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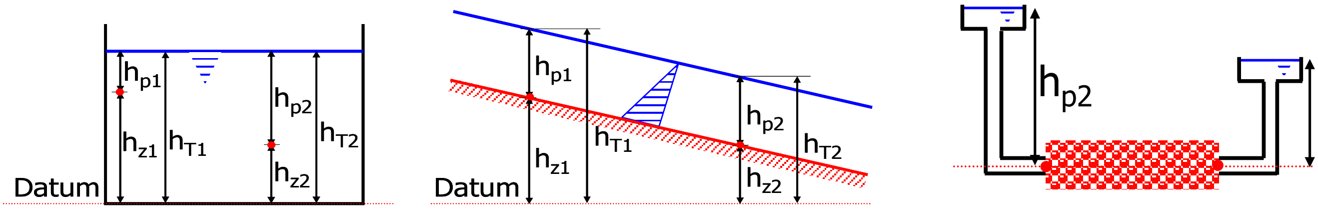
Introduction

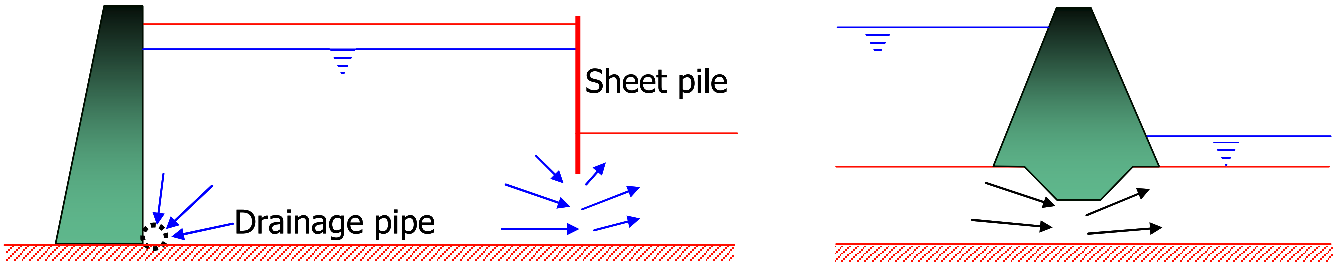
Swimming pool — Open Channel



Objectives

- To obtain pore pressure (stability analysis)
- To calculate flow
- To verify piping conditions

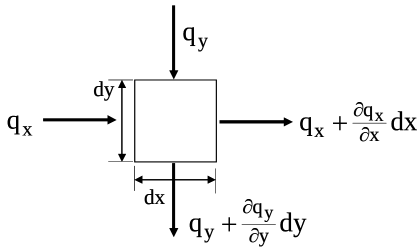
Retaining wall — Cofferdam — Hydraulic dam



Laplace's Equation

Elemental Cube:

- Saturation S=100 %
- Void ratio e= constant
- Laminar flow



$$q_{in} = q_{out}$$

Continuity:

$$q_x + q_y - \left(q_x + \frac{\partial q_x}{\partial x} dx + q_y + \frac{\partial q_y}{\partial y} dy \right) = 0$$

$$\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$$

$$\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$$

- Darcy's law:

$$q_x = k_x . i . A = k_x . \frac{\partial h_T}{\partial x} . dy . 1$$

- Replacing:

$$0 = k_x . \frac{\partial^2 h_T}{\partial x^2} dx . dy . 1 + k_y . \frac{\partial^2 h_T}{\partial y^2} dy . dx . 1$$

$$0 = k_x . \frac{\partial^2 h_T}{\partial x^2} + k_y . \frac{\partial^2 h_T}{\partial y^2}$$

- if $k_x = k_y$

$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$$

Laplace's Equation! (Isotropy):

Typical cases

1 Dimensional:

$$0 = \frac{\partial^2 h_T}{\partial x^2} \quad , \qquad \text{constant} = \frac{\partial h_T}{\partial x} = i$$

linear variation!!

$$h_T = a + b . x$$

2-Dimensional:

$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$$

3-Dimensional:

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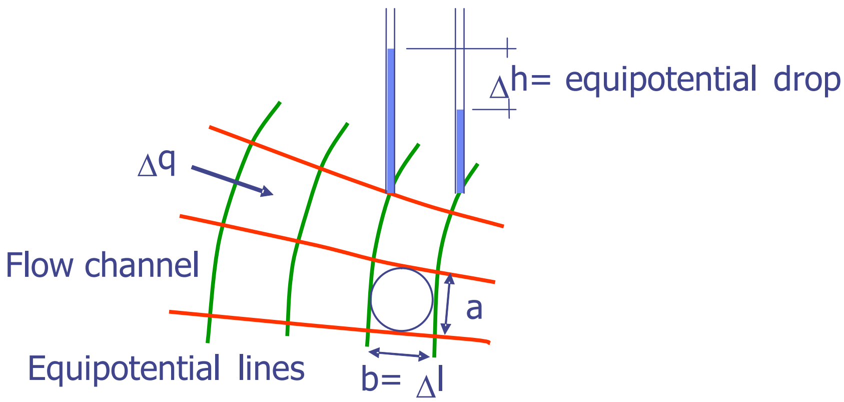
$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} + \frac{\partial^2 h_T}{\partial z^2}$$

Laplace's Equation Solutions

- Exact solutions (for simple B.C.'s)
- Physical models (scaling problems)
- Approximate solutions: method of fragments
- Graphical solutions: flow nets
- Analogies: heat flow and electrical flow
- Numerical solutions: finite differences

Flow Nets

- The procedure consists on drawing a set of perpendicular lines: equi-potentials and flow lines.
- These set of lines are the solution to the Laplace's equation.
- It is an iterative (and tedious!) process.
- Identify boundaries:
  - First and last equi-potentials
  - First and last flow lines



- gradient:

$$i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b} = \frac{h/N_e}{b}$$

- flow per channel:

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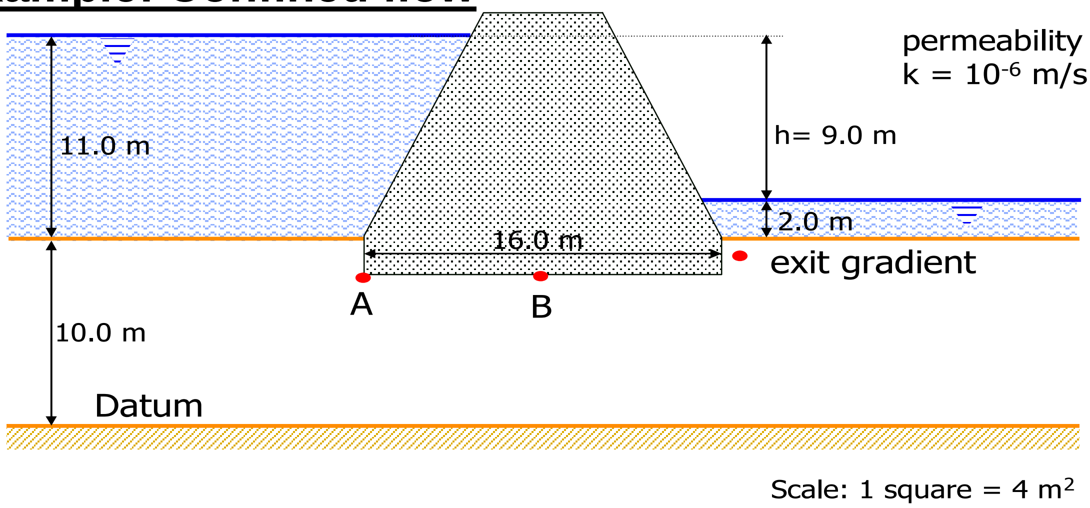
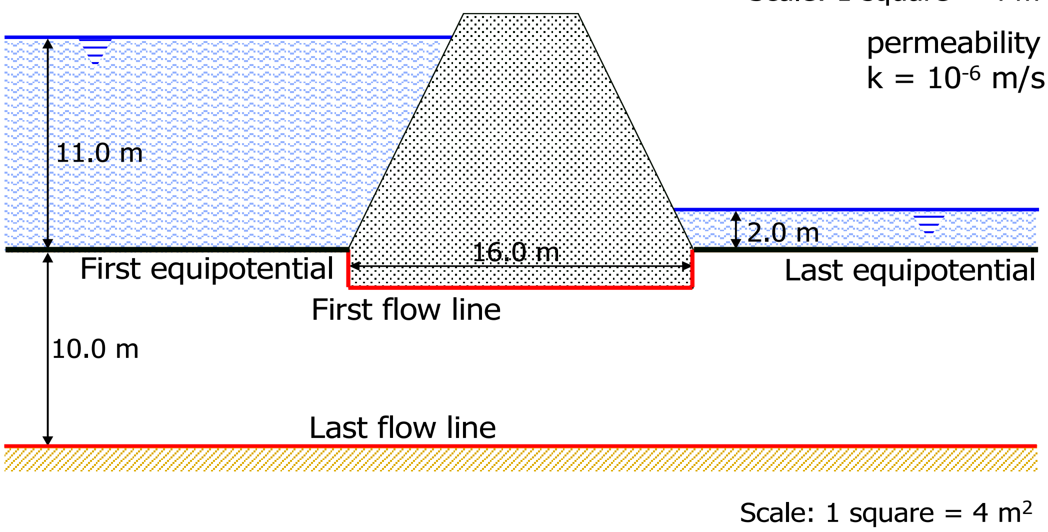
$$\Delta q = k \cdot \frac{\Delta h}{\Delta l} \cdot A = k \cdot \frac{h/N_e}{b} \cdot A$$

- total flow:

$$q = \Delta q \cdot N_f = k \cdot h \cdot \frac{a}{b} \cdot \frac{N_f}{N_e}$$

Example: Confined flow





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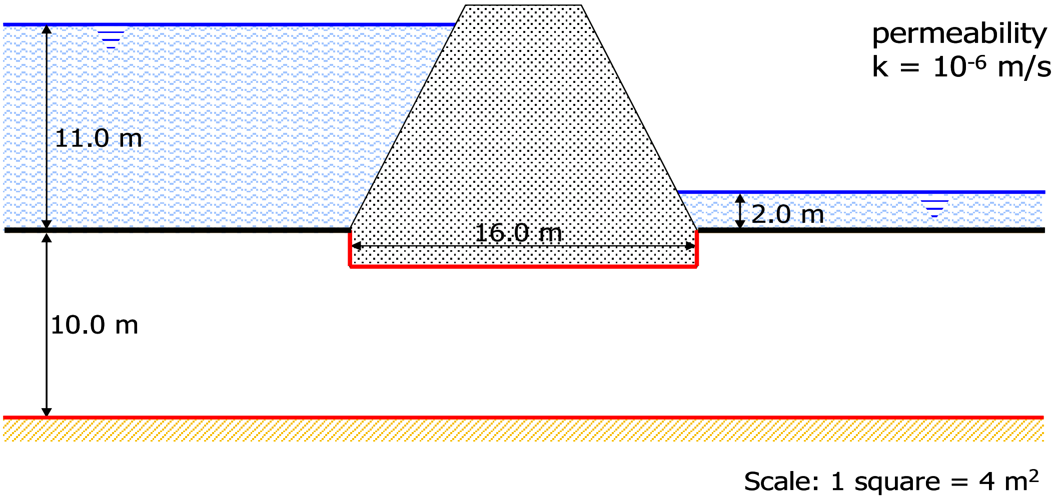
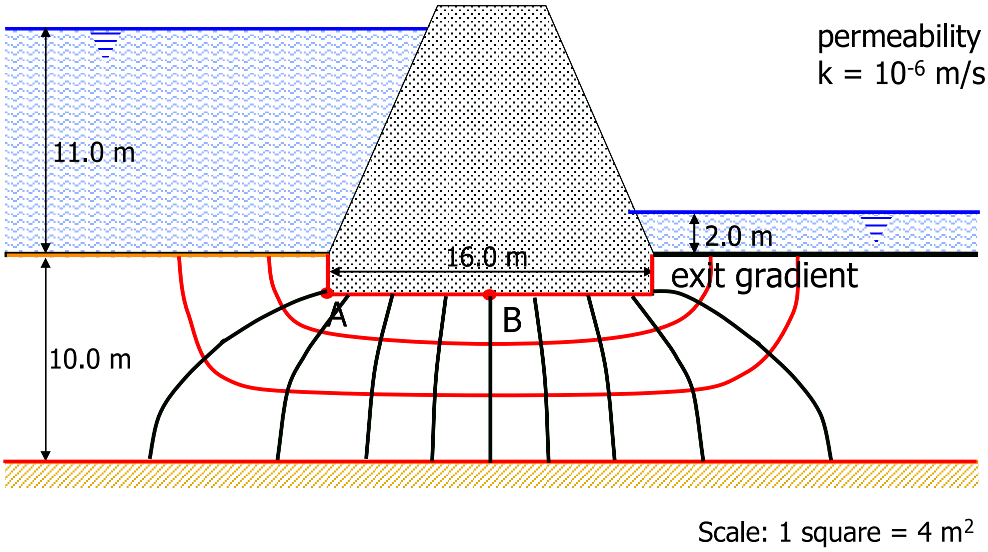




Seepage loss under the dam:

$$q = k \cdot \Delta h \cdot \left(\frac{N_f}{N_e} \right) = 10^{-6}\,\frac{m}{s} \cdot 9m \cdot \frac{3}{10} = 2 \cdot 10^{-4}\,\frac{m^3}{s \cdot m}$$

Exit gradient:

$$i_e = \frac{\Delta h}{\Delta l} = \frac{h/N_e}{\Delta l} = \frac{0.9\ m}{2\ m} = 0.45$$

$$i_{crit} = 1 \therefore \Delta h \cdot \gamma_w = \Delta l \cdot (\gamma_{sat} - \gamma_w) \Rightarrow \text{piping !!}$$

Total head at points A and B:

$$h_{TA} = h_{To} - h \cdot \frac{N_A}{N_f} = 21\,m - 9\,m \cdot \frac{1}{10} = 20.1\,m$$

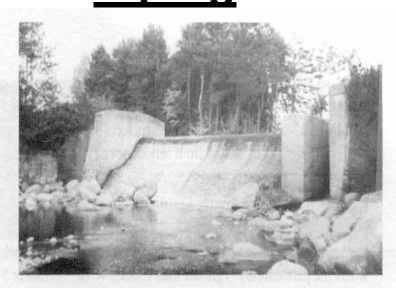
$$h_{TB} = h_{To} - h \cdot \frac{N_B}{N_f} = 21\,m - 9\,m \cdot \frac{5}{10} = 16.5\,m$$

Pressure head at points A and B:

$$h_{PA} = h_{TA} - h_{zA} = 20.1\,m - 8\,m = 12.1\,m \Rightarrow\approx 121\ kPa$$

$$h_{PB} = h_{TB} - h_{zB} = 16.5\,m - 8\,m = 8.5\,m \Rightarrow\approx 85\,kPa$$

◈ **Piping**



Example: Unconfined flow

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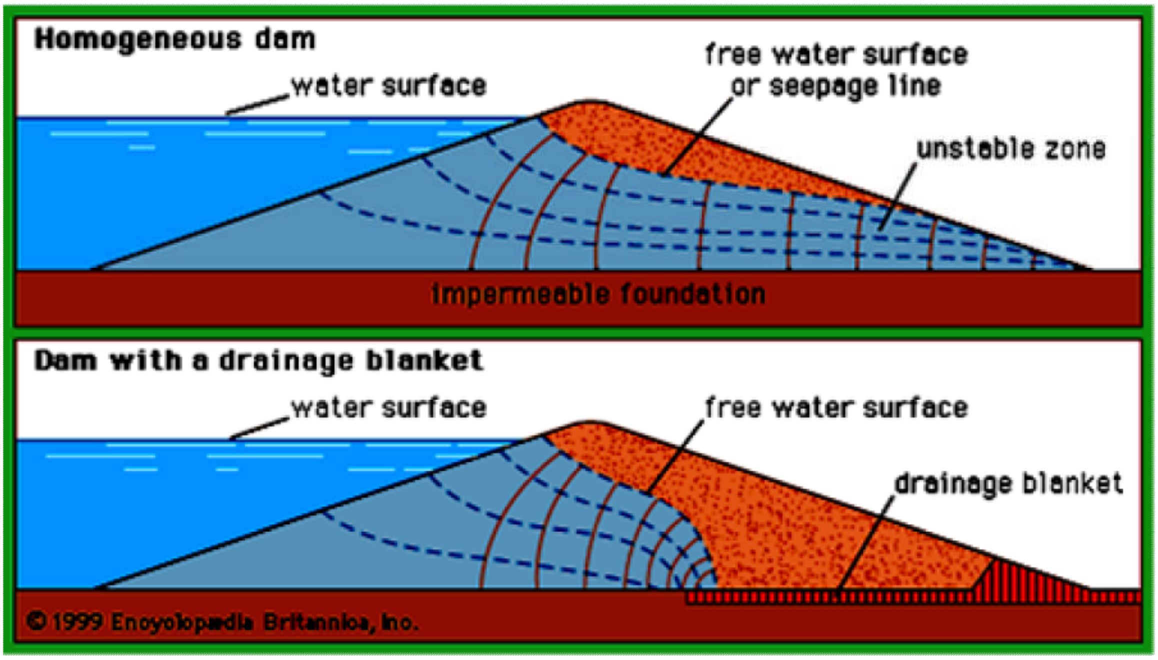

Seepage Control - Filters

- Seepage: Cut-off walls
  Impervious blankets
- Erosion and piping: Filters

Requirements

Piping: D15 (filter) =< 5 D85 (soil)

Permeability: D15 (filter) >= 5 D15 (soil)

Uniformity: D50 (filter) =< 25 D50 (soil)

Example**.** Impervious concrete dam with cut-off.

Sketch a flow net for the situation shown below, and hence calculate the seepage quantity per unit width of dam per day. The permeability of the soil, k = 10^{-3} mm/sec.

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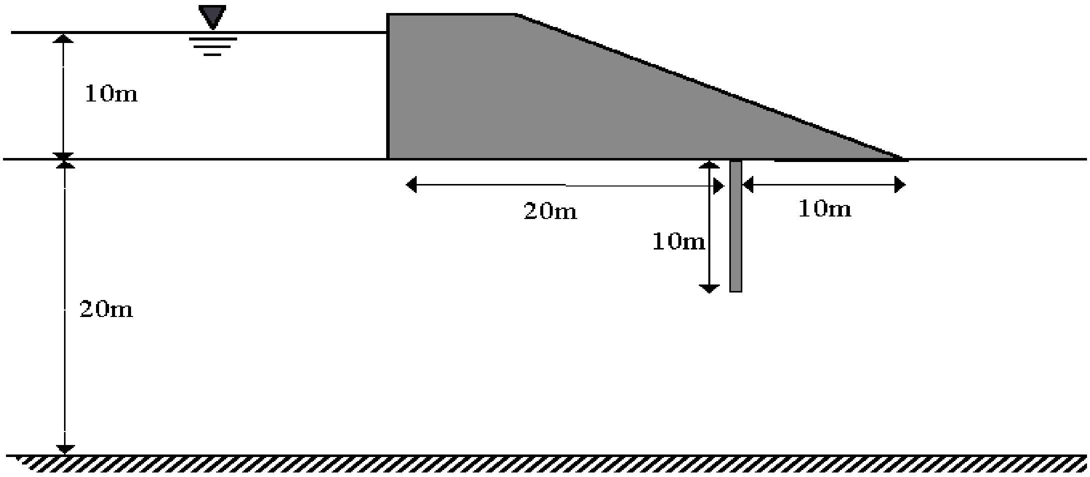

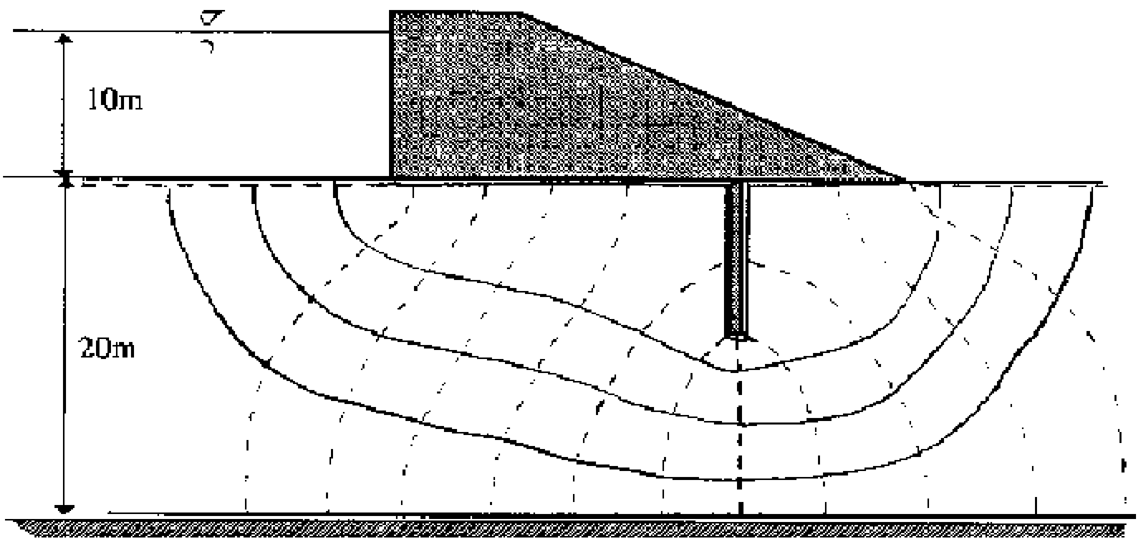

$$q = kh\frac{N_f}{N_d} = \frac{10^{-3}}{1000} \times 10 \times \frac{3.33}{12} = 2.78 \times 10^{-6}\ \text{m}^3/\text{sec}$$

(This is q/unit length)

To m^3/day: $2.78 \times 10^{-6} \times 60 \times 60 \times 24 = \underline{\underline{0.24\ \text{m}^3/\text{day}}}$

Tutorial

Ex1:

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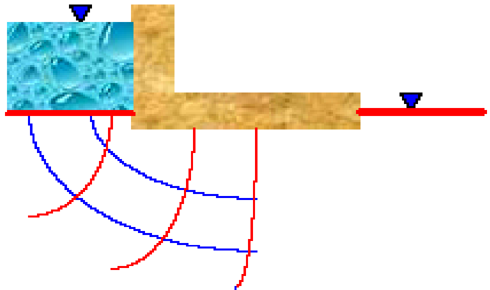
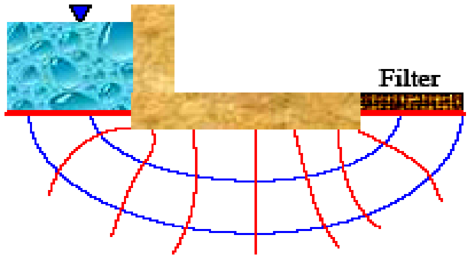
State what is wrong, if anything, with each of the following flow nets.

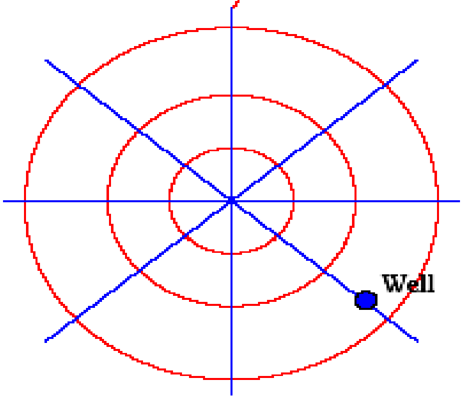
a)



b)



c)



Solution:

a) Impossible mesh, because two equipotential lines intersect.

b) Impossible mesh, because two flow-lines intersect.

c) The well should be at the center of the net (a sink or a source point).

Ex2:

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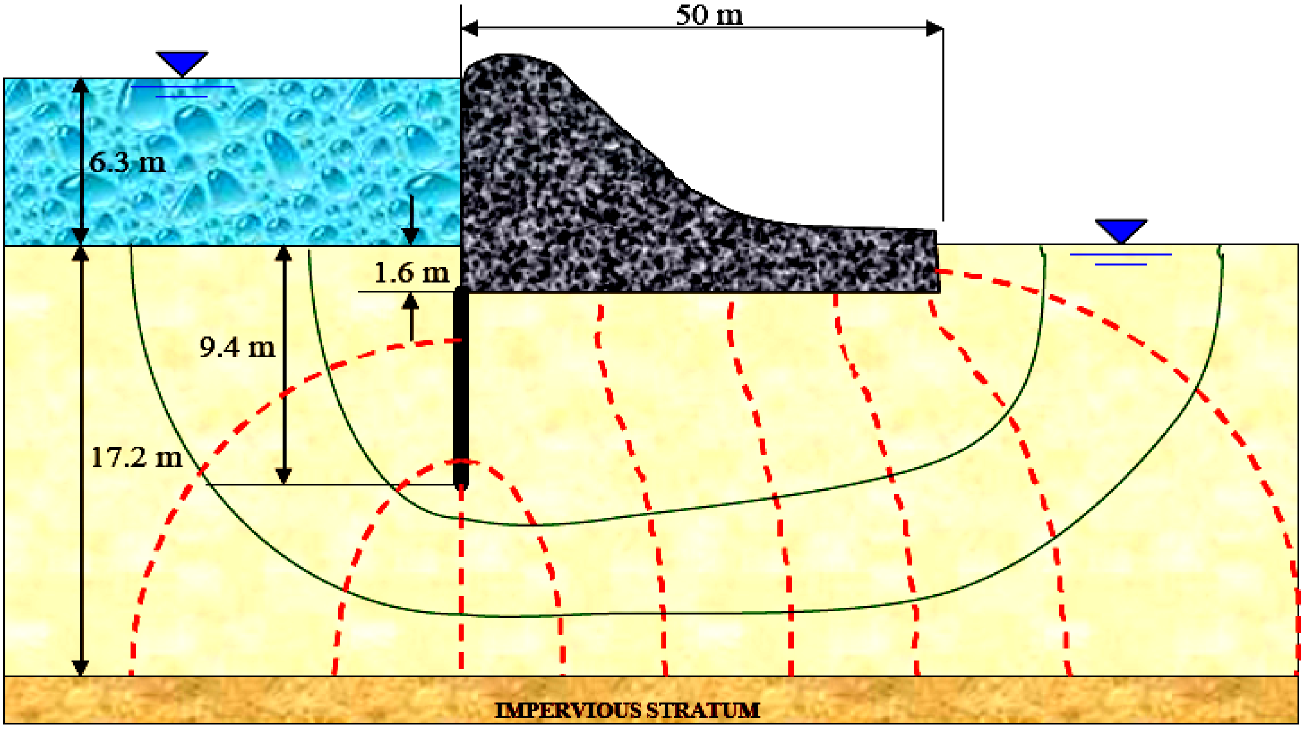
The completed flow net for the dam shown below includes a steel sheet-pile cutoff wall located at the head (head-water side) of the dam, in order to reduce the seepage loss. The dam is half a kilometer in width (shore to shore), and the permeability of the under-laying silty sand is 3.5×10^{-4} cm/s. Find the total seepage loss under the dam in liter per year. Would the dam be more stable if the cutoff wall was placed under its toe (tail-water side)?



Solution:

a) Using ***Forcheimer's*** equation:

$$q = k\Delta h \frac{N_f}{N_p} = 3.5 \times 10^{-4} \frac{cm}{\sec}\left(\frac{m}{100\ cm}\right)(6.3m)\left(\frac{3}{10}\right) = 6.6 \times 10^{-6}\ m^2/\sec/m$$

b) $$Q = Lq = 500\ m\left[6.6 \times 10^{-6}\ m^2/\sec/m\left(\frac{lts}{10^{-3}\ m^3}\right)\left(31.5 \times 10^6 \frac{\sec}{year}\right)\right] = 104 \frac{million\ liters}{year}$$

c) No. Placing the cutoff wall at the toe would allow high uplift hydrostatic pressures under the dam, thereby decreasing the dam's stability against sliding.

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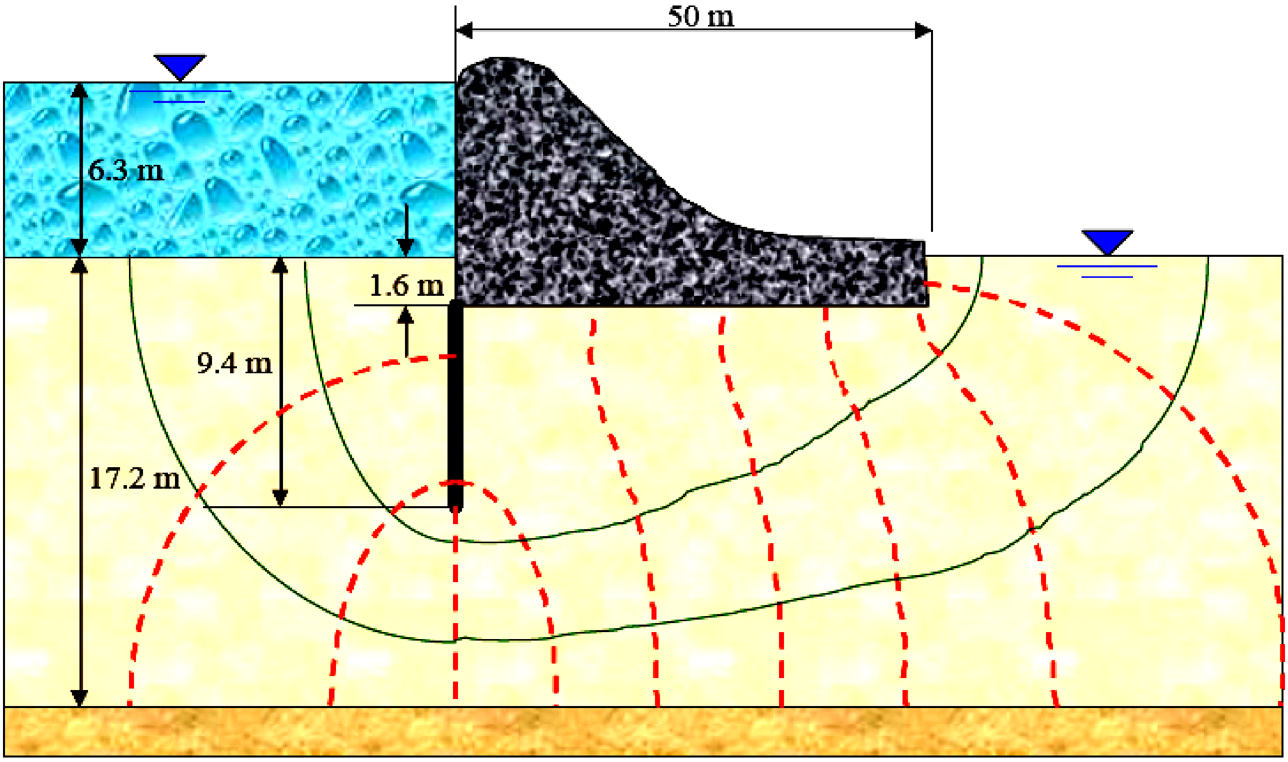

Solution:

a) Using ***Forcheimer's*** equation:

$$q = k\Delta h \frac{N_f}{N_p} = 3.5\times10^{-4}\ \frac{cm}{\sec}\left(\frac{m}{100\,cm}\right)(6.3m)\left(\frac{3}{10}\right) = 6.6\times10^{-6}\ m^2/\sec/m$$

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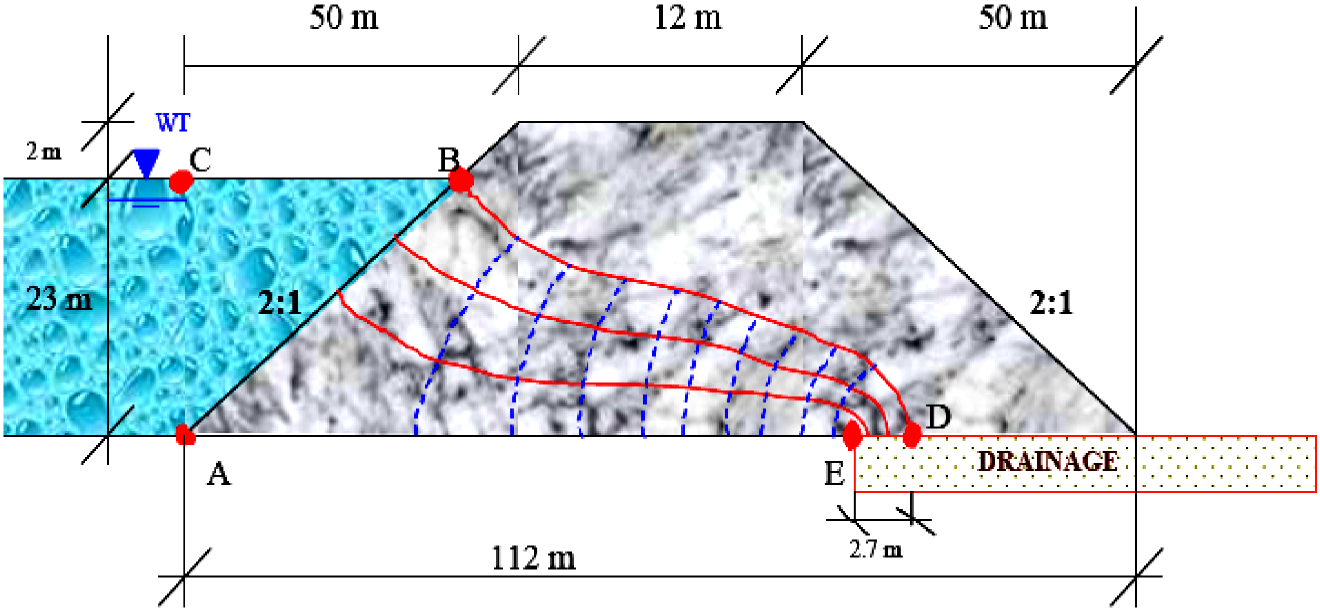
In Miami-Dade County, the Everglades are kept as wetlands by containing their runoff with levees. Levee #111 runs North-South, and is 2 kilometers west of Krome Avenue (its cross section is show below). If your laboratory tests indicate that the permeability of the fill of the 80-year old levee is now 0.30 m/day, what is the volume of water lost through the levee along each kilometer of length, in m^3/day?



Section of levee looking North

$$Q = qL = k\Delta h \frac{N_f}{N_{eq}} \bullet L = \left(0.30\ \frac{m}{day}\right)(23\ m)\left(\frac{3}{9}\right)1000\ m$$

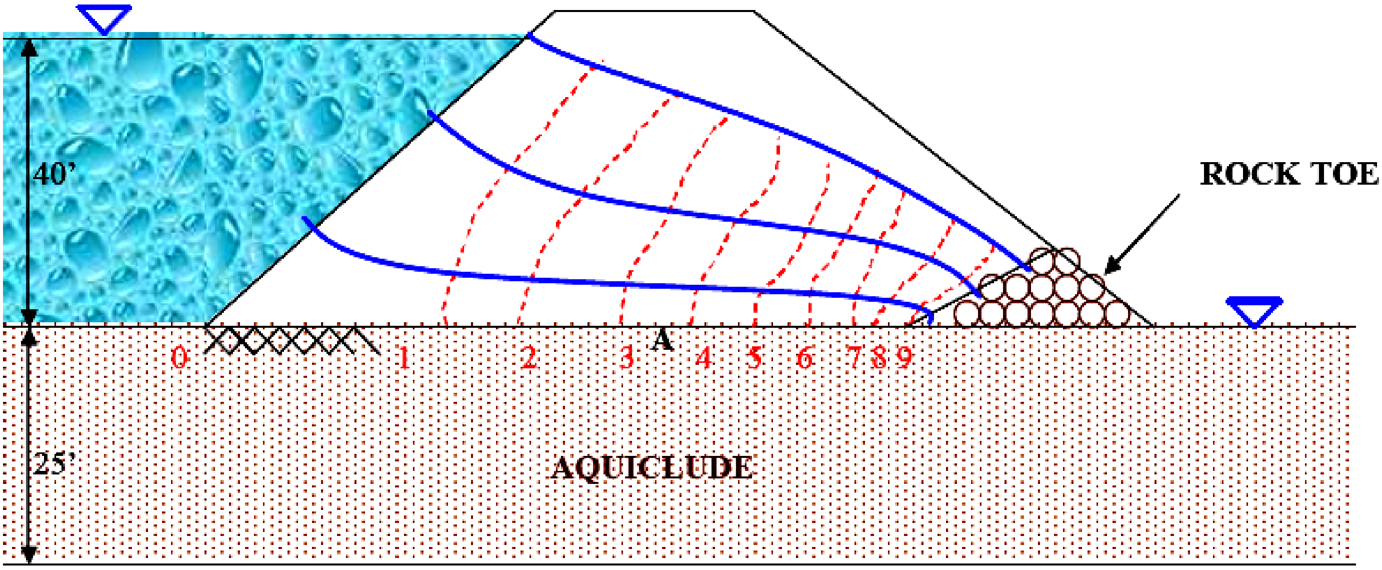
$$\boxed{Q = 2{,}300\ m^3/day}$$

Find the seepage through the earth dam shown below in gallons/day, if the sieve analysis shows the D_{10} to be 0.17 mm, and the dam is 1200 feet wide. What is the pressure head at the top of the aquiclude and at mid-dam (point A)?

$$N_f = 3$$

$$N_p = 9$$

$$k = 15 D_{10}^2 = 15(0.17\,mm)^2 = 0.43\,\frac{mm}{\sec}$$



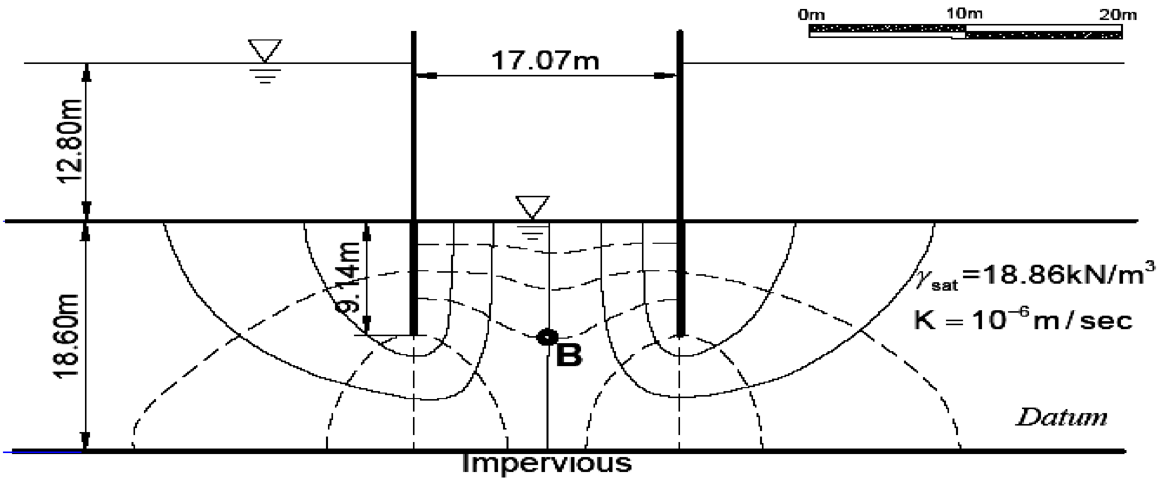
$$Q = q \times h = k \times i \times A \times h = \frac{\left(0.43\frac{mm}{\sec} \times 1\,in \times 40\,ft \times 86{,}400\,\sec \times 7.5\,gallons \times 1200\,ft \times \frac{3}{9}\right)}{(25.4\,mm \times 12\,in \times day)}$$

$$Q = 14.6 \times 10^6\,\frac{\text{gal}}{day}$$

At point "A" the dynamic pressure head is $\frac{3.4}{9}(40\,ft) = 15\,ft.$

Problem 2. Cofferdam

Solution 1) The datum line is impervious base.



1) The pore water pressure at B (u_b)

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{12.8}{8} = 1.6m$$

- The total head

$$h_t = \Delta H_t - \Delta h \times (N_d)_B = (12.8 + 18.6) - 1.6 \times 5 = 23.4m$$

- The elevation head at B : h_e = 18.6-9.14=9.46m
- The pressure head at B

$$(h_p)_B = h_t - h_e = 23.4 - 9.46 = 13.94m$$

- The pore water pressure at B

$$u_B = h_p \times \gamma_w = 13.94 \times 9.81 = 136.75kN/m^2 = 136.75kPa$$

2) The maximum hydraulic gradient(i_e)

$$i_{max} = \frac{\Delta h}{L_{min}} = \frac{1.6}{1.75} = 0.91$$

The critical hydraulic gradient(i_c)

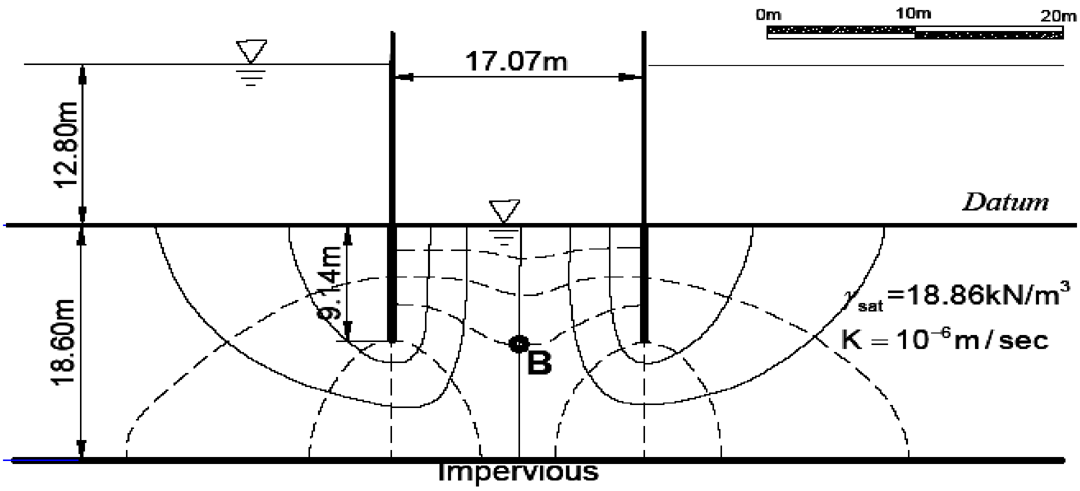
$$i_c = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{18.86 - 9.81}{9.81} = 0.92$$

3) Determine the factor of safety against quick sand and explain it

$$FOS = \frac{i_c}{i_{max}} = \frac{0.92}{0.91} = 1.01 < 4$$

The piping may occur.

Solution 2) The datum line is the right surface.



1) The pore water pressure at B (u_b)

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{12.8}{8} = 1.6m$$

- The total head

$$h_t = \Delta H - \Delta h \times (N_d)_B = 12.8 - 1.6 \times 5 = 4.8m$$

- The elevation head at B : h_e = -9.14m
- The pressure head at B

$$(h_p)_B = h_t - h_e = 4.8 - (-9.14) = 13.94m$$

- The pore water pressure at B

$$u_B = h_p \times \gamma_w = 13.94 \times 9.81 = 136.75kN/m^2 = 136.75kPa$$

4) The maximum hydraulic gradient(i_e)

$$i_{\max} = \frac{\Delta h}{L_{\min}} = \frac{1.6}{1.75} = 0.91$$

The critical hydraulic gradient(i_c)

$$i_c = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{18.86 - 9.81}{9.81} = 0.92$$

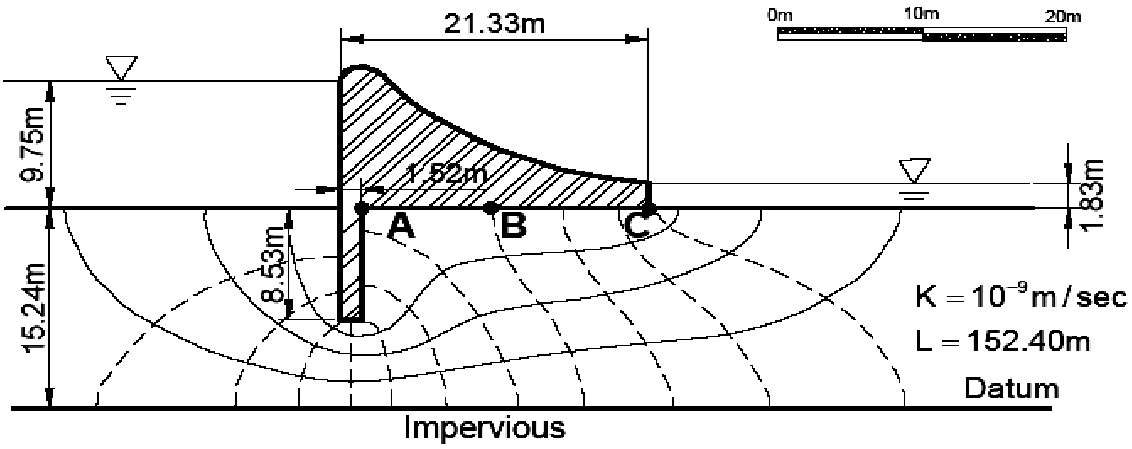
5) Determine the factor of safety against quick sand and explain it

$$FOS = \frac{i_c}{i_{\max}} = \frac{0.92}{0.91} = 1.01 < 4$$

The piping may occur.

Problem 3. Spillway

Solution 1) The datum line is impervious base.



1) Determine the pore water pressure distribution at A, B ,C

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{(9.75-1.89)}{12} = 0.66m$$

- The pressure head at each point : $h_p = h_t - h_e = \Delta H - N_d \times \Delta h - h_e$

$$(h_p)_A = (9.75+15.24) - 7.2\times 0.66 - 15.24 = 5m$$

$$\therefore u_A = 5\times 9.81 = 49.05kPa$$

$$(h_p)_B = (9.75+15.24) - 8\times 0.66 - 15.24 = 4.47m$$

$$\therefore u_B = 43.85kPa$$

$$(h_p)_C = (9.75+15.24) - 11\times 0.66 - 15.24 = 2.49m$$

$$\therefore u_C = 24.43kPa$$

h_e at end of wall =(15.24-8.53)m

$$(h_p)_{end\ of\ wall} = (9.75+15.24) - 4\times 0.66 - (15.24-8.53) = 15.64m$$

$$\therefore u_{end\ of\ wall} = 15.64\times 9.81 = 153.43kPa$$

2) The resultant uplift force (Hint : $F_{up} = A_{bottom} * u_{aver}$)

$$F_{up} = (A_{wall}\times u_{wall}) + A_{AB}\times u_{AB} + A_{BC}\times u_{BA}$$

$$= ((1.52\times 152.4)\times 153.43) + (9\times 152.4)\times \frac{49.05+43.85}{2} + (11\times 152.4)\times \frac{43.85+24.43}{2}$$

$$= 156484.9kN = 156.5MN \ (When\ consider\ the\ wall)$$

$$= 120943.1kN = 120.9\ MN \ (When\ ignore\ the\ wall)$$

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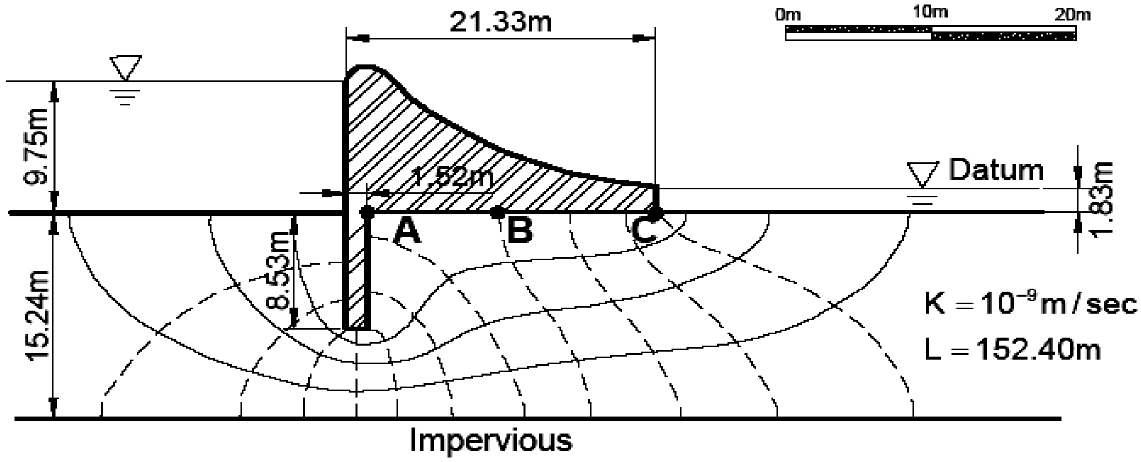
Solution 2) The datum is the surface of downstream.



3) Determine the pore water pressure distribution at A, B ,C

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{(9.75-1.89)}{12} = 0.66m$$

- The pressure head at each point: $h_p = h_t - h_e = \Delta H - N_d \times \Delta h - (-1.83)$

$$(h_p)_A = (9.75-1.83) - 7.2\times 0.66 + 1.83 = 5m$$

$$\therefore u_A = 5\times 9.81 = 49.05kPa$$

$$(h_p)_B = (9.75-1.83) - 8\times 0.66 + 1.83 = 4.47m$$

$$\therefore u_B = 43.85kPa$$

$$(h_p)_C = (9.75-1.83) - 11\times 0.66 + 1.83 = 2.49m$$

$$\therefore u_C = 24.43kPa$$

h_e at end of wall =-(8.53+1.83)m

$$(h_p)_{end\ of\ wall} = (9.75-1.83) - 4\times 0.66 + (8.53+1.83) = 15.64m$$

$$\therefore u_{end\ of\ wall} = 15.64\times 9.81 = 153.43kPa$$

4) The resultant uplift force (Hint : $F_{up} = A_{bottom} * u_{aver}$)

$$F_{up} = (A_{wall}\times u_{wall}) + A_{AB}\times u_{AB} + A_{BC}\times u_{BA}$$

$$= ((1.52\times 152.4)\times 153.43) + (9\times 152.4)\times \frac{49.05+43.85}{2} + (11\times 152.4)\times \frac{43.85+24.43}{2}$$

$$= 156484.9kN = 156.5MN\ (When\ consider\ the\ wall)$$

$$= 120943.1kN = 120.9\ MN\ (When\ ignore\ the\ wall)$$

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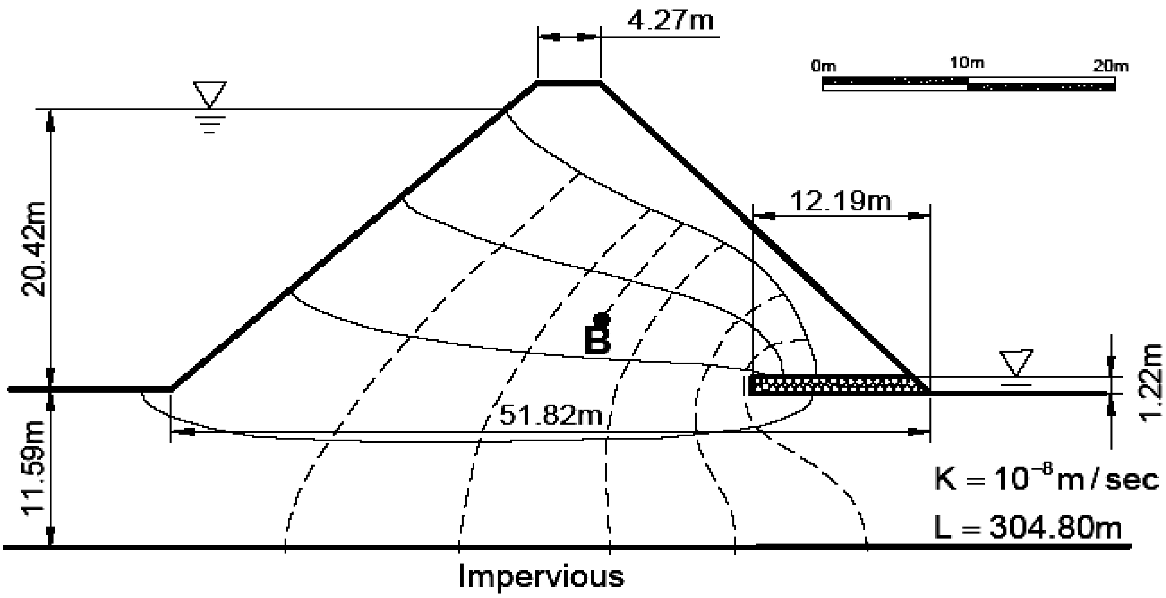
Problem 4. Evaluate the flow rate of the earth dam



$$q = 10^{-8} \times (20.42 - 1.22) \times \frac{4}{6} = 1.28 \times 10^{-7} m^3 / \sec / m$$

$$Q = q \times L = k\Delta H \frac{N_f}{N_d} L = 10^{-8} \times (20.42 - 1.22) \times \frac{4}{6} \times 304.8$$

$$= 3.9 \times 10^{-5} m^3 / \sec$$