $$k_{20^0C} = k_{T^0C} \frac{h_{T^0C}}{h_{20^0C}}$$

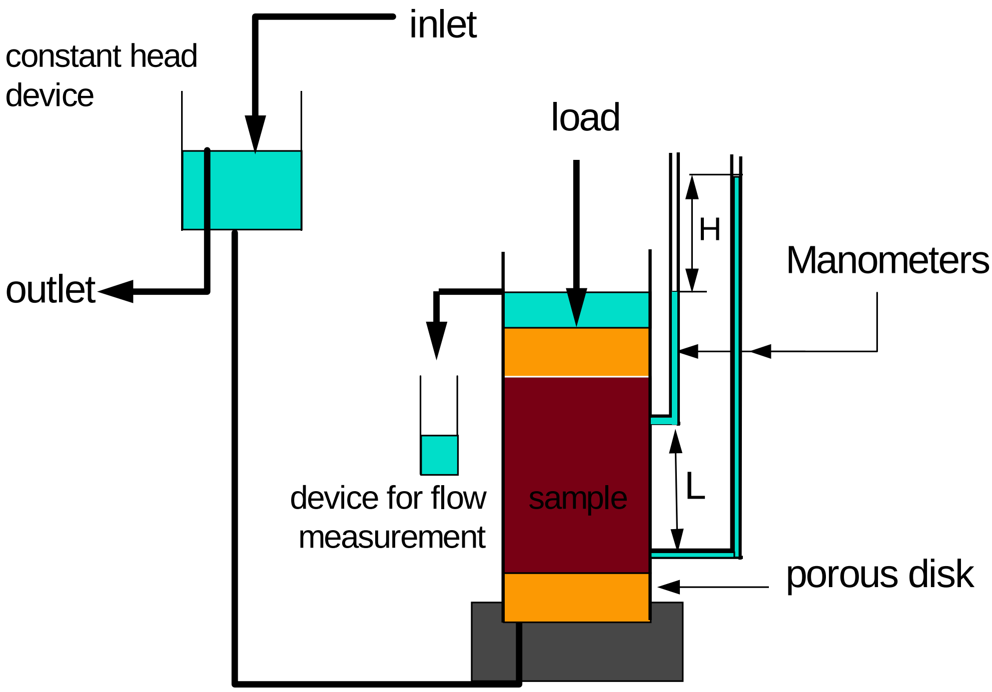


Fig. 4 Constant Head Permeameter

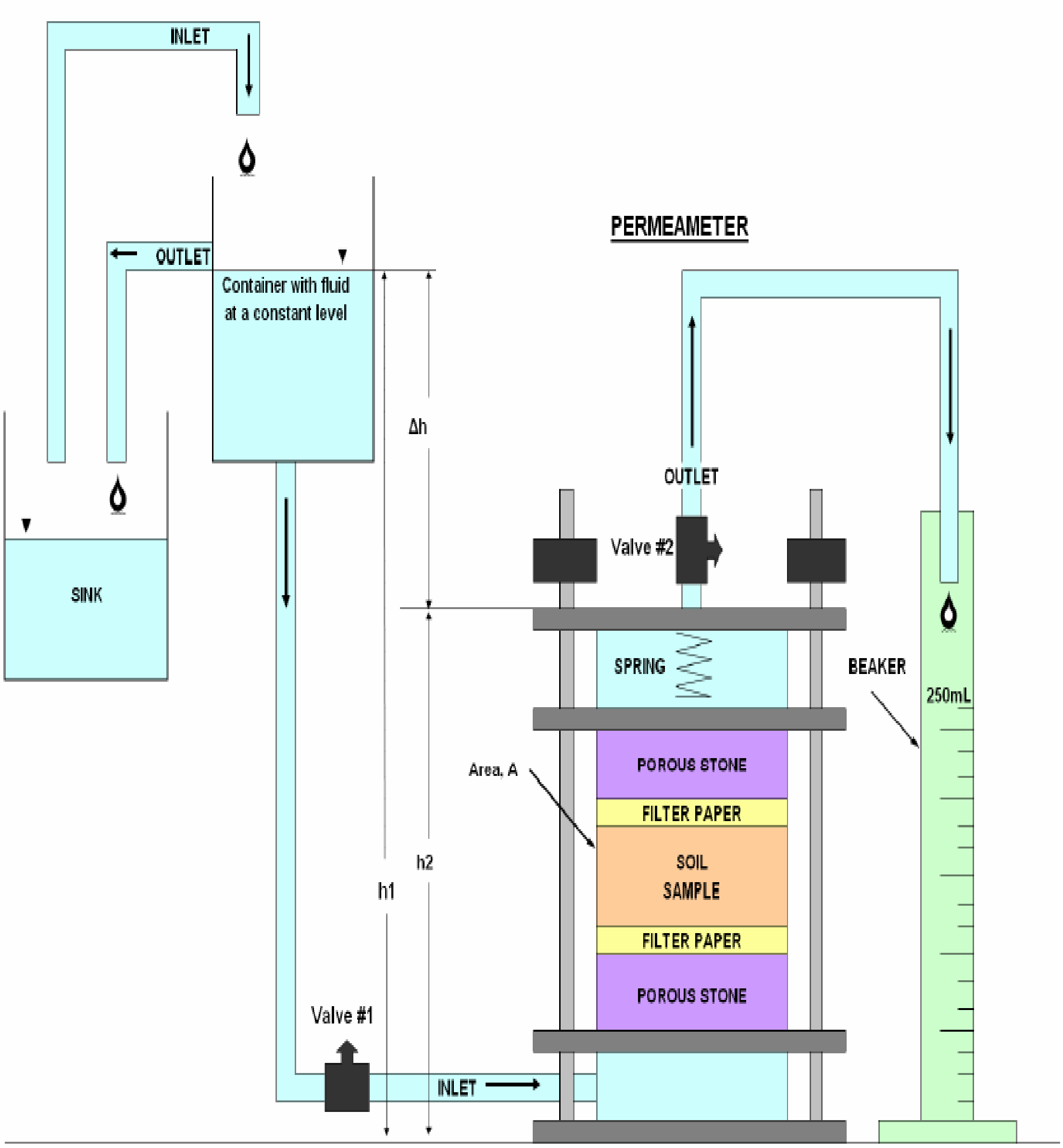
2-The Falling head test

- The falling head test is used both for coarse-grained soils as well as fine-grained soils
- Same procedure in constant head test except:
  - Record initial head difference, h_1 at t = 0
  - Allow water to flow through the soil specimen
  - Record the final head difference, h_2 at time $t = t_2$
  - Collect water at the outlet, Q ml at time t ≈ 60 sec
- Calculate *k* as
- Where;

$$k = \frac{aL}{At} \ln \frac{h_1}{h_2}$$

a = inside cross sectional area of the water tank
h_1 = distance to bottom of the beaker before the test
h_2 = distance to bottom of the beaker after the test

- Calculate $$k_{20^0C} = k_{T^0C} \frac{h_{T^0C}}{h_{20^0C}}$$



Typical permeability ranges

Soils exhibit a very wide range of permabilities and while particle size may vary by about 3-4 orders of magnitude, permeability may vary by about 10 orders of magnitude.

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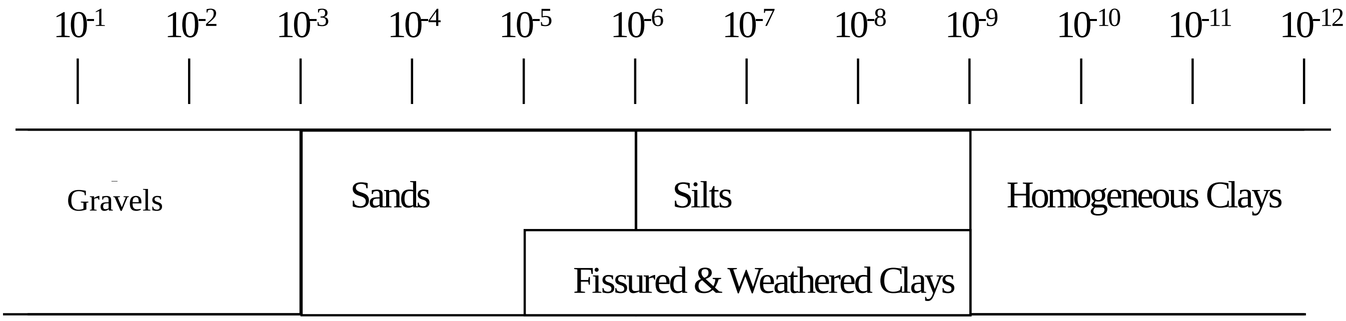
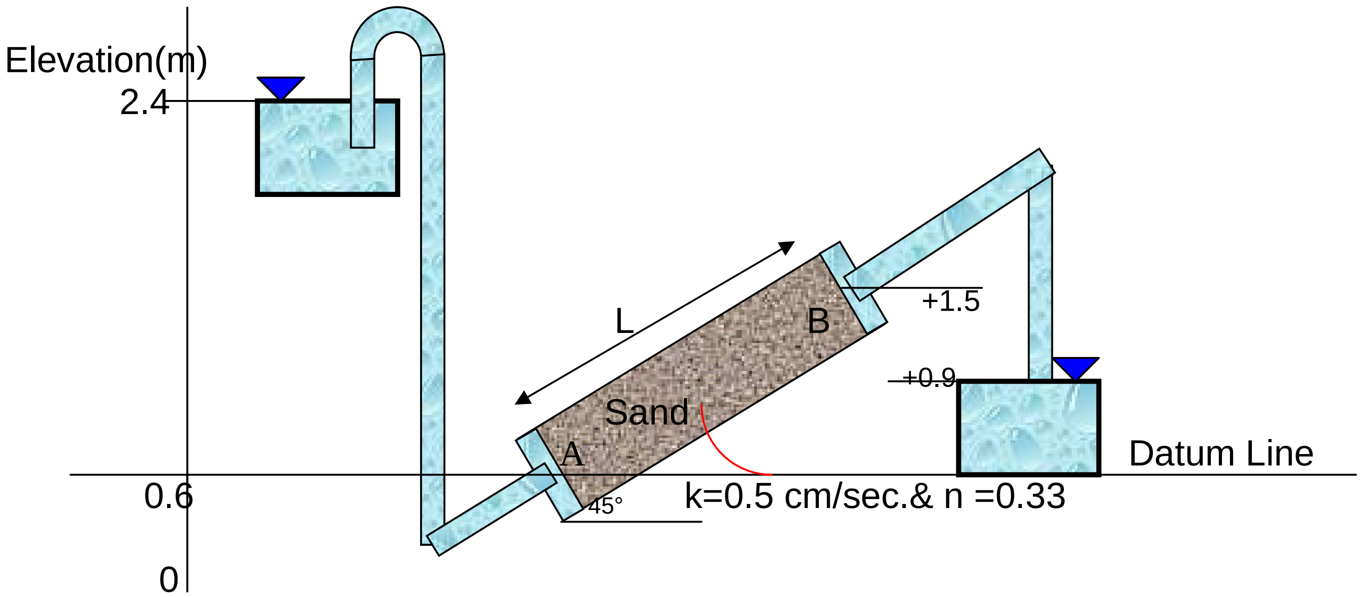


Fig 6 Typical Permeability Ranges

Q. For the setup shown below; plot to scale elevation head, pressure head, total head , and seepage velocity versus distance along the sample axis.



Solution:

$$\sin 45 = \frac{0.9}{L} \Rightarrow L = 1.272\text{m}$$

Point	he	hp	ht	v
A	0	1.8	1.8	0.589
B	0.9	- 0.6	0.3	0.589

$$i = \frac{\Delta h_t}{L} = \frac{1.8-0.3}{1.272} = 1.18$$

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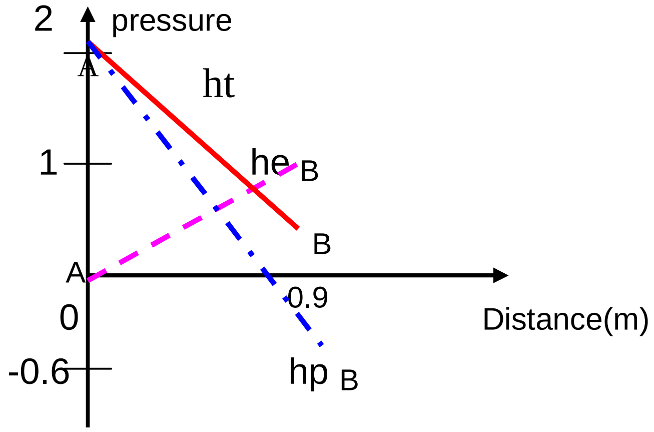
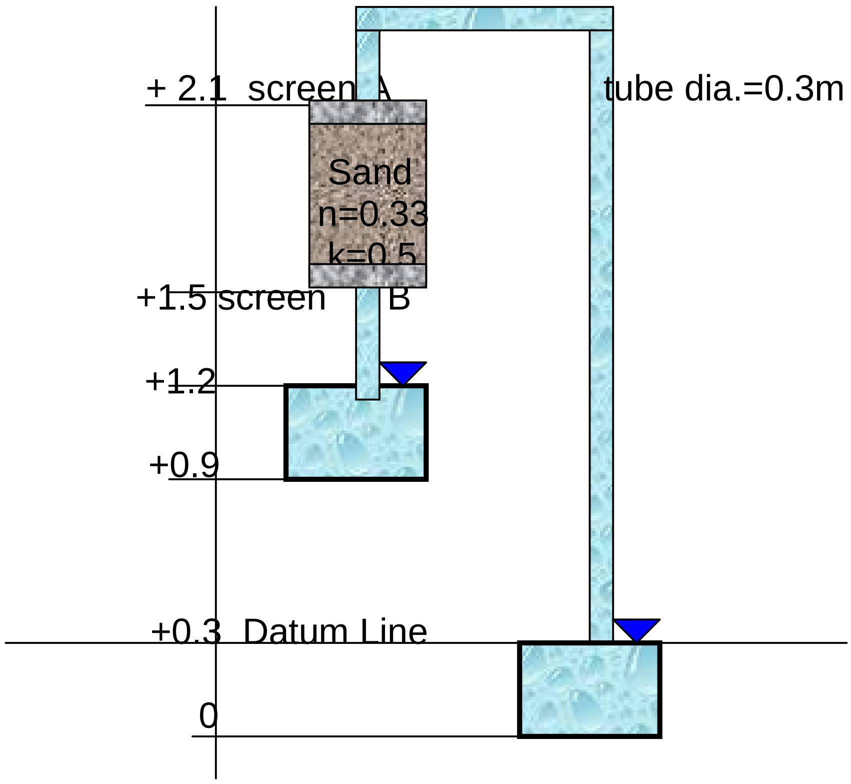
V= ki= 1.18 x 0.5 = 0.589 cm/sec.

$$V_s = \frac{V}{n} = \frac{0.589}{0.33} = 1.77 cm/sec.$$



Q. For the setup shown below; compute the vertical force exerted by the soil on screen A and that on screen B. Neglect friction between the soil and tube. G= 2.75



Solution:

$$Area = \frac{\pi}{4}(0.3)^2 = 0.07 m^2$$

$$e = \frac{n}{1-n} = \frac{0.33}{1-0.33} = 0.5$$

$$\gamma_{sand} = \frac{G+e}{1+e}\gamma_w = \frac{2.75+0.5}{1+0.5}(9.81) = 21.25 kN/m^2$$

Point	he	hp	ht
A	1.8	-1.8	0
B	1.2	- 0.3	0.9

$$i_c = \frac{G-1}{1+e} = \frac{2.75-1}{1+0.5} = 1.1666$$

$$i = \frac{\Delta h_t}{L} = \frac{1.2-0.3}{0.6} = 1.5$$

$$i = \frac{\Delta h_t}{L} = \frac{1.2-0.3}{0.6} = 1.5 > i_c = 1.1666 \Rightarrow \therefore \text{Boiling}$$

∴ the soil stratum don't effect on the screen B.

For Screen A the force effected by the soil on it is Seepage force – weight of the soil

$$\frac{(\text{Seepage Force})}{(\text{Soil Volume})} = i\gamma_w$$

∴

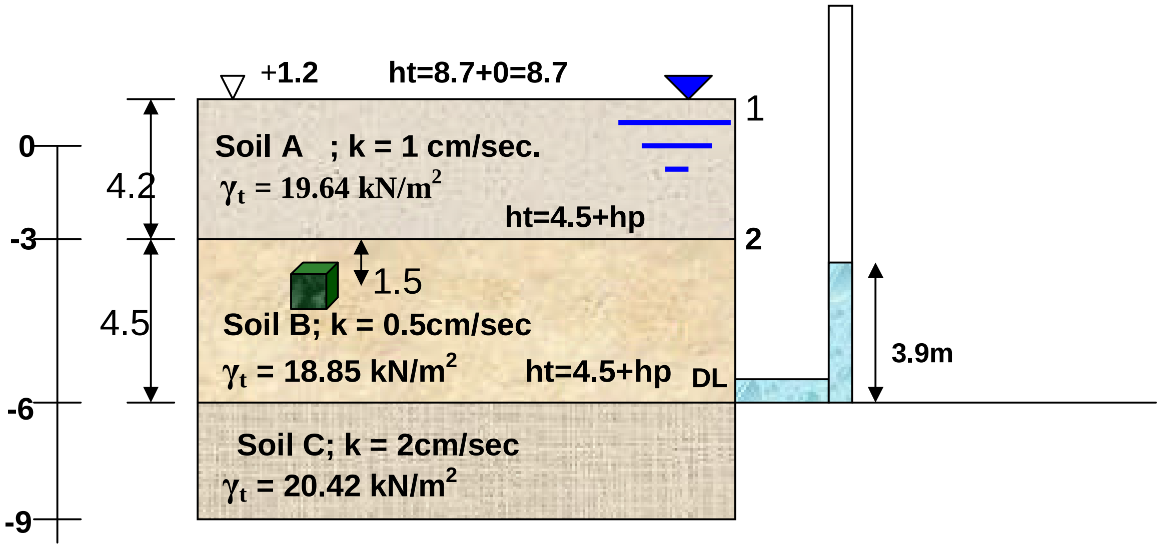
$$(\text{Seepage Force}) = i\gamma_w(\text{Soil Volume}) = \frac{0.9}{0.6}(9.81)\left(0.6 x \frac{\pi}{4}(0.3)^2\right) = 8.829\left(\frac{\pi}{4}(0.3)^2\right)$$

$$(\text{Soil Weight}) = (\gamma_{sat} - \gamma_w)(\text{Soil Volume})$$

$$(\text{Soil Weight}) = (21.25-9.81)\left(0.6 x \frac{\pi}{4}(0.3)^2\right) = 6.864\left(\frac{\pi}{4}(0.3)^2\right)$$

∴ Force on the screen A $= 8.829\left(\frac{\pi}{4}(0.3)^2\right) - 6.864\left(\frac{\pi}{4}(0.3)^2\right) = 0.14137$ kN.

Q. In the profile shown below , steady vertical seepage is occurring. Make a scaled plot of elevation versus pressure head pore pressure, seepage velocity, and vertical effective stress. Determine the seepage force on a 0.3m cube whose center is at elevation -4.5m. G for all soil = 2.75.



Datum line at point 3

point	he	hp	ht	u	σ_v	σ'_v	v
1	8.7	0	8.7	0	0	0	0.375
2	4.5	2.7	7.2*	27	82.48	55.48	0.375
3	0	3.9	3.9	39	167.3	128.3	0.366

$$* \; v = ki = k\frac{\Delta h_t}{L}$$

$$q_A = q_B$$

$$A_A i_A k_A = A_B i_B k_B$$

$$A_A = A_B \Rightarrow i_A k_A = i_B k$$

$$\therefore (1)\left(\frac{8.7-(4.5+hp)}{4.2}\right) = (0.5)\left(\frac{(4.5+hp)-3.9}{4.5}\right) \Rightarrow hp = 2.7m$$

Or

$$\therefore (1)\left(\frac{8.7-h_{t2}}{4.2}\right) = (0.5)\left(\frac{h_{t2}-3.9}{4.5}\right) \Rightarrow h_{t2} = 7.2m$$

$$J = i\gamma_w = \frac{7.2-3.9}{4.5}(1) = 7.2 kN/m^3$$

J for the cube = 7.2 x 0.3 x 0.3 x 0.3 = 0.2 kN

$$v = ki$$

$$v_2 = (1)\frac{8.7-7.2}{4.2} = 0.357 cm/sec.$$

$$v_3 = (0.5)\frac{7.2-3.9}{4.5} = 0.366 cm/sec.$$

Q. for each of cases shown below ; determine the discharge velocity, the seepage velocity, and the seepage force per unit volume for

a. a permeability of 0.1 cm/sec and a porosity of 50%, and

b. a permeability of 0.001 cm/sec and a void ratio of 0.67.

solution:

For a k= 0.1 cm/sec & n=0.5

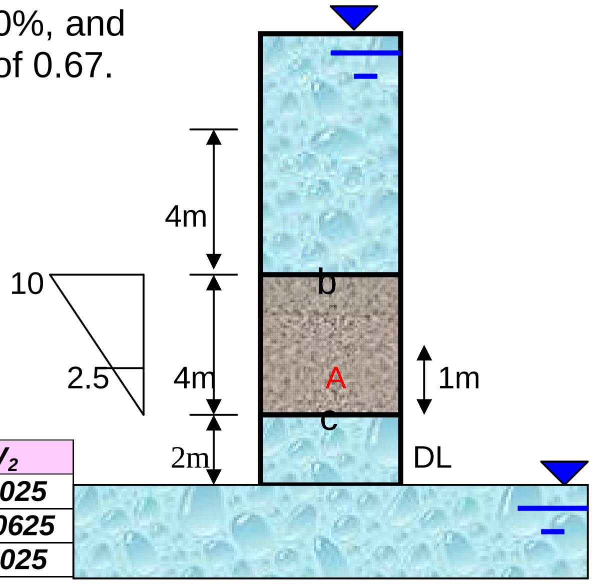
For b k =0.001 cm/sec & $n = \frac{e}{1+e} = \frac{0.67}{1+0.67} = 0.4$

Case 1

Let $\gamma_{sat.}$ = 20 kN/m^2

Let γ_w =10 kN/m^2

c



point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	6	4	10	40	40	0	0.25	0.0025
A	3	-0.5	2.5	-5	100	+105	0.5	0.00625
C	2	-2	0	-20	120	+140	0.25	0.0025

$$i = \frac{\Delta h_t}{L} = \frac{10-0}{4} = \frac{10}{4} = 2.5$$

For K = 0.1cm / sec

V = K x i = 0.1 x 2.5 = 0.25 cm / sec

$$V = \frac{v}{n} = \frac{0.25}{0.5} = 0.5 cm/sec$$

For K = 0.001 cm /sec

V = K x I = 0.001 x 2.5 = 0.0025 cm / sec

$$Vs = \frac{v}{n} = \frac{0.0025}{0.4} = 0.00625 cm/sec$$

Seepage force / unit volume = i x γ_w = 2.5 x 10 = 25 kN/m^3

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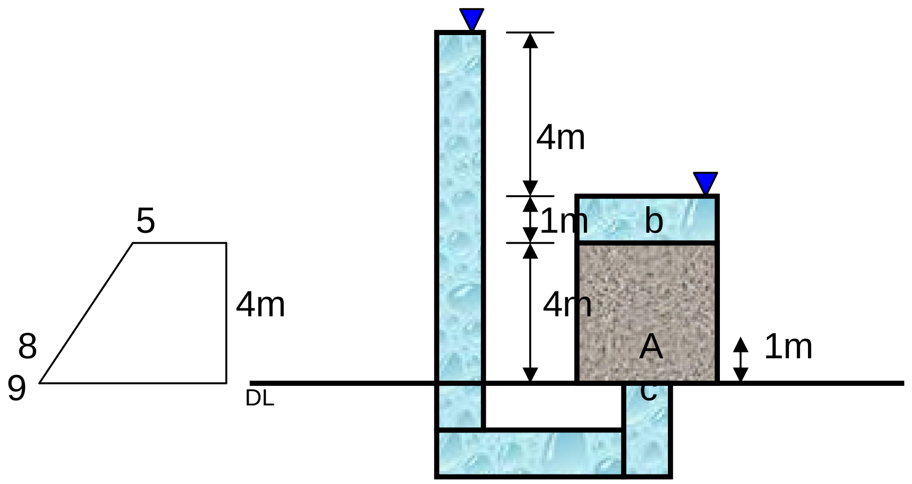
Case 2



point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	4	1	5	10	10	0	0.1	0.001
A	1	7	8	70	70	0	0.2	0.0025
C	0	9	9	90	90	0	0.1	0.001

$$i=\frac{\Delta h_t}{L}=\frac{9-5}{4}=\frac{4}{4}=1 \quad \therefore \text{ Boiling}$$

$$V_1=Ki=0.1(1)=0.1\text{cm/sec}$$

$$Vs=\frac{V}{n}=\frac{0.1}{0.5}0.2\text{cm/sec}$$

$$V_2=K_2(i)=0.001(1)=0.001\text{cm/sec}$$

$$Vs_2=\frac{0.001}{0.4}=0.0025\text{cm/sec}$$

Seepage force / unit volume = I x γ_w = 1 x10 = 10 kN/m^3

Case 3

point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	0	9	9	90	90	0	0.1	0.001
A	0	6	6	60	60	0	0.2	0.0025
c	0	5	5	50	50	0	0.1	0.001

$$i=\frac{\Delta h_t}{L}=\frac{9-5}{4}=\frac{4}{4}=1 \quad \therefore \text{ Boiling}$$

$$V_1=Ki=0.1(1)=0.1\text{cm/sec}$$

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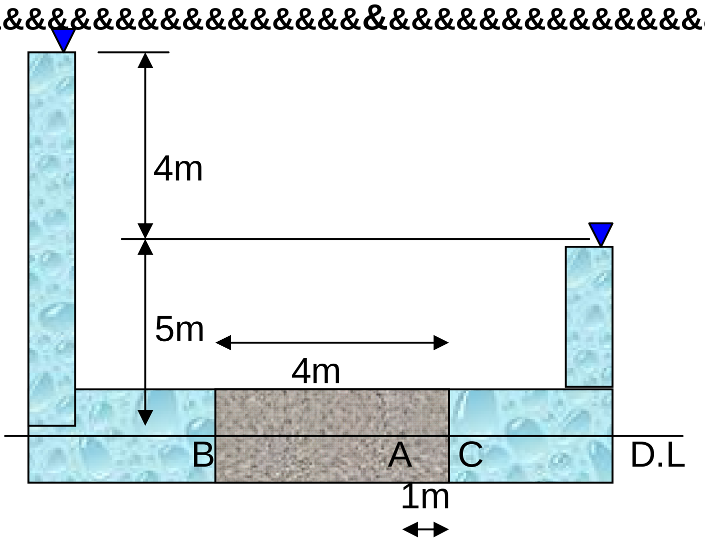
$$Vs_1 = \frac{V_1}{n} = \frac{0.1}{0.5} = 0.2 cm/sec$$

$$V_2 = K_2(i) = 0.001(1) = 0.001 cm/sec$$

$$Vs_2 = \frac{V_2}{n} = \frac{0.001}{0.4} = 0.0025 cm/sec$$



Q. For the soil profile shown ; find (H) that make the soil in Boiling condition

$\sigma'_v = \sigma_v - 4$

= (3 x 18 + 3 x 20 + 6 x 18) – 16 x 10 = 62 kN/m^2

$\sigma_v = 4$

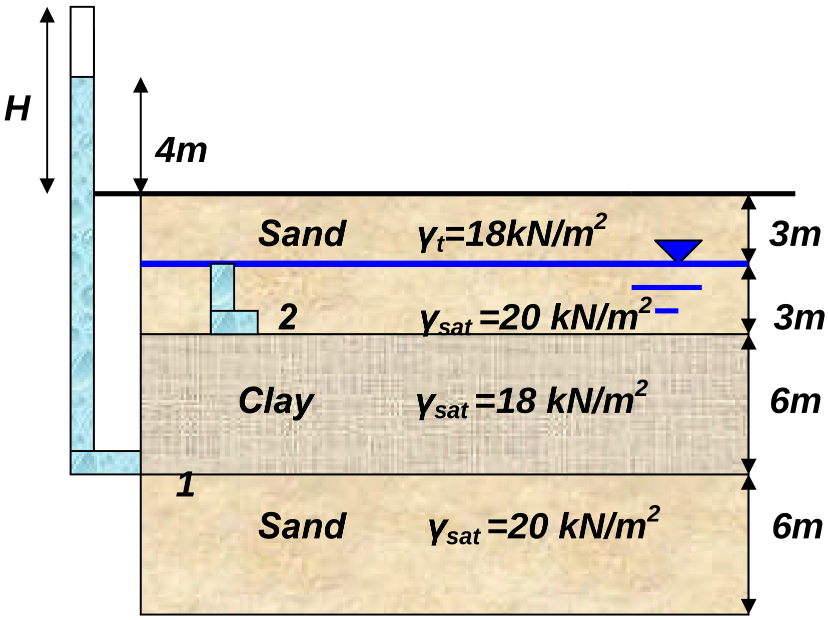
222 = (6 + 6 + H) x 10

\ H = 10.2 m

Or

$$i_c = \frac{g_{sat} - g_w}{g_w} = \frac{18-10}{10} = 0.8 \Rightarrow i_c = \frac{\Delta h_t}{L} = \frac{12+H-9}{6}$$

\ H = 1.8m



Example**1.** A sample of sand was tested in a constant head permeameter. The results were:

Diameter of sample = 100mm

Length between manometer tappings = 120mm

Head difference measured by manometer = 80mm

Quantity of water passing through sample in 10 minutes = 150ml

Determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 100^2}{4} = 7.85 \text{ x } 10^3 \text{ mm}^2$$

$$Q = 150 \text{ ml} = 150 \text{ cc} = 150 \text{ x } 10^3 \text{ mm}^3$$

$$k = \frac{Ql}{At\Delta h} = \frac{150 \times 10^3 \times 120}{7.85 \times 10^3 \times (10 \times 60) \times 80} = \underline{\underline{4.78 \text{ x } 10^{-2} \text{ mm/s}}}$$

Example**2.** A 100mm diameter sample of fine sand was tested in a falling head permeameter. The length of the sample was 150mm. Water in the standpipe fell from 1000 to 400mm in 44 seconds. If the diameter of the standpipe was 10mm, determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 100^2}{4} = 7.85 \text{ x } 10^3 \text{ mm}^2$$

$$a = \frac{\pi d^2}{4} = \frac{\pi \times 10^2}{4} = 78.5 \text{ mm}^2$$

$$k = \frac{al}{At}\ln\left(\frac{h_1}{h_2}\right) = \frac{78.5 \times 150}{7.85 \times 10^3 \times 44}\ln\left(\frac{1000}{400}\right) = 0.0312 \text{ mm/s} = \underline{\underline{3.12 \text{ x } 10^{-2} \text{ mm/s}}}$$

Example**3.** A sample of coarse sand, 55mm in diameter, was tested in a constant head permeameter. Water percolated through the soil and a head loss of 100mm was recorded over a length of sample of 150mm. The discharge water, collected after 6.0 seconds had a mass of 400g.

Determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 55^2}{4} = 2375.8 \text{ mm}^2$$

$$k = \frac{Ql}{At\Delta h} = \frac{400 \times 10^3 \times 150}{2375.8 \times 6 \times 100} = \underline{\underline{42 \text{ mm/s}}}$$

N.B. *400g water has volume 400 ml*

Example**4.** A falling head permeability test is to be performed on a soil whose permeability is estimated at 3.0 x 10-3 mm/s. What **diameter of standpipe** should be used if the head is to drop from 275mm to 200mm in 5 minutes?

Assume that the area of the sample is 1500mm and its length is 85mm.

$$k = \frac{al}{At}\ln\left(\frac{h_1}{h_2}\right)$$

$$3\times10^{-3} = \frac{a\times 85\times \ln\left(275/200\right)}{1500\times(5\times60)}$$

$$\Rightarrow \underline{a = 49.87\ \text{mm}^2}$$

$$\text{Now,}\quad a = \frac{\pi d^2}{4}$$

$$\Rightarrow \qquad d^2 = \frac{a\times 4}{\pi} = \frac{49.87\times 4}{\pi} = 63.5\ \text{mm}^2$$

$$\Rightarrow d = \sqrt{63.5} = 7.97\ \text{mm},\ \underline{\underline{\text{say } 8\ \text{mm}}}$$

Example**5.** A pumping out test was carried out on a soil stratum which extended to a depth of 20m where an impermeable layer was encountered. Ground water level originally occurred at 0.5m below the ground level. Observation wells were placed at 5m and 10m from the pumping well.

During steady pumping conditions water was discharged at the rate of 250 kg/minute and the drawdowns in the two wells were 1.5 and 0.2m

Determine the **coefficient of permeability** of the soil in metres/hour.

$z_2 = 19.5 - 0.2 = 19.3m$

$z_1 = 19.5 - 1.5 = 18.0m$

$q = 250\ kg/min \equiv 0.25\ m^3/min = 0.25 \times 60\ m^3/hr = 15\ m^3/hr$

$$k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{\pi\left(z_2^2 - z_1^2\right)} = \frac{15 \times \ln\left(\frac{10}{5}\right)}{\pi\left(19.3^2 - 18^2\right)} = \underline{\underline{68.3 \times 10^{-3}\ m/hr}}$$

Tutorial

1. If the flow out is 200 mm^3/sec in figure 1, find K_1 and K_2
2. In figure 2, when $\frac{dh}{dt} = 0.01\ mm/\sec$,
   1) Derive the equation giving K
   2) Calculate K

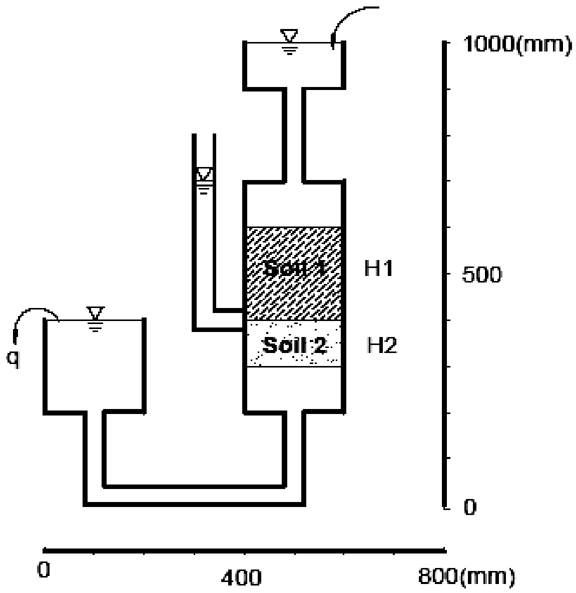


Figure 1.

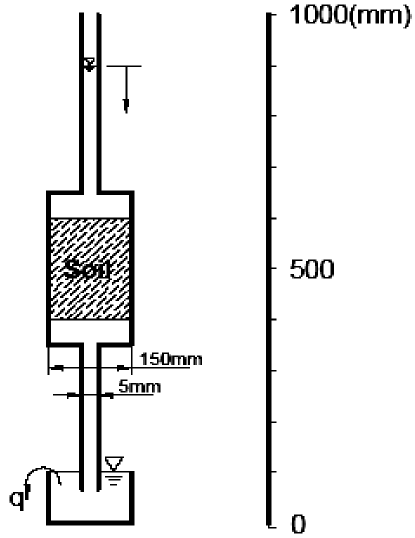
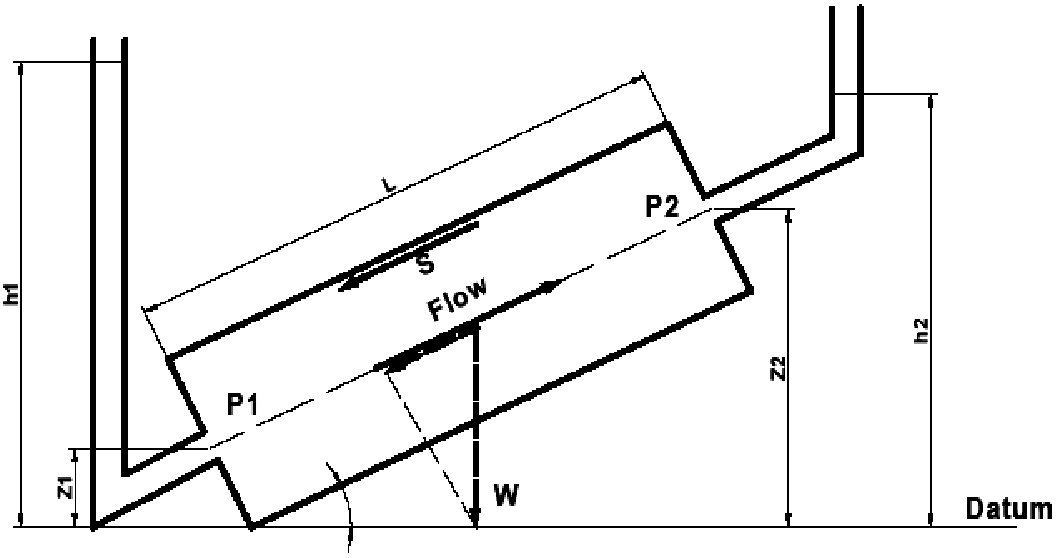
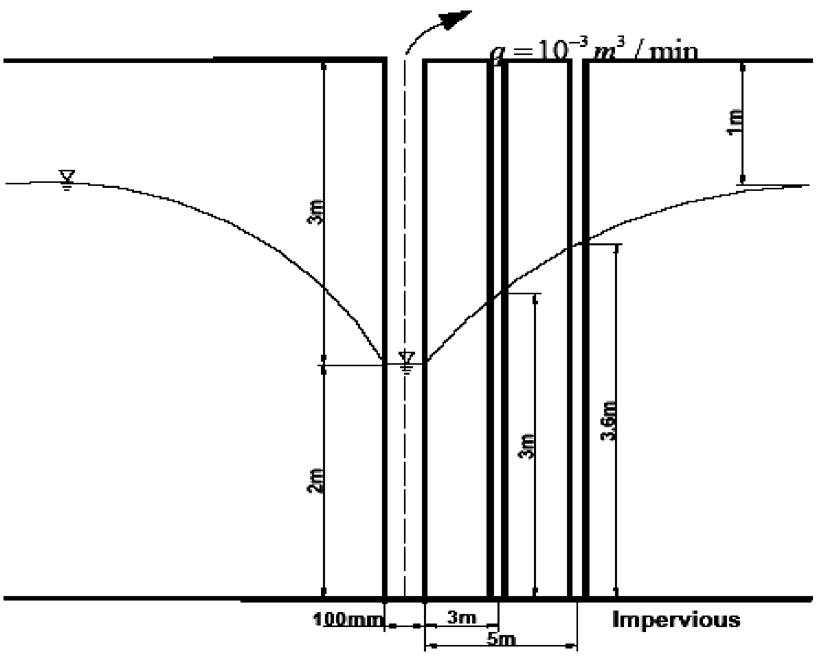


Figure 2.

3. Develop the equation for the seepage force per unit soil volume in the Figure below (Hint : Do a free body diagram of the water in the tube)



4. Referring to the Figure below, derive the equation linking q to K and th dimensions involved and calculate K for the numbers given.



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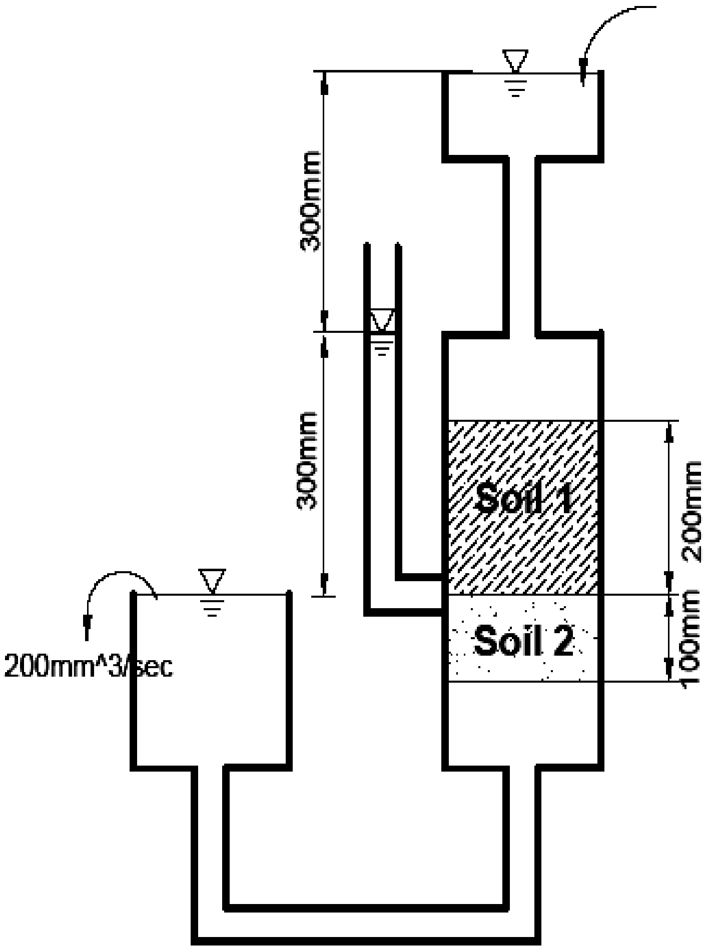
Problem 1.



$$Q_{Soil\,1} = Q_{Soil\,2}$$

$$Q = A \times V \qquad A_{Soil\,1} = A_{Soil\,2}$$

$$\therefore V_{Soil\,1} = V_{Soil\,2}$$

$$V = \frac{Q}{A} = \frac{200}{\frac{\pi}{4} \times 200^2}$$

$$= 6.37 \times 10^{-3}\, mm/\sec$$

1) Calculation K_1

$$V_{Soil\,1} = K_{Soil\,1} i = K_{Soil\,1} \times \frac{\Delta h_1}{\ell_1}$$

$$\therefore K_{Soil\,1} = V_{Soil\,1} \times \frac{\ell_1}{\Delta h_1}$$

$$= 6.37 \times 10^{-3} \times \frac{200}{300}$$

$$K_{Soil\,1} = 4.24 \times 10^{-3}\, mm/\sec$$

$$or \;\; 4.24 \times 10^{-6}\, m/\sec$$

2) Calculation K_2

$$V_{Soil\,2} = K_{Soil\,2} i = K_{Soil\,2} \times \frac{\Delta h_2}{\ell_2}$$

$$\therefore K_{Soil\,2} = V_{Soil\,2} \times \frac{\ell_2}{\Delta h_2}$$

$$= 6.37 \times 10^{-3} \times \frac{100}{300}$$

$$K_{Soil\,2} = 2.12 \times 10^{-3}\, mm/\sec \quad or \; 2.12 \times 10^{-6}\, m/\sec$$

Problem 2.

1) Derivation

$$q = -K\frac{h}{L}A = a\frac{dh}{dt}$$

$$dt = -\frac{aL}{KA}\frac{dh}{h}$$

$$\int_0^t dt = -\frac{aL}{KA}\int_0^t \frac{1}{h}dh$$

$$t = -\frac{aL}{KA}\ln(h)\Big]_0^t$$

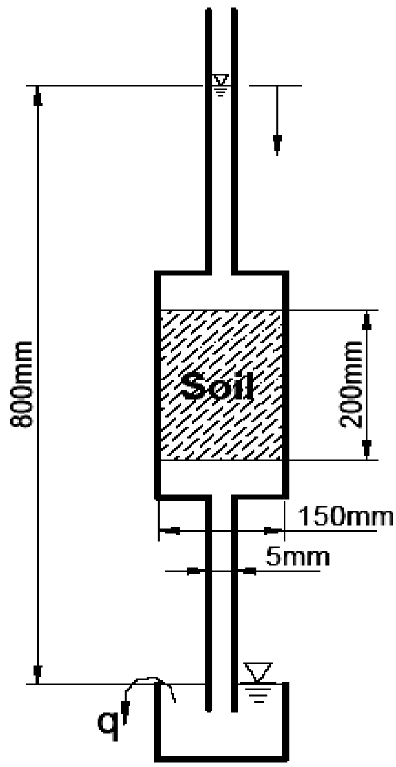
Boundary conditions

$t = 0 \quad h = h_1 \; , t = t \quad h = h_2$

$$t = -\frac{aL}{KA}\left[\ln(h_2) - \ln(h_1)\right]$$

$$= \frac{aL}{KA}\ln\left(\frac{h_1}{h_2}\right)$$

$$\therefore\; K = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$



2) Calculate K

Solution 1)

$$K = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

If $t = 100\sec, \quad dh = 0.01\times100 = 1mm$

$\therefore\; h_1 = 800mm,\; h_2 = 799mm$

$$K = \frac{\frac{\pi}{4}\times5^2\times200}{\frac{\pi}{4}\times150^2\times100}\ln\left(\frac{800}{799}\right)$$

$$= 2.78\times10^{-6}mm/\sec \quad or \quad 2.78\times10^{-9}m/\sec$$

Solution 2)

$$Q_{tube} = Q_{Soil}$$

$$Q \; in \; d = 5mm \; tube, Q = AV = \frac{\pi}{4}\times5^2\times0.01 = 0.196mm^3/\sec$$

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$Q \ in \ d = 150mm \ tube$

$$Q = \frac{\pi}{4} \times 150^2 \times V = 0.196 mm^3/\sec$$

$$V = 1.11 \times 10^{-5} mm/\sec$$

$$= Ki = K\frac{\Delta h}{l} = K\frac{800}{200}$$

$$\therefore K = 2.78 \times 10^{-6} mm/\sec, \ or \ 2.78 \times 10^{-9} m/\sec$$

Problem 3.

$$P_1 A = P_2 A + S + \gamma_w A L \sin\theta$$

$$\gamma_w (h_1 - z_1) A = \gamma_w (h_2 - z_2) A + S + \gamma_w A L \frac{z_2 - z_1}{L}$$

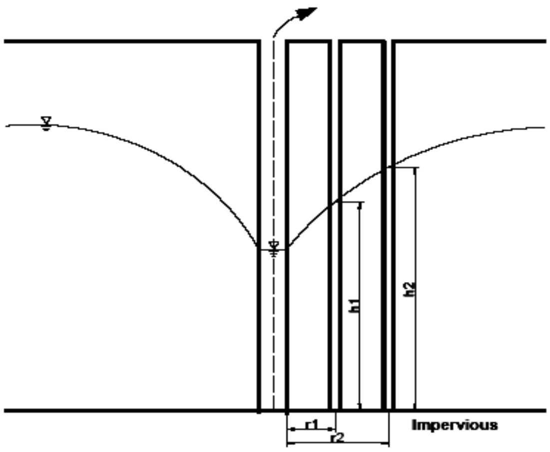
$$h_1 - z_1 = h_2 - z_1 + \frac{S}{\gamma_w A} + z_2 - z_1$$

$$\frac{h_1 - h_2}{L} = \frac{S}{\gamma_w A L} = i$$

The seepage force $= \gamma_w i = \dfrac{S}{AL}$ (per unit soil volume)

Problem 4.

1) Derivation



$$q = K\left(\frac{dh}{dr}\right) \times 2\pi r h$$

$$\frac{dr}{r} = \frac{2\pi K}{q} h dh$$

$$\int_{r_1}^{r_2} \frac{1}{r} dr = \frac{2\pi K}{q} \int_{h_1}^{h_2} h dh$$

$$\ln R\Big]_{r_1}^{r_2} = \frac{2\pi K}{q} \times \frac{1}{2} H^2 \Big]_{h_1}^{h_2}$$

$$\ln\left(\frac{r_2}{r_1}\right) = \frac{\pi K}{q}(h_2^2 - h_1^2)$$

$$\therefore K = \frac{q}{\pi(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right)$$

2) Calculation

$$K = \frac{q}{\pi(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right) = \frac{10^{-3}/60}{\pi(3.6^2 - 3^2)} \ln\left(\frac{5.05}{3.05}\right)$$

$$= 6.76 \times 10^{-7} m/\sec$$