University of Al Anbar | Soil Phsics | Mr. Ahmed Amin Al Hity
Collage of Engineering | 2^{nd} Stage | Lecture no. 9
Water Resources& Dams Eng. Dept. | 2019-2020 | Date 15 / 05/ 2020

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Total Stresses and Effective stresses

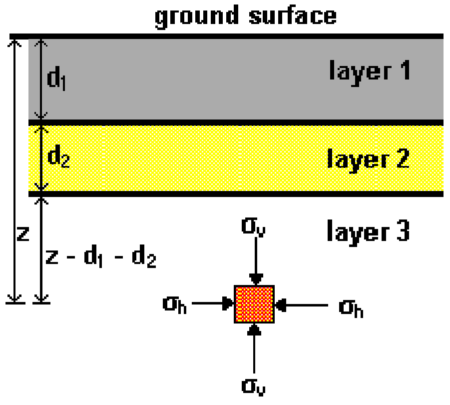
Total stress:

The total stress at depth z is the sum of the weights of soil in each layer thickness above.
Vertical total stress at depth z,

$$\sigma_v = \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 (z - d_1 - d_2)$$

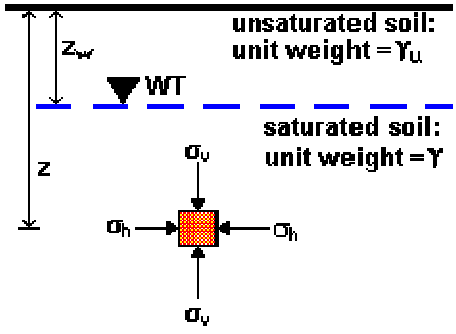
where

$\gamma_1, \gamma_2, \gamma_3$, etc. = unit weights of soil layers 1, 2 , 3, etc. respectively



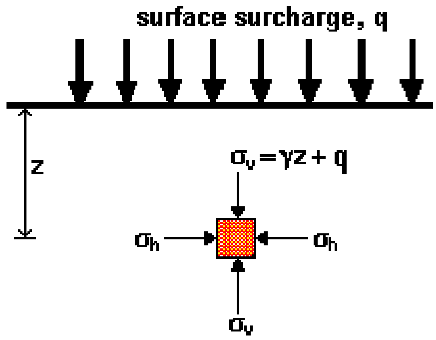
Just above the water table the soil will remain saturated due to capillarity, but at some distance above the water table the soil will become unsaturated, with a consequent reduction in unit weight (unsaturated unit weight = γ_u)

$$\sigma_v = \gamma_u \cdot z_w + \gamma_s (z - z_w)$$

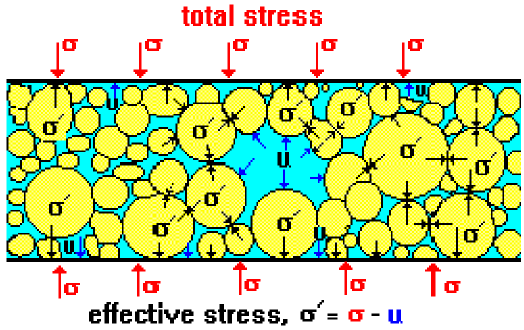


The addition of a surface surcharge load will increase the total stresses below it. If the surcharge loading is extensively wide, the increase in vertical total stress below it may be considered constant with depth and equal to the magnitude of the surcharge.
Vertical total stress at depth z,

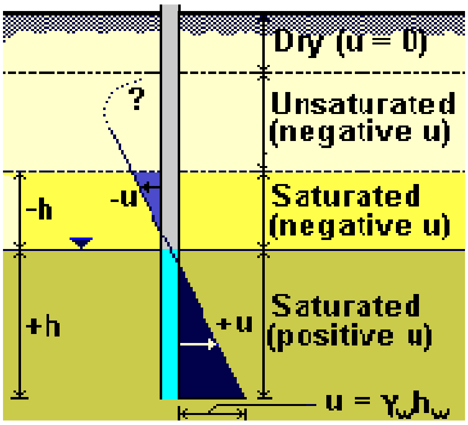
$$\sigma_v = \gamma \cdot z + q$$

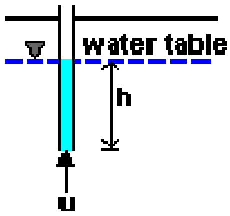


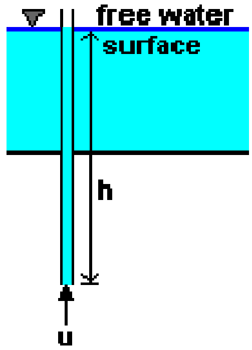
Pore Pressure:



- For saturated soil: *pore pressure = pore water pressure*
- For partially saturated soil: *pore pressure = pore water pressure + pore air pressure*
- For dry soil :*pore pressure = pore air pressure*
- In the case of partially saturated soil : *pore pressure depends on degree of saturation (S).*
- At level x-x *: pore water pressure (u) =h_2 γw (saturated soil)*







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Under hydrostatic conditions (no water flow) the pore pressure at a given point is given by the **hydrostatic pressure:**

$$u = \gamma_w \cdot h_w$$

-The natural static level of water in the ground is called the **water table** or the **phreatic surface** (or sometimes the **groundwater level**). Under conditions of no seepage flow, the water table will be horizontal, as in the surface of a lake. The magnitude of the pore pressure at the water table is zero. Below the water table, pore pressures are positive.

$$u = \gamma_w \cdot h_w$$

-Below the water table, pore pressures are **positive**. In dry soil, the pore pressure is **zero**. Above the water table, when the soil is saturated, pore pressure will be **negative**.

$$u = - \gamma_w \cdot h_w$$

-The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and type (and thus the size of pores):
· in coarse soils capillary rise is very small
· in silts it may be up to 2m
· in clays it can be over 20m

- In conditions of seepage in the ground there is a change in pore pressure. Consider seepage occurring between two points P and Q.

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The hydraulic gradient, **i**, between two points is the head drop per unit length between these points. It can be though of as the "potential" driving the water flow.

Hydralic gradient P-Q $i = -\frac{\delta h}{\delta s} = \frac{\delta u}{\delta s} \cdot \frac{1}{\gamma_w}$

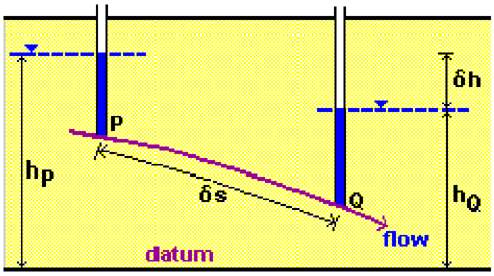
Thus $\delta u = i . \gamma_w . \delta s$

But in **steady-state** seepage, i = constant

Therefore the change in pore pressure due to seepage alone, $\delta u_s = i . \gamma_w . s$

For seepage flow vertically downward, i is negative

For seepage flow vertically upward, i is positive.



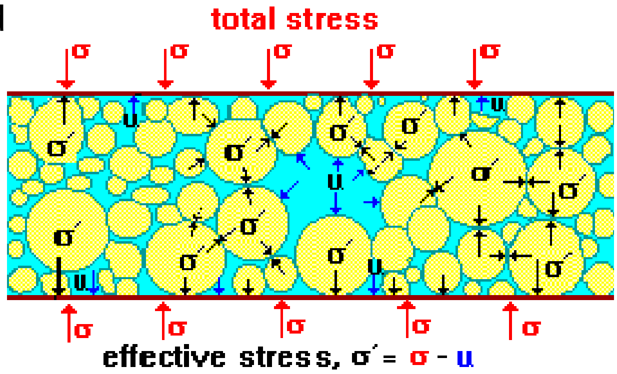
Effective Stress :

Ground movements and instabilities can be caused by changes in total stress (such as loading due to foundations or unloading due to excavations), but they can also be caused by changes in pore pressures (slopes can fail after rainfall increases the pore pressures).

In fact, it is the combined effect of total stress and pore pressure that controls soil behaviour such as shear strength, compression and distortion. The difference between the total stress and the pore pressure is called the effective stress:

effective stress = total stress - pore pressure

or σ´ = σ - u



Note that the prime (dash mark ´) indicates effective stress.

Capillarity in Soils

Water can raise and remain above the line of atmospheric pressure (pheriatic line) in a very fine pores due to attraction (surface tension)between adjacent molecules in the surface.

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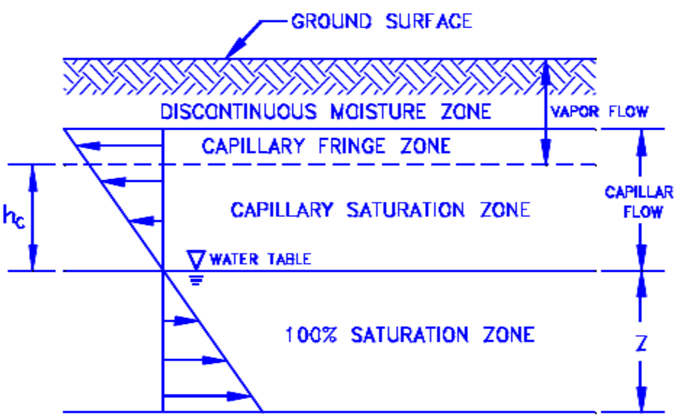
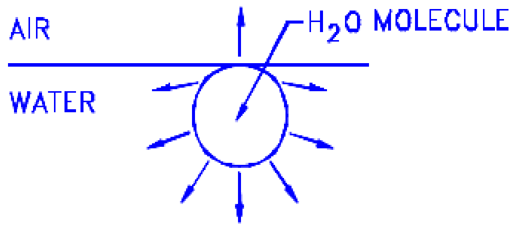




Capillary Head or Capillary Rise

$W = F_T COS\alpha$

$\gamma_w \pi r^2 h = T_s . 2\pi r . COS\alpha$

Where Ts= Surface tension

$\therefore h = \frac{2Ts}{\gamma_w r} \cos\alpha$ = Capillary head

$\Rightarrow$ **therefore** $h_C = \frac{-4T_o}{\gamma_w d}$

= Capillary rise $h \propto \frac{1}{r}$

For example, how much does the water rise above the water table in a very fine sand (d = 0.1 mm) ?

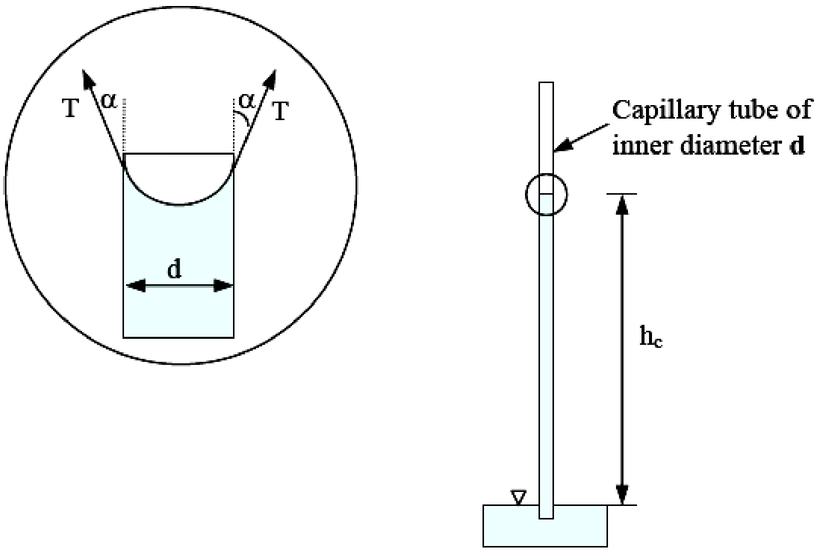
$$h_C = \frac{-4T_o}{\gamma_w d} = \frac{-4(0.073 \, N/m)}{9.81 \, kN/m^3 \, (0.1mm)} = \underline{0.30 \text{ m}}$$

Using typical values of T = 0.073 N/m, $\alpha = 0^\circ$ and $\gamma_w = 9810 \text{ N/m}^3$ in Eq. 6.6, it can be shown that:

$$h_c(m) \approx \frac{0.03}{d(mm)}$$

What do these have to do with soils? The interconnected voids within the soil can act like capillary tubes (not straight though) and allow the water to rise well above the water table. The "capillary tube" diamater of a soil is approximately 1/5 of D_{10}. Therefore, the capillary rise within a soil can be written as:

$$h_c(m) \approx \frac{0.15}{D_{10}(mm)}$$



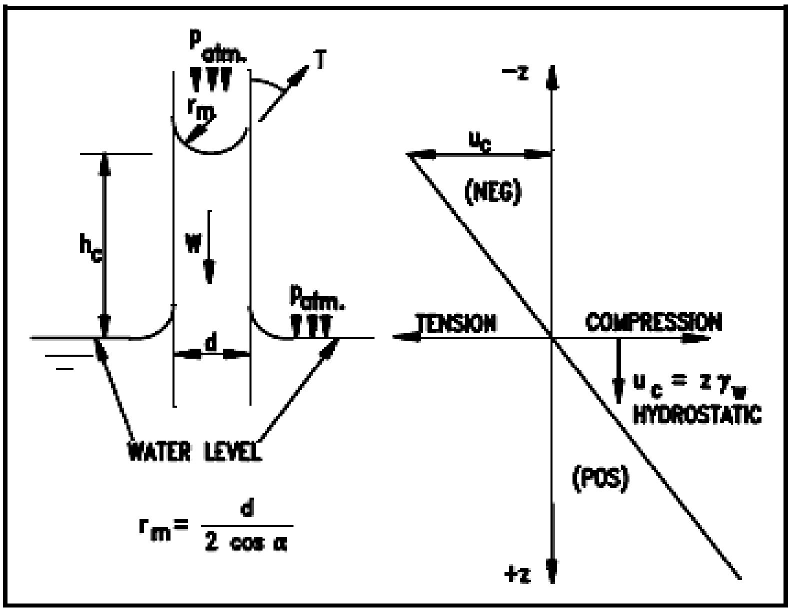


Figure 1. Capillarity water rise and pressure in capillary tube.

Effective Stress in the zone of Capillary Rise

The pore water pressure at a point above W.T depends on degree of saturation due to capillary rise.

-For fully saturated zone by capillary rise $u = -h\,\gamma_w$

-For partially saturated zone by capillary rise $u = -s\,h\,\gamma_w/100$

-his measured from and above W.T.

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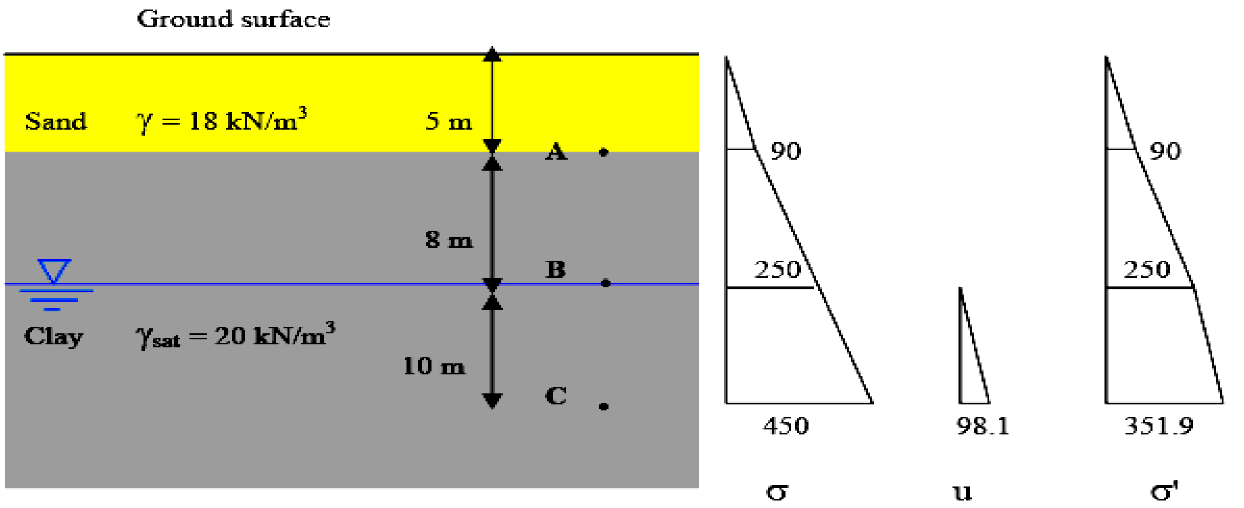
1) For the cross section given in the figure, calculate the total vertical stresses, pore pressures, and effective vertical stresses at points A, B, C and sketch these stress distributions. Assume that the whole clay layer is fully saturated.



At point A:
$\sigma = 18x5 = 90$ kN/m^2 = 90 kPa   u = 0 $\sigma' = 90-0 = 90$ kPa

At point B:
$\sigma = 18x5+20x8 = 250$ kPa   u = 0 $\sigma' = 250-0 = 250$ kPa

At point C:
$\sigma = 18x5+20x(10+8) = 450$ kPa   u = 9.81x10=98.1 kPa
$\sigma' = 450-98.1 = 351.9$ kPa

2) The basin of a lake consists of uniform clay with saturated unit weight 19 kN/m^3. Calculate the effective stress at a depth of 20 m below ground surface under the lake when the water depth is 5 m. The elevation of water in the lake changes throughout the year, it rises to 10 m in the rain season. How does this affect the effective stress you have previously calculated?

When water depth = 5 m
$\sigma = 9.81x5+19x20= 429$ kPa
u = 9.81x(5+20)= 245 kPa
$\sigma' = 429-245 = 184$ kPa

When water depth = 10 m
$\sigma = 9.81x10+19x20= 478$ kPa
u = 9.81x(10+20)= 294 kPa
$\sigma' = 478-294 = 184$ kPa

Note that effective stress does not change with increasing water depth in the lake while total stress and pore water pressure increase. Effective stress on the ground surface is zero.

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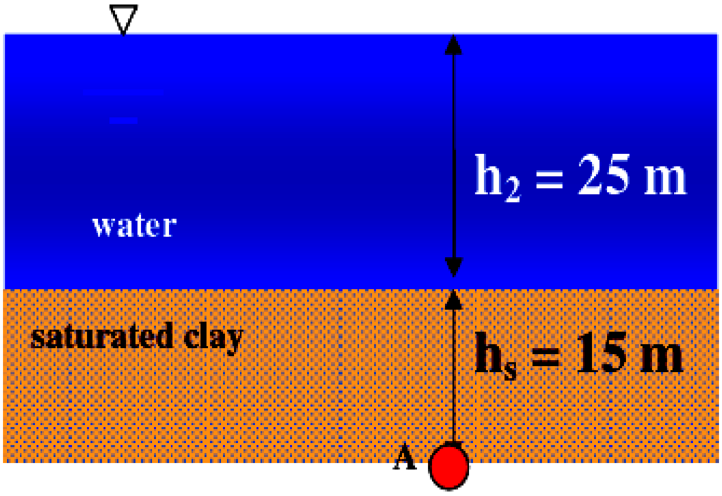
A sample was obtained from point A in the submerged clay layer shown below, and it was determined that it had a w = 54%, and a Gs = 2.78. What is the effective vertical stress at A?



Solution:

The effective stress σ' at the point A consists solely of the depth of the soil (not the water) multiplied by the soil buoyant unit weight.

$$\sigma' = \gamma' h_{soil} \quad where \quad \gamma' = \gamma_{SAT} - \gamma_W$$

In order to find γ' there are a number of derivation, such as this one,

$$\gamma' = \frac{(G_s + e)\gamma_W}{1+e} - \gamma_W$$ where the voids ratio e can be replace with $Se = wG_s$

and noticing that $S = 1$ because the soil is 100% saturated, $e = wG_s$

$$\sigma' = \left[\frac{(G_s + wG_s)}{1+wG_s}\gamma_W - \gamma_W\right]h_{soil} = \left[\frac{2.78+(0.54)(2.78)}{1+(0.54)(2.78)}(9.81) - 9.81\right](15\,m)$$

$$\sigma' = 105\,kPa$$

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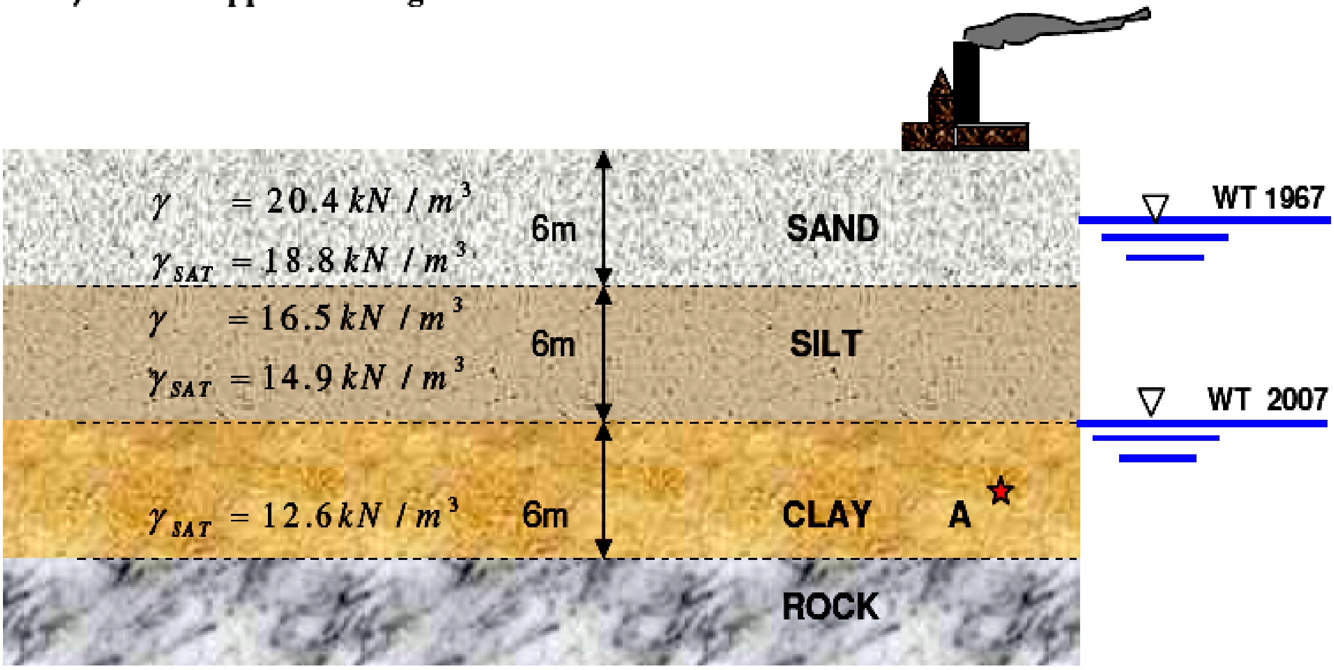
The city of Houston, Texas has been experiencing a rapid lowering of its phreatic surface during the past 40 years, due to large volumes of water pumped out of the ground by industrial users.

a) What was the effective vertical stress at a depth of 15 m in 1967?
b) What is the effective stress at the same depth in 2007?
c) What happens to the ground surface as a result of the draw downs?



Solution:

a)

$$\sigma'_V = [\gamma h + \gamma' h']_{SAND} + [\gamma' h']_{SILT} + [\gamma' h']_{CLAY}$$

$$\sigma'_V = \left[(20.4)(3) + (18.8 - 9.81)(3)\right] + \left[(14.9 - 9.81)(6)\right] + \left[(12.6 - 9.81)(3)\right]$$

$$\sigma'_V = 128\ kPa$$

b) $\sigma'_V = \left[(20.4)(6) + (16.5)(6)\right] + \left[(12.6 - 9.81)(3)\right] = 230\ kPa$

This is an 80% increase in stress due solely to a dropping water table.

c) The ground surface has also been lowered due to the decreasing thickness of the clay and the silt strata due to their loss volume previously occupied by the water.

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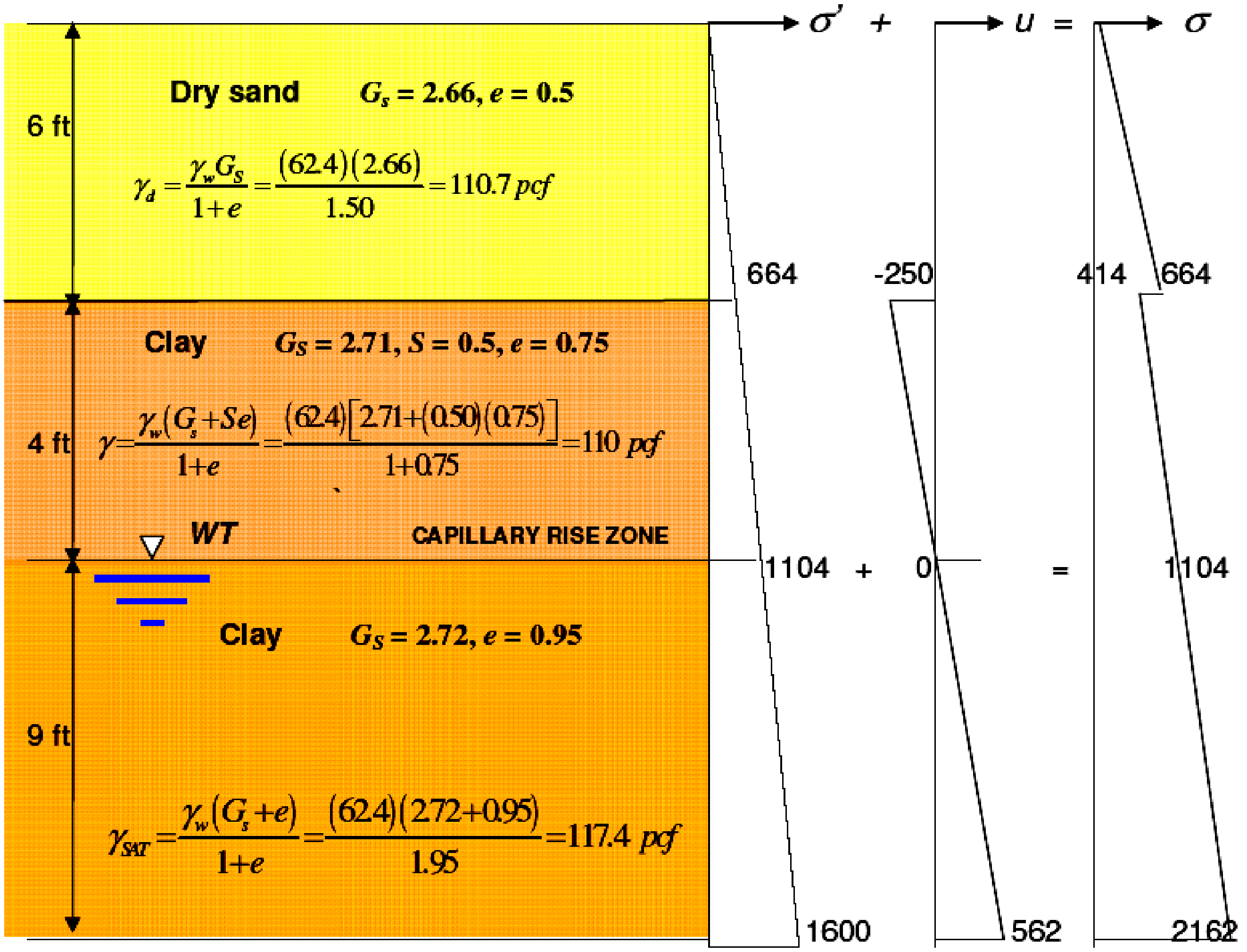
In the soil profile shown below, show a plot of the *pore water pressure* and the *effective stress* along the right margin of the figure with numerical values at each interface. Pay heed to the capillarity in the upper clay (d = 0.001 mm), where S = 50% in the upper clay stratum.



Depth, ft	σ' +	u =	σ
0'	0	0	0
6'	(110.7)(6) = 664	0	664
		(62.4)(-4) = -250	414
10'	664+ (110)(4)=1104	0	1104
19'	1104+(117.4-62.4)(9)=1600	9(62.4) = 562	2162

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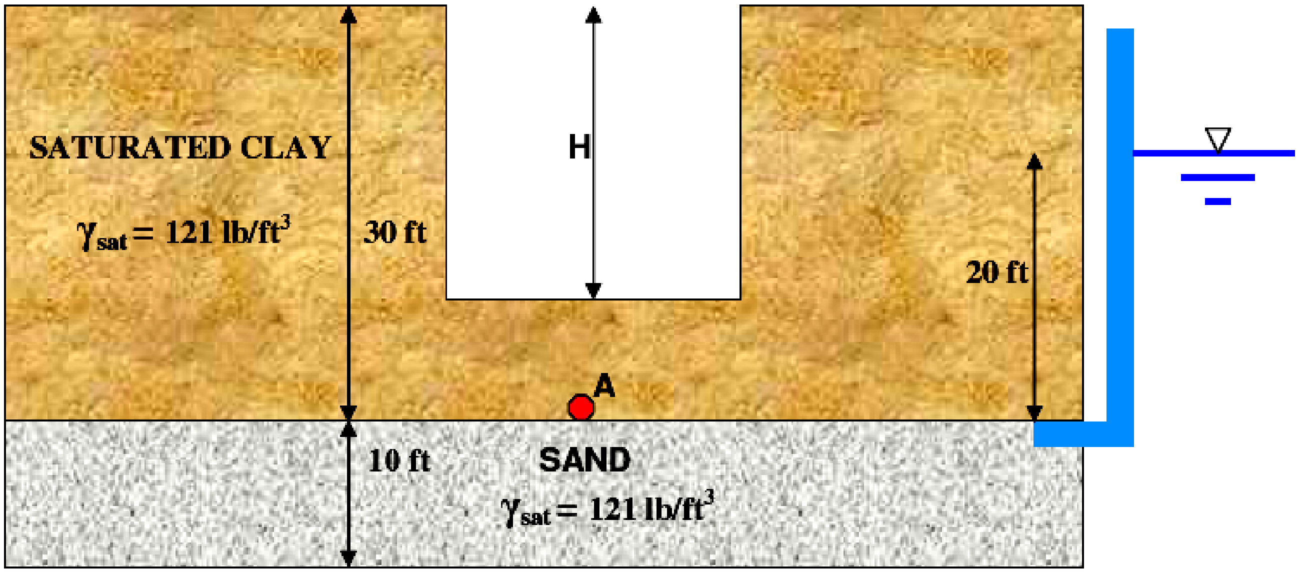
Calculate the maximum theoretical depth of excavation *H* below, before the remaining clay layer is uplifted by the vertical seepage pressure.

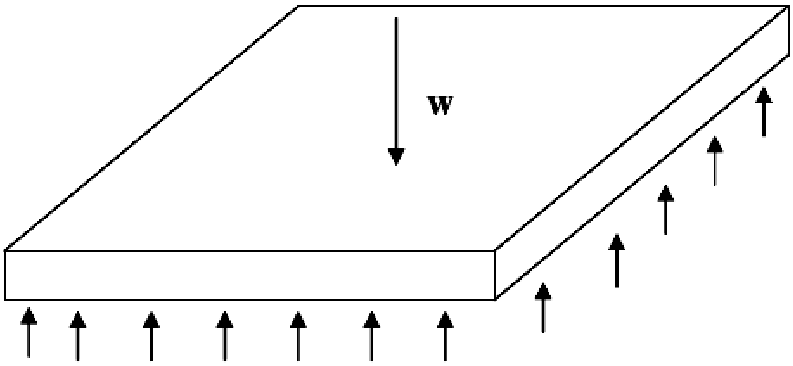


Solution:

The maximum depth of excavation *H* is reached when the effective stress σ' = 0 (that is, the upward seepage force is equal to the downward weight of the soil. Mathematically,

$$\sigma'_A = \sigma_A - u_A = \left(121\frac{lb}{ft^3}\right)(30' - H) - \left(62.4\frac{lb}{ft^3}\right)(20') = 0$$

$$30' - H = \frac{(62.4)(20')}{(121)} = 10.3\ feet \quad \therefore \quad H = 19.7\ feet$$



Seepage hydrostatic uplift

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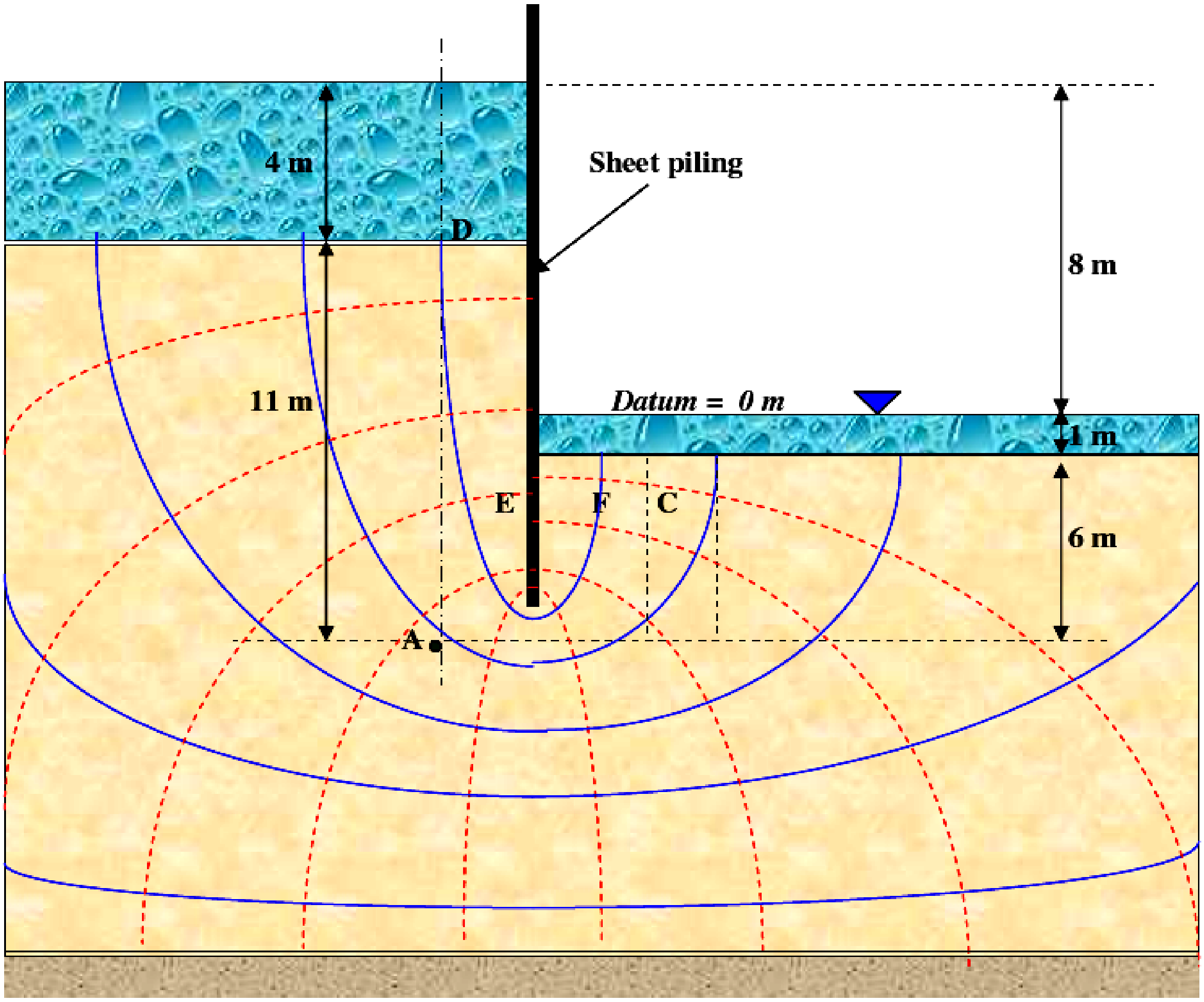
If the saturated unit weight of the soil in this shallow bay bottom is 20 kN/m^3, what is the effective vertical stress σ' at point A?



Solution:

$$\text{The total stress } \sigma_A = \gamma_w h_w + \gamma_{SAT} h_{SOIL} = \left(9.81\frac{kN}{m^3}\right)(4\,m) + \left(20\frac{kN}{m^3}\right)(11\,m) = 259\frac{kN}{m^2}$$

$$\text{The pore water pressure } u_A = \gamma_w (h_A - z_A) = \left(9.81\frac{kN}{m^3}\right)\left[\left(\frac{5.7}{10}\right)(8\,m) - (-7\,m)\right] = 122\frac{kN}{m^2}$$

$$\text{Therefore, the effective stress at } A: \ \sigma_A^{'} = \sigma_A - u_A = 259 - 122 = 137\frac{kN}{m^2} = 137\,kPa$$

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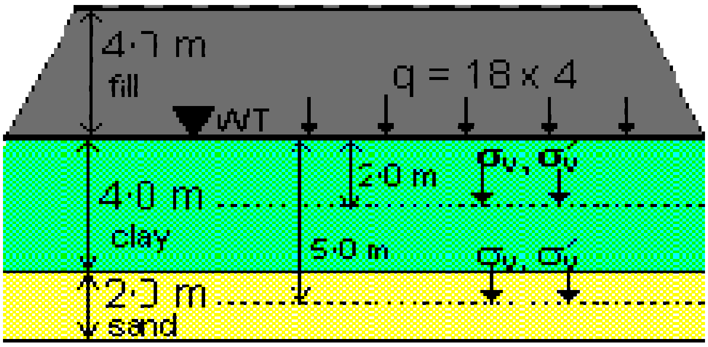
<u>Example:</u> The figure shows how an extensive layer of fill will be placed on a certain site. The unit weights are: clay and sand = 20kN/m³, rolled fills 18kN/m³, assume water = 10 kN/m³.

Calculations are made for the total and effective stress at the mid-depth of the sand and the mid-depth of the clay for the following conditions: initially, before construction; immediately after construction; many years after construction



<u>Initially, before construction</u>

Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

σ_v = 20.0 x 2.0 = **40.0kPa**

Pore pressure

u = 10 x 2.0 = 20.0kPa

Vertical effective stress

$\sigma'_v = \sigma_v$ - u = **20.0kPa**

Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress

σ_v = 20.0 x 5.0 = **100.0 kPa**

Pore pressure

u = 10 x 5.0 = 50.0 kPa

Vertical effective stress

$\sigma'_v = \sigma_v$ - u = **50.0 kPa**

<u>immediately after construction</u>

The construction of the embankment applies a surface surcharge:

q = 18 x 4 = 72.0 kPa.

The sand is drained (either horizontally or into the rock below) and so there is no increase in pore pressure. The clay is undrained and the pore pressure increases by 72.0 kPa.

Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

σ_v = 20.0 x 2.0 + 72.0 = **112.0kPa**

Pore pressure

u = 10 x 2.0 + 72.0 = 92.0 kPa

Vertical effective stress

$\sigma'_v = \sigma_v - u$ = **20.0kPa**

(i.e. no change immediately)

Initial stresses at mid-depth of sand (z = 5.0m)

Vertical total stress

σ_v = 20.0 x 5.0 + 72.0 = **172.0kPa**

Pore pressure

u = 10 x 5.0 = 50.0 kPa

Vertical effective stress

$\sigma'_v = \sigma_v - u$ = **122.0kPa**

(i.e. an immediate increase)

Many years after construction

After many years, the excess pore pressures in the clay will have dissipated. The pore pressures will now be the same as they were initially.

Initial stresses at mid-depth of clay (z = 2.0 m)

Vertical total stress

σ_v = 20.0 x 2.0 + 72.0 = **112.0 kPa**

Pore pressure

u = 10 x 2.0 = 20.0 kPa

Vertical effective stress

$\sigma'_v = \sigma_v - u$ = **92.0 kPa**

(i.e. a long-term increase)

Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress

σ_v = 20.0 x 5.0 + 72.0 = **172.0 kPa**

Pore pressure

u = 10 x 5.0 = 50.0 kPa

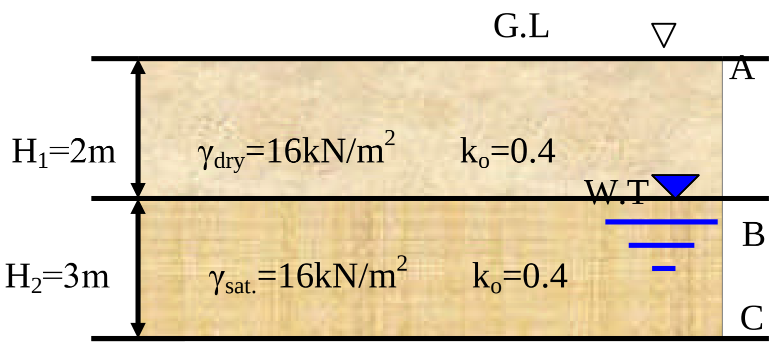
Vertical effective stress

$\sigma'_v = \sigma_v$ - u = **122.0 kPa**

(i.e. no further change)

Example1. For the soil profile shown below find $\Delta\sigma_v$; $\Delta\sigma_h$; **u** ;$\Delta\sigma'_v$; $\Delta\sigma'_h$; at points A,B and C.

Solution:

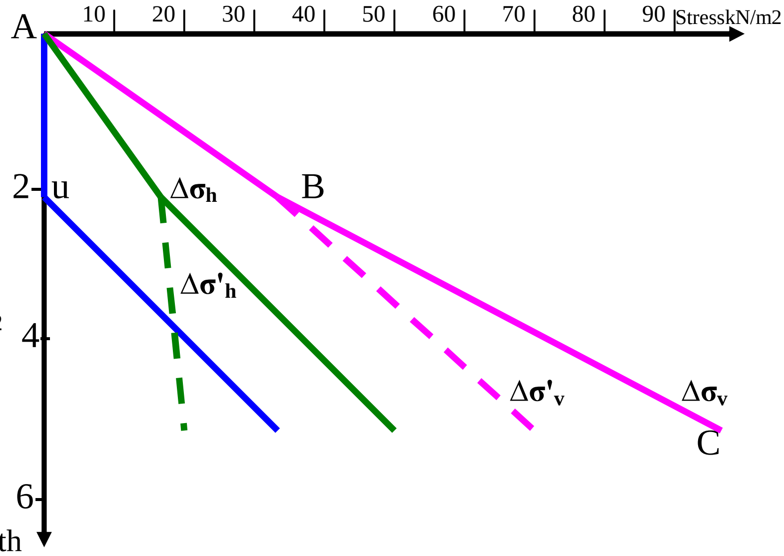


1. at point A

$\Delta\sigma_v = 0$

$\Delta\sigma_h = 0$; **u** = 0 ;$\Delta\sigma'_v = 0$

$\Delta\sigma'_h = 0$



2. At point B

$\Delta\sigma_v = \gamma_{dry}$ x H_1=16 x 2= 32 kN/m^2

$\Delta\sigma_h = \Delta\sigma_v$ x k_o= 32 x 0.4 = 12.8 kN/m^2

u = γ_wx H_1=0 **;**

$\Delta\sigma'_v = \Delta\sigma_v$ = 32 kN/m^2

$\Delta\sigma'_h = \Delta\sigma_h$= 12.8 kN/m^2

3. At point C

$\Delta\sigma_v = \gamma_{dry}$ x H_1 + $\gamma_{sat.}$ x H_2 =16 x 2 + 3 x 20 = 92 kN/m^2

$\Delta\sigma_h = \Delta\sigma_{v1}$ x k_o + $\Delta\sigma_{v2}$ x k_o = 32 x 0.4 + 60 x 0.5 = 42.8 kN/m^2

u = γ_wx H_2= 9.81 x 3 = 29.43 kN/m^2

$\Delta\sigma'_v = \Delta\sigma_v$ **- u** = 92 – 29.43 = 62.57 kN/m^2

$\Delta\sigma'_h = \Delta\sigma_h$ – u = 42.8 – 29.43=13.37 kN/m^2