Rigid Pavement

Postgraduate Studies
Highways Engineering

Prepared By:

Dr Talal H. Fadhil

Dr Taher M. Ahmed

Department of Civil Engineering

University of Anbar

Syllabus of:
Advanced Pavement Design

3. Rigid Pavement ....(2 Weeks)

3.1. Analysis of: Stress, Strain and Deflection in Rigid Pavement,

3.2. Type of Stresses

3.2.1. Due to Temperature Variations

3.2.2 Due to Load (Westergard Method)

3.2.3. Effect of Dual Tires.

3.2.4. Due to Subgrade Friction.

3.2.5. Steel Stress

3.2.6. Tie-Bars.

3.3. Design of Dowel-Bars

3.4. Design of Joints.

3. Rigid Pavement

- Rigid pavements consist of Portland concrete slabs resting on a base course or directly on the subgrade. The modulus elasticity of the Portland concrete, which is about of 28 000 MPa, is much higher than the moduli of the foundation materials (subbase, subgrade) which typically range from 80 to 600 Mpa. The major portion of the load carrying capacity is mainly due to the rigidity of high modulus of elasticity of the slab. This is often been referred to plate action and when designed considered as a plate on elastic foundation.
- One of the most common simplifications used in analyzing concrete pavements concerns the subgrade and the way it supports the slab. The subgrade is modeled either **as a series of non interacting linear springs** (Figure 3.1a) or as an **homogeneous and isotropic continuum of infinite depth** (Figure 3.1b).

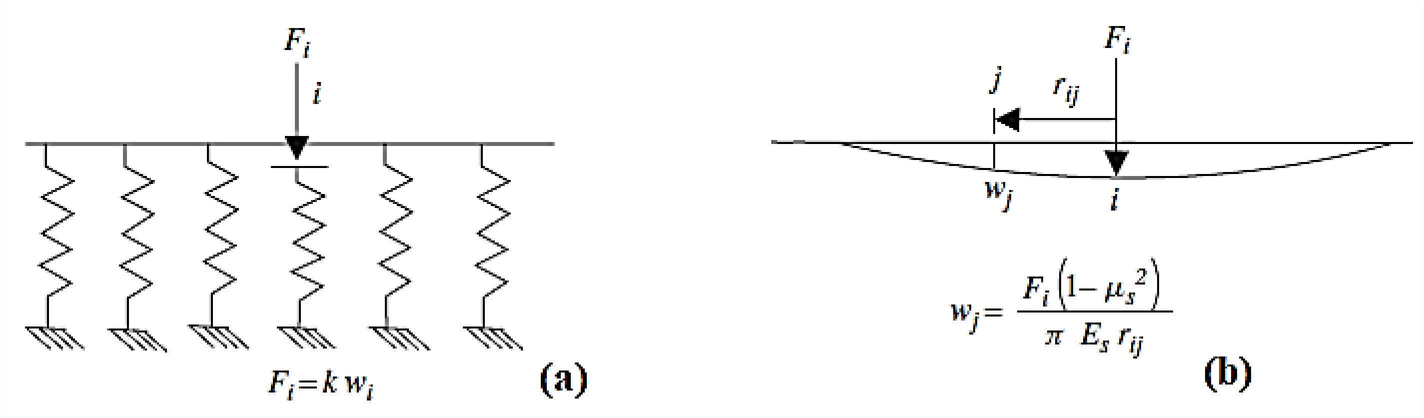

Figure. 3.1. Types of subgrade foundation model.

❖The first foundation model (Fig.3.1a) is characterized by the elastic constant of the springs, referred to as the *modulus of subgrade reaction*, denoted by *k*. This foundation, referred to as Winkler, or "liquid," implies that load at a particular point generates subgrade deflection only directly underneath that point.

❖ The second model (Fig.3.1b) is characterized by the foundation elastic modulus and the Poisson's ratio of the subgrade, denoted by E_s and μ_s, respectively (to differentiate them from the elastic constants of the slab itself, denoted by E and μ).

3.1. Analysis of: Stress, ~~Strain~~ and Deflection in Rigid Pavement,

➢ Stresses in concrete pavements are the result of the interaction of a number of factors, which can be grouped into three main categories:

1. Effect of temperature and moisture changes in the slab.
2. External applied load
3. Volume changes of supporting materials including frost action.

3.1.1. Stress Due to Curling (Temperature variation)

➢ **During the day, when the temperature on the top of the slab is greater than that at the bottom**, the top tends to expand with respect to the neutral axis, while the bottom tends to contract . However, the weight of the slab restrains it from expansion and contraction ; thus, compressive stresses are induced at the top, tensile stresses at the bottom . **At night, when the temperature on the top of the slab is lower than that at the bottom**, the top tends to contract with respect to the bottom; thus, tensile stresses are induced at the top and compressive stresses at the bottom .

➢ Another explanation of curling stress can be made in terms of the theory of a plate on a Winkler, or liquid, foundation . A Winkler foundation is characterized by a series of springs attached to the plate, as shown in Figure 3.2. When the temperature on the top is greater than that at the bottom, the top is longer than the bottom and the slab curls downward . The springs at the outside edge are in compression and push the slab up, while the springs in the interior are in tension and pull the slab down .

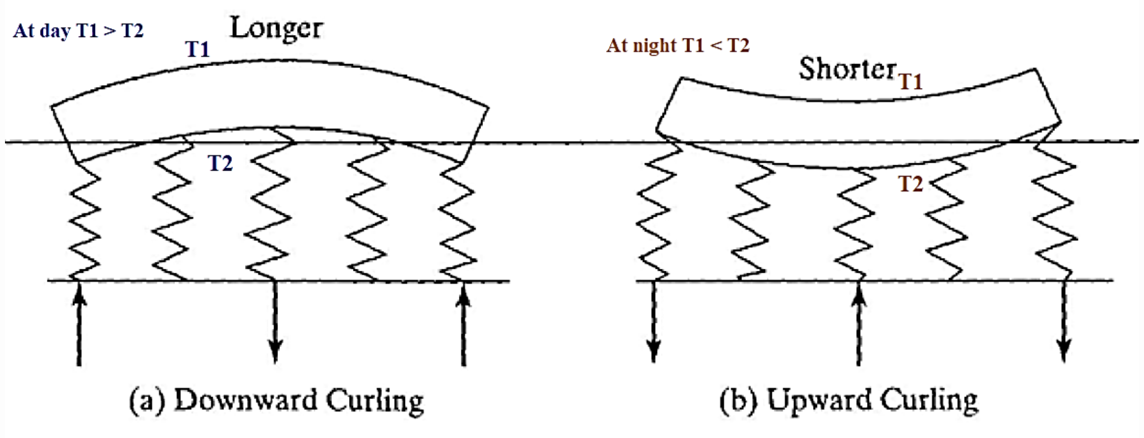

Figure 3.2. Effect of temperature

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} \quad \ldots\ldots\ldots\ldots \text{(3.1) strain in y direction due to stresses.}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} \quad \ldots\ldots\ldots\ldots \text{(3.2) strain in x direction due to stresses.}$$

If plate is bent in the x direction → ε_y=0 (substituting in Equation 3.1)

$$\sigma_y = \mu\sigma_x \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.3)$$

Substituting Equation 3.3. in Equation 3.2 yields:

$$\sigma_x = \frac{E\varepsilon_x}{1-\mu^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

- When bending occurs in both the x and y directions, as is the case for temperature curling, the stresses in both directions must be superimposed to obtain the total stress. The maximum stress in an infinite slab due to temperature curling can be obtained by assuming that the slab is completely restrained in both x and y directions.

- Let **Δt** be the temperature differential between the top and the bottom of the slab (T1-T2) and $\boldsymbol{\alpha_t}$ be the coefficient of thermal expansion of concrete . If the slab is free to move and the temperature at the top is greater than that at the bottom, the top will expand by a strain of **(α_t Δt/2)** and the bottom will contract by the same strain, as shown in Figure 3.3 . If the slab is completely restrained and prevented from moving, a compressive strain will result at the top and a tensile strain at the bottom . The maximum strain is

$$\epsilon_x = \epsilon_y = \frac{\alpha_t \Delta t}{2} \quad \ldots\ldots\ldots\ldots \text{(3.5)} \quad \text{substituting in Equation (3.4)}$$

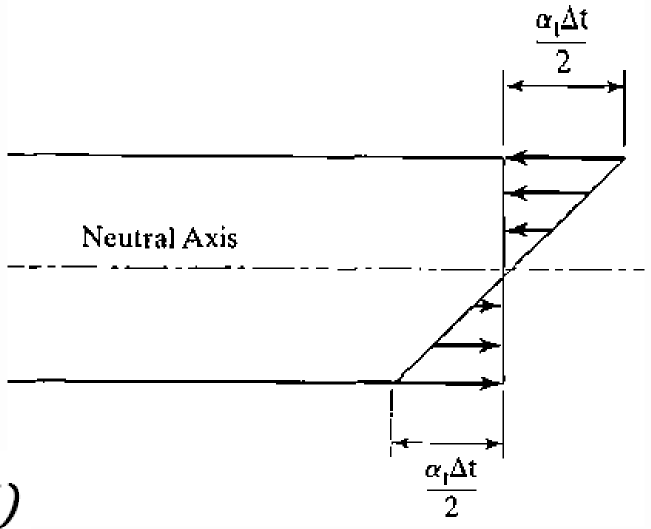

Figure 3.3. *Temperature gradient*

<u>Note</u>: α_t for concrete is 9 X 10 $^{-6}$ mm/mm/°C

$$\sigma_x = \frac{E\alpha_t\Delta t}{2(1-\mu^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

- The stress in the x direction due to also bending in the y direction is:

$$\sigma_x = \frac{\mu E\alpha_t\Delta t}{2(1-\mu^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.7)$$

The total stress is the sum of Equations 3.6 and 3.7 :

$$\sigma_x = \frac{E\alpha_t \Delta t}{2(1-\mu^2)}(1+\mu) = \frac{E\alpha_t \Delta t}{2(1-\mu)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

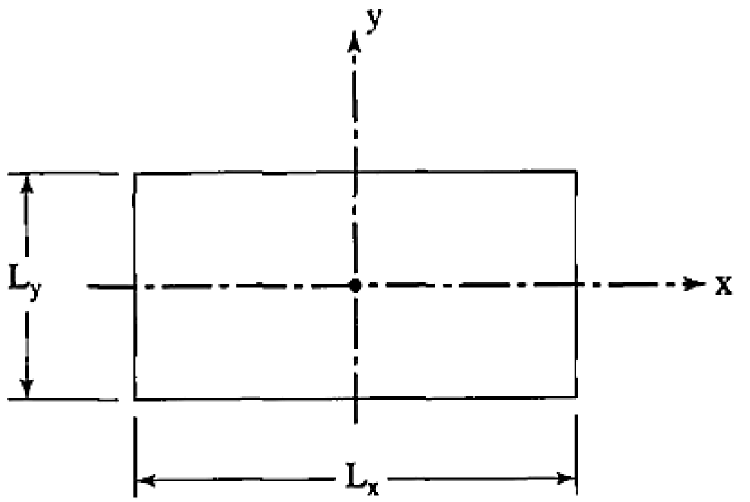

Figure 3.4. A finite slab.

➢ For finite slab as shown in Figure 3.4. finite slab with lengths Lx in the x direction and Ly in the y direction. The total stress in the x direction can be **expressed as shown in Equations 3.9 and finally in Equation 3.10:**

$$\sigma_x = \frac{C_x E\alpha_t \Delta t}{2(1-\mu^2)} + \frac{C_y \mu E\alpha_t \Delta t}{2(1-\mu^2)} \quad \ldots\ldots\ldots\ldots (3.9)$$

$$\sigma_x = \frac{E\alpha_t \Delta t}{2(1-\mu^2)}(C_x + \mu C_y) \quad \ldots\ldots\ldots\ldots (3.10)$$

➢ Note: C_x and C_y are correction factors for a finite slab. The first term in Eq.3.9 is the stress due to bending in the x direction and the second term is the stress due to bending in the y direction. **Equation 3.10 gives the maximum interior stress at the center of a slab.**

➢ Bradbury (1938) developed a simple chart for determining C_x and C_y, as shown in Figure 3.5. The correction factor C_x depends on L_x/ℓ and the correction factor C_y depends on L_y/ℓ, where ℓ is the radius of relative stiffness defined in Equation 3.11.

$$\ell = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.11)$$

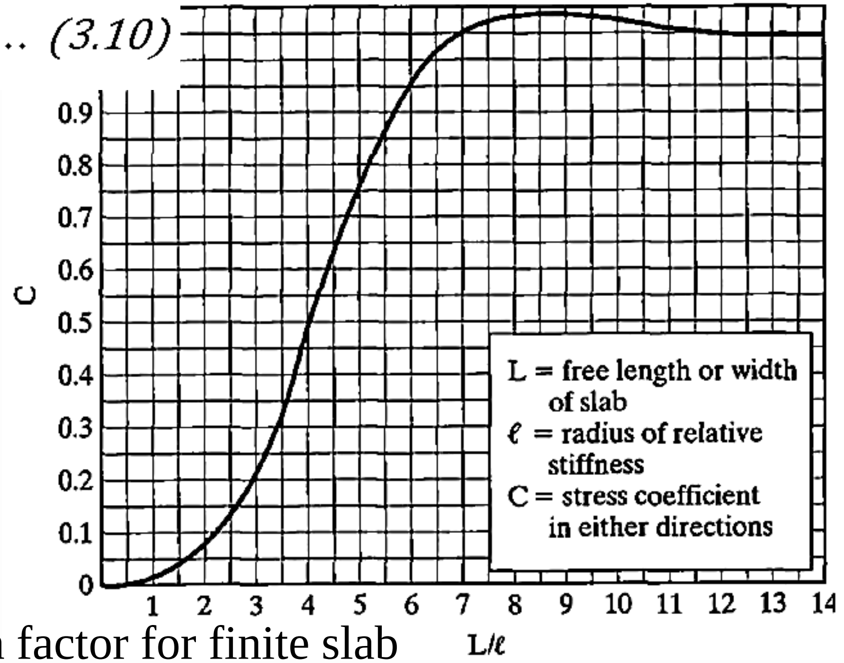

Fig. 3.5. Stress correction factor for finite slab

➢ The stress at the edge of the mid span of the slab can be determined by Equation 3.12.

$$\sigma = \frac{CE\alpha_t \Delta t}{2} \quad \dots\dots\dots\dots (3.12)$$

Note:

1. in which σ may be σ_x or σ_y depending on whether C is C_x or C_y.
2. Eq. 3.12 is the same as Eq. 3.10 when the Poisson ratio at the edge is taken as 0.

Example 1:

Figure 3.6 shows a concrete slab, 25 ft (7 .62 m) long, 12 ft (3 .66 m) wide, and 8 in. (203 mm) thick, subjected to a temperature differential of 20°F (11 .1°C). Assuming that k = 200 pci (54.2 MN/m3), E of concrete is 27.6 Gpa (4×10^6 psi) , μ of concrete is 0.15, and $\boldsymbol{\alpha_t} = 5 \times 10^{-6}$ in./in./°F (9×10^{-6} mm/mm/°C), determine the maximum curling stress in the interior and at the edge of the slab .

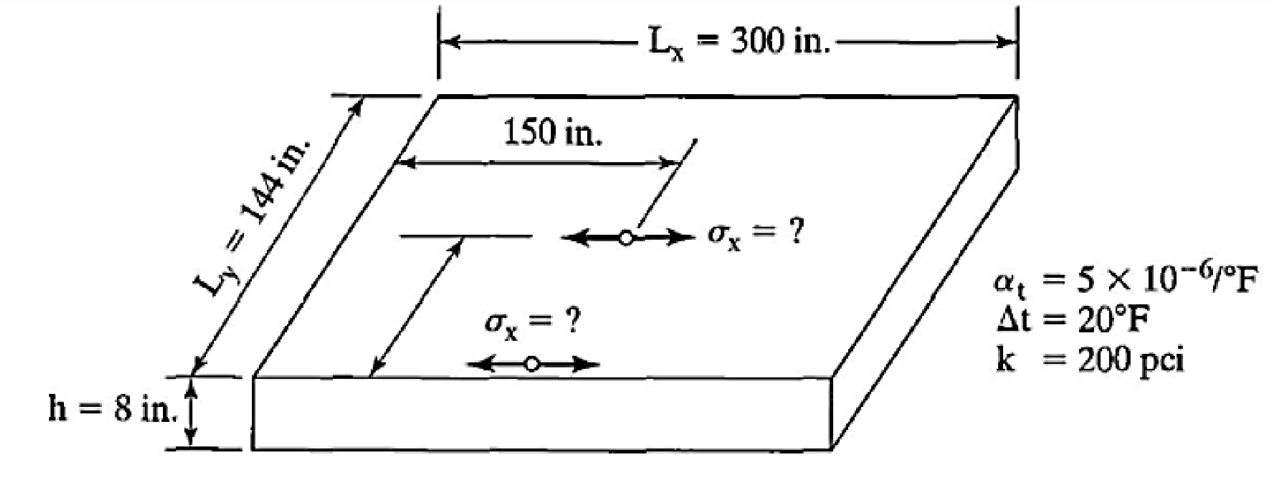

Figure 3.6. Example 1.

3.2.2 Due to Load (Westergard Method)

➢ The stresses caused by traffic loading on a concrete pavement slab depend on the location of the load, load configuration, slab thickness ***h***, modulus of subgrade reaction ***k***, radius of a circular contact area ***a***, magnitude of the wheel load ***P***, and radius of relative stiffness ℓ.

➢ There are three different types of stresses and deflections can be computed based on the locations of the load (contact area), i.e. interior loading, edge loading, and corner loading, on the slab, as shown in Figure 3.10.

➢ Corner stresses are associated with corner breaks, while edge and interior stresses are associated with mid-slab transverse cracking.

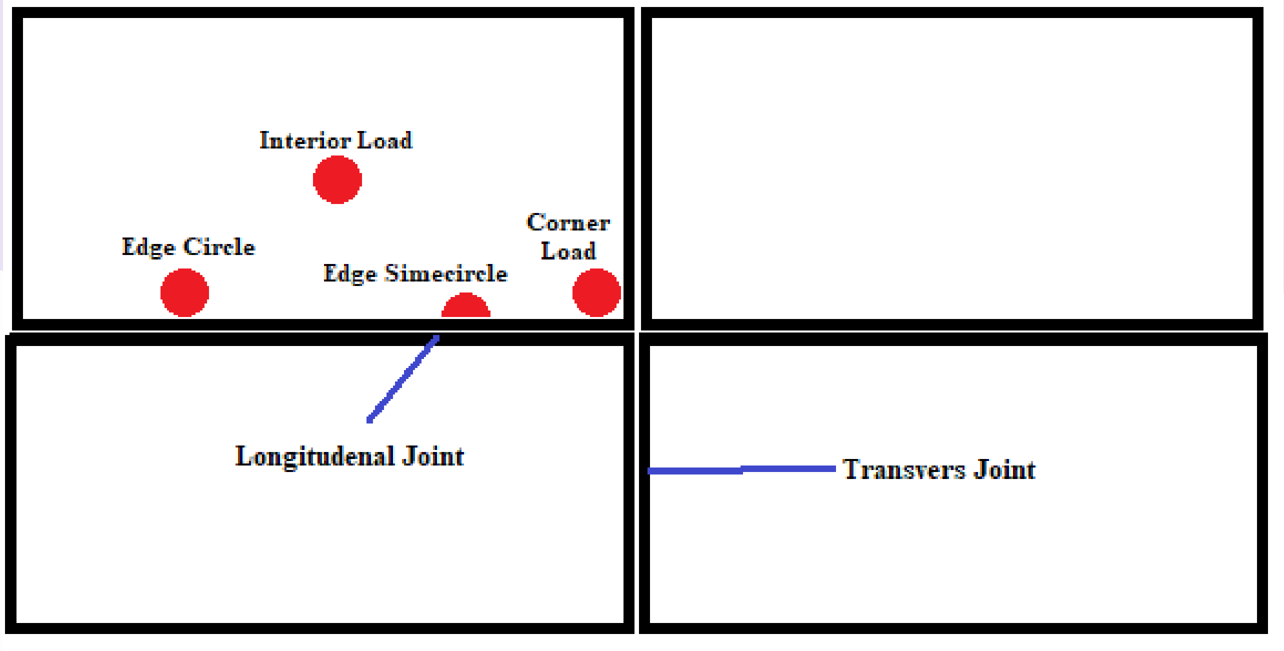

Figure 3.10. Corner, edge, and interior loading.

3.2.2.1. Stress and Deflection Due to Corner Load

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right] \dots\dots\dots\dots\dots\dots\dots\dots (3.13)$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right] \dots\dots\dots\dots\dots\dots\dots (3.14)$$

Where: σ_c & Δ_c are the stress and deflection at the corner respectively.

And the modified formulas are :

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{c}{\ell}\right)^{0.72}\right] \dots\dots\dots\dots\dots\dots (3.15)$$

$$\Delta_c = \frac{P}{k\ell^2}\left[1.205 - 0.69\left(\frac{c}{\ell}\right)\right] \dots\dots\dots (3-16)$$

c =1.772a

3.2.2.2. Stress and Deflection Due to interior Load

$$\sigma_i = \frac{3(1+\mu)P}{2\pi h^2}\left[ln\left(\frac{\ell}{b}\right) + 0.6159\right] \quad \text{-----------3.17}$$

For a Poisson ratio (μ) of 0.15 and in terms of base-10 logarithms, Eq . 3.17 can be written as Eq.3.18

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right] \quad \text{-----------3.18}$$

b = *a* when *a* ≥ *1.724h or* $b = \sqrt{1.6a^2 + h^2} - 0.675h$ when $a < 1.724h$ -----------3.19

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}. \quad \text{-----------3.20}$$

3.2.2.3. Stress and Deflection Due to edge Load

$$\underset{(\text{circle})}{\sigma_e} = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + \frac{1.18(1+2\mu)a}{\ell}\right] \quad \text{-----------3.21}$$

$$\underset{(\text{semicircle})}{\sigma_e} = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\ln\left(\frac{Eh^3}{100ka^4}\right) + 3.84 - \frac{4\mu}{3} + \frac{(1+2\mu)a}{2\ell}\right] \quad \text{-----------3.22}$$

$$\underset{(\text{circle})}{\Delta_e} = \frac{\sqrt{2+1.2\mu}P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.76+0.4\mu)a}{\ell}\right] \quad \text{-----------3.23}$$

$$\underset{(\text{semicircle})}{\Delta_e} = \frac{\sqrt{2+1.2\mu}P}{\sqrt{Eh^3k}}\left[1 - \frac{(0.323+0.17\mu)a}{\ell}\right] \quad \text{-----------3.24}$$

For μ = 0 .15, Eqs. 3.21 to 3.24 can be written as follow:

$$\sigma_{e\,(circle)} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right] \quad \text{-----------3.25}$$

$$\sigma_{e\,(semicircle)} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.282\left(\frac{a}{\ell}\right) + 0.650\right] \quad \text{-----------3.26}$$

$$\Delta_{e\,(circle)} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right] \quad \text{-----------3.27}$$

$$\Delta_{e\,(semicircle)} = \frac{0.431P}{k\ell^2}\left[1 - 0.349\left(\frac{a}{\ell}\right)\right] \quad \text{-----------3.28}$$

Example 2:

Figure 3.11 shows a concrete slab subjected to a edge loading. Given k = 100 pci (27 .2 MN/m^3), h = 10 in. (254 mm), a = 6 in. (152 mm), and P = 10,000lb (44.5 kN), determine the maximum stress and deflection due to edge loading.

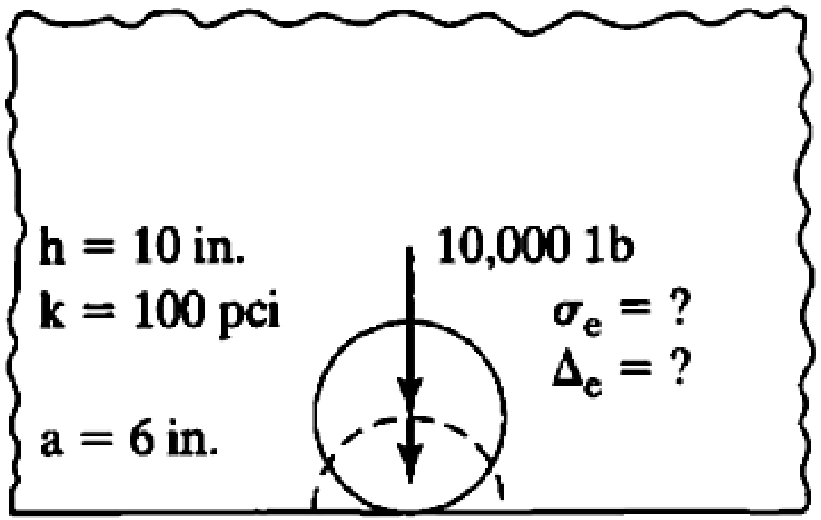

Figure 3.11. Example 2.

Solution:

For a circular area, from Eq . 3 .25,

$$\sigma_{e\ (circle)} = 0.803 \times 10{,}000/100 \times [4 \log(42.97/6) + 0.666(6/42.97) - 0.034] = 279.4 \text{ psi};$$

From Eq. 3.27

$$\Delta_{e(circle)} = 0.431 \times 10{,}000/(100 \times 1846.4) \times [1 - 0.82(6/42.97)] = 0.0207 \text{ in.}$$

For a semicircular area, from Eq . 3 .26,

$$\sigma_{e\ (simecircle)} = 0.803 \times 10{,}000/100 \times [4 \log(42.97/6) + 0.282(6/42.97) + 0.650] = 330.0 \text{ psi};$$

From Eq . 3.28,

$$\Delta_{e\ (simecircle)} = 0.431 \times 10{,}000/(100 \times 1846.4) \times [1 - 0.349(6/42.97)] = 0.0222 \text{ in.}$$

Note:

1. It can be seen that the maximum stress due to edge loading is greater than that due to corner and interior loadings and that the maximum deflection due to edge loading is greater than that due to interior loading but much smaller than that due to corner loading. The fact that both the stress and deflection are greater under a semicircular loaded area than those under a circular area is reasonable: The centered of a semicircle is closer to the pavement edge than is that of a circle.
2. Two examples (4.2 and 4.3 in Pages 155 to 157) from the text book by Yang are homework.

3.2.3. Effect of Dual Tires.

With the **exception of Eqs. 3.22, 3.24, 3.26 and 3.28 for a semicircular loaded area**, all of the closed-form formulas (**Westergard' formulas**) presented so far are based on a circular loaded area . When a load is applied over a set of dual tires, it is necessary to convert it into a circular area, so that the equations based on a circular loaded area can be applied.

So the radius of contact area can be find using Equation 3.29.

$$a = \sqrt{\frac{0.8521 P_d}{q\pi} + \frac{S_d}{\pi}\left(\frac{P_d}{0.5227q}\right)^{1/2}} \quad \ldots\ldots\ldots\ldots (3.29)$$

H.W. :Drive the Equation 3.29.

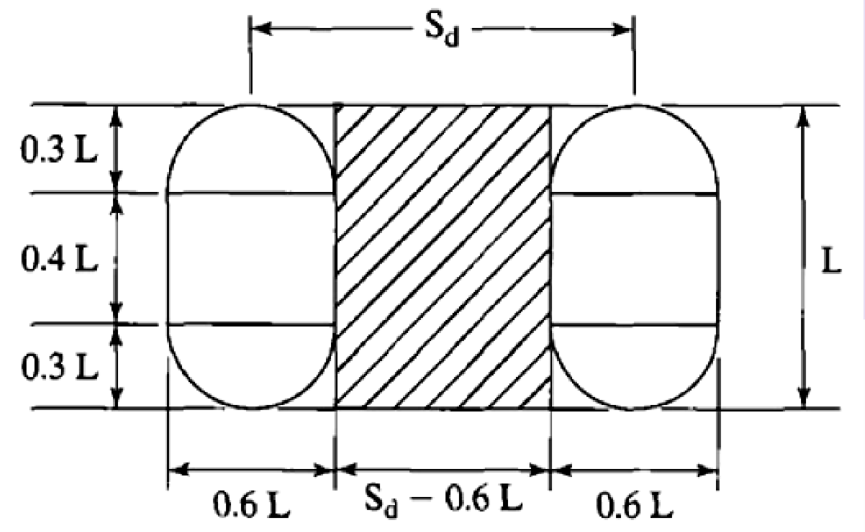

Figure 3.12.Method for converting duals into a circular area.

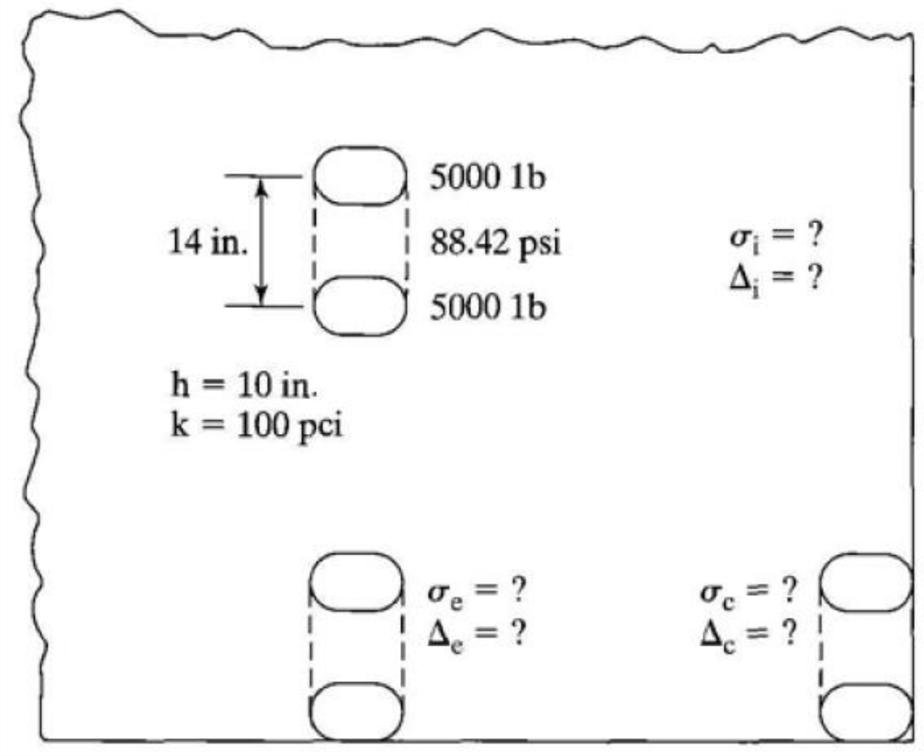

Figure 3.13. Example 3.

Example 3:

Using Westergard' formulas, determine the maximum stress in the interior, edge and corner if the 10,000-lb load is applied on a set of duals spaced at 14 in. on centers, as shown in Figure 3.13, instead of over a 6-in. circular area .

Solution:

With S_d = 14 in., q = 10,000/(36π) = 88.42 psi, and P_d = 5000 lb, from Eq . 3.29.

$$a = \sqrt{\frac{0.8521 \times 5000}{88.42\pi} + \frac{14}{\pi}\left(\frac{5000}{0.5227 \times 88.42}\right)^{1/2}} = 7.85 \text{ in. } (199 \text{ mm})$$

3.2.4. Stresses Due to Subgrade Friction.

- ➢ Frictional stresses between a concrete slab and the foundation (subgrade) can be determined using a simple mechanics approach. Consider a concrete slab subject to a decrease in temperature as shown in Figure 3.14. The concrete slab will tend to shrink toward the center of the slab from both ends. The frictional resistance between the concrete slab and the foundation will prevent the slab from moving and the stresses in the concrete will develop. The magnitude of the frictional forces developed depends on the relative movement between the slab and the foundation (subgrade). Frictional forces will range from zero at the center where no movement occurs to a maximum some distance away from the center where movement is fully mobilized as shown in Figure 3.14. **this movement is the criteria of limiting the length of the slab panel.**

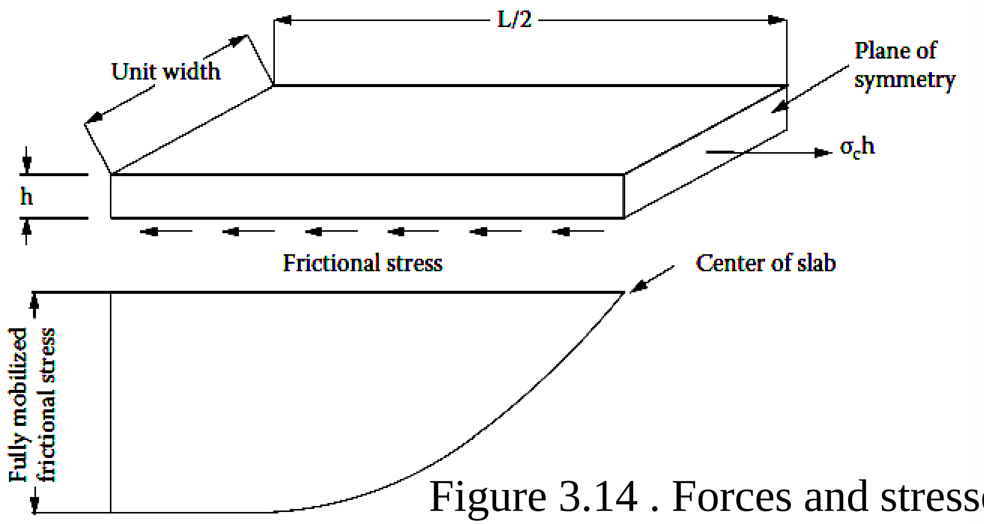

Figure 3.14 . Forces and stresses due to friction in a concrete slab.

- ➢ Tensile stress due to friction can be resisted based on the type of PCC. For **plain concrete pavements**, the spacing between **contraction joints** must be designed so that the stresses due to friction will eliminate the concrete to crack. For **longer joint spacings**, **steel reinforcements** must be provided to take care of the stresses caused by friction. The number of **tie bars** required is also controlled by the friction. On the other hand, Polyethylene sheet can be used underneath the PPC for reducing the frictional stresses.

- The maximum tensile stress in the concrete (**σ_c**) is achieved at the center of the slab and is determined by equating the frictional force per unit width (or **b**) of the slab **($\gamma_c Lbhf_a/2$)** to the tensile force **($\sigma_c hb$)** in the concrete. Thus, the stress in concrete can be determine by Equation 3.30. where: γ_c is density of concrete, f_a is coefficient of friction between slab and subgrade (**it is taken as 1.5**)

$$\sigma_c = \frac{\gamma_c L f_a}{2} \quad \ldots\ \ldots\ \ldots\ 3.30$$

Example 4:

Given a concrete pavement with a joint spacing of 25 ft and a coefficient of friction of 1.5, as shown in Figure 3.15. determine the stress in concrete due to friction .

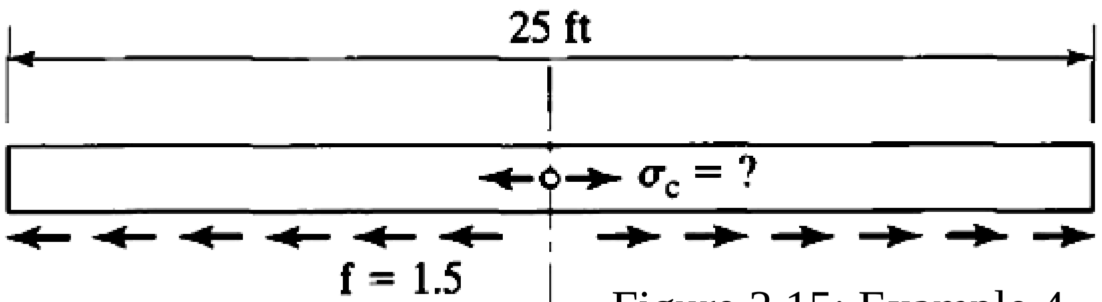

Figure 3.15: Example 4.

Solution :

With γ_c = 150 pcf = 0 .0868 pci , L = 25 ft = 300 in., and f_a = 1.5,

From Equation 3.30, σ_c = 0.0868 × 300 × 1 .5/2 = 19 .5 psi.

Note: The tensile strength of concrete ranges from $3\sqrt{f'_c}$ **to** $5\sqrt{f'_c}$ where f_c', is the compressive strength of cubic concrete specimens.

3.2.4.1. Joint Opening

The spacing of joints in plain concrete pavements depends more on the shrinkage characteristics of the concrete rather than on the stress in the concrete. Longer joint spacings cause the joint to open wider and decrease the efficiency of load transfer. Joint opening **ΔL** can be computed approximately by Equation 3.31.

$$\Delta L = CL(\alpha_t \Delta T + \epsilon) \quad \ldots\ \ldots\ \ldots\ \ldots\ 3.31$$

α_t is the coefficient of thermal expansion of concrete (5 to 6 × 10^{-6} /°F (9 to 10.8 × 10^{-6}/°C); **ϵ** is the drying shrinkage coefficient of concrete (0.5 to 2 .5 × 10^{-4}; **L** is the joint spacing or slab length ; **ΔT** is the temperature range, which is the temperature at placement (during casting) minus the lowest mean monthly temperature ; and **C** is the adjustment factor due to slab-subbase friction, (C = 0.65 for stabilized base and C = 0.8 for granular subbase).

Example 5:

Given ΔT = 60°F, a_t = 5.5×10^{-6}/°F, ϵ= 1.0×10^{-4}, C = 0.65, and the allowable joint openings for undoweled and doweled joints are 0.05 and 0.25 in. respectively, determine the maximum allowable joint spacing (slab length).

Solution:

From Equation 3.31: $\Delta L = CL(\alpha_t \Delta T + \epsilon)$

$L = \Delta L/[C(\alpha_t \Delta T + \epsilon)] = \Delta L/[0.65(5.5 \times 10^{-6} \times 60 + 0.0001)]$ **= ΔL/0.00028**

For the undoweled joint, L = 0.05/0 .00028 = 178.6 in. = 14 .9 ft (4.5 m).

For the doweled joint, L = 0 .25/0.00028 = 892 .9 in. = 74 .4 ft (22.7 m).

3.2.5. Steel Stress

- Steel is used in concrete pavements as three categories: **reinforcements**, **tie bars**, **and dowel bars**.
- Due to frictional stresses, PCC is reinforced in a longitudinal and transverse in addition to the tie bars.
- The design of dowels in transverse joints.

3.2.5.1.Reinforcements

- Wire fabric or bar mats may be used in concrete slabs for control of temperature cracking.
- These reinforcements do not increase the structural capacity of the slab but are used for two purposes: to increase the joint spacing (length of the panel) and to tie the cracked concrete together and maintain load transfers through aggregate interlock.
- When steel reinforcements are used, it is assumed that all tensile stresses are taken by the steel alone, (there is no effect for the concrete). From Figure 3.14.

$$A_s f_s = \frac{L}{2} \gamma_c h b f_a \text{ ....(3.31)}$$

$$A_s = \frac{L}{2} \frac{\gamma_c h b f_a}{f_s} \dots \dots \dots \dots \dots \dots . (3.32)$$

Where: A_s is the area of steel required per unit width (b =1)

f_s is the allowable stress in steel.

So Equation 3.32 become as shown in Equation 3.33.

$$A_s = \frac{L}{2} \frac{\gamma_c h f_a}{f_s} \dots \dots \dots \dots \dots \dots \dots \dots . (3.33)$$

Note: The steel is usually placed at the mid depth of the slab and discontinued at the joint. The amount of steel obtained from Equation 3.33 is at the center of the slab and can be reduced toward the end. However, in actual practice the same amount of steel is used throughout the length of the slab. The sizes and weights of different standard reinforcing bars are listed in Table 3.1.

Table 3.1. Weights and Dimensions of Standard Reinforcing Bars

Bar size designation	Weight (lb/ft)	Nominal dimensions, round sections: Diameter (in.)	Cross-sectional area ($in.^2$)	Perimeter (in.)
No. 3	0.376	0.375	0.11	1.178
No. 4	0.668	0.500	0.20	1.571
No. 5	1.043	0.625	0.31	1.963
No. 6	1.502	0.750	0.44	2.356
No. 7	2.044	0.875	0.60	2.749
No. 8	2.670	1.000	0.79	3.142
No. 9	3.400	1.128	1.00	3.544
No. 10	4.303	1.270	1.27	3.990
No. 11	5.313	1.410	1.56	4.430

Example 6:

Determine the wire fabric required for a two-lane concrete pavement, 8 in. thick, 60 ft long, 24 ft wide, f_s 43000 psi, γ_c is 0.0868 pci, and with a longitudinal joint at the center, as shown in Figure 3.16.

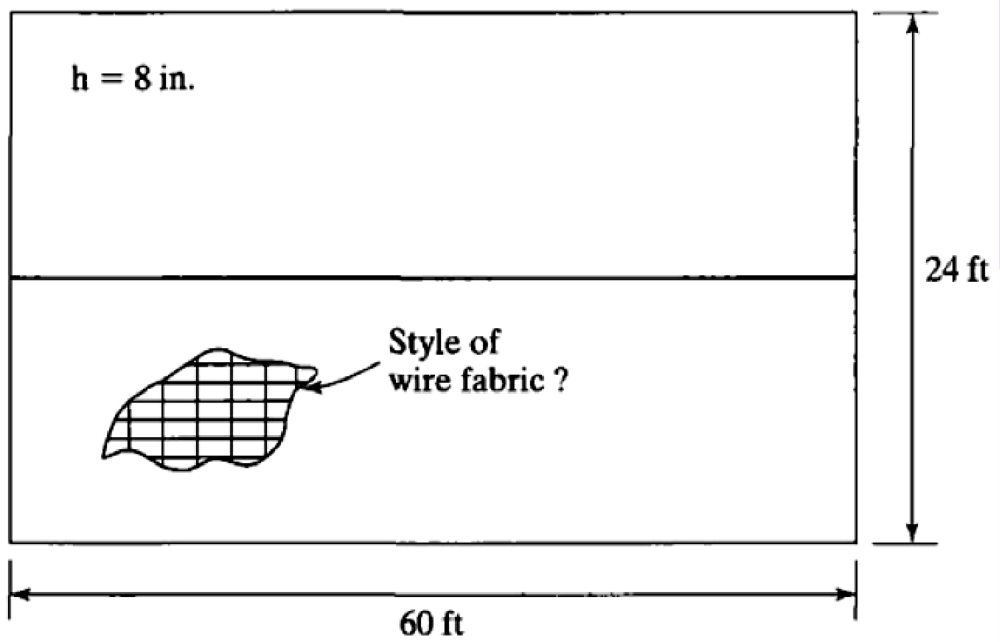

Figure 3.16. Example 6.

Solution:

from Eq. 3.33, the required longitudinal steel is :

A_S = 0.0868 × 8 × 720 × 1 .5/(2 x 43,000) = 0 .00872 in.^{2}/in . = 0 .105 in.^{2}/ft .

The required transverse steel is:

A_S = 0 .0868 × 8 × 24 × 12 X 1 .5/(2 x 43,000) = 0 .00349 In.^{2}/in. = 0 .042 in.^{2} /ft.

From Table 4.3, use 6 x 12 – W5 .5 X W4.5 with cross sectional areas of 0 .11 in.^{2} for longitudinal wires and 0 .045 in .^{2} for transverse wires .

Note: If the concrete pavement is used for a four-lane highway with all four slabs tied together at the three longitudinal joints, the transverse reinforcements in the two inside lanes should be doubled, because the length L in Eq . 3.33. should be 48 ft (14.6 m) instead of 24 ft (7 .3 m).

3.2.6. Tie-Bars.

Tie Bars are placed along the longitudinal joint to tie the two slabs together so that the joint will be tightly closed and the load transfer across the joint can be ensured. **The amount of steel required for tie bars can be determined in the same way as the longitudinal or transverse reinforcements by slightly modifying Equation 3.33 to be as shown in Equation 3.34.**

$$A_s = \frac{\gamma_c h L' f_a}{f_s} \quad \ldots\ldots\ldots\ldots\ 3.34$$

Where: A_S is the area of steel required per unit length of slab and L' is the distance from the longitudinal joint to the free edge where no tie bars exist

Notes:

- For two- or three lane highways, L' is the lane width (L_w).
- If tie bars are used in all three longitudinal joints of a four-lane highway, L' is treated as follow:
  - **L' = lane width (L_w)** equal to the lane width for the two outer joints adjacent to the free edges as shown in Figure 3.17.
  - **L' = $2L_w$** for the lane width for the inner joint as shown in Figure 3.17.

- The length of tie bars is governed by the allowable bond stress. For **deformed bars, an allowable bond stress of 350 psi (2400 kPa**) may be assumed. The length of bar should be based on the full strength of the bar, namely,

$$t = 2\left(\frac{A_1 f_s}{v \Sigma o}\right) \quad \ldots\ldots\ldots\ 3.35$$

Where: **t** is the length of the tie bar, ***v*** is \the allowable bond stress, $\mathbf{A_1}$ is the area of one bar, and **∑o** is the bar perimeter. For a given bar diameter **d**, $\mathbf{A_l = \pi\, d^2/4}$ and $\mathbf{\sum o = \pi d}$, so Eq. 3.35 can be simplified to be as Equation 3.36.

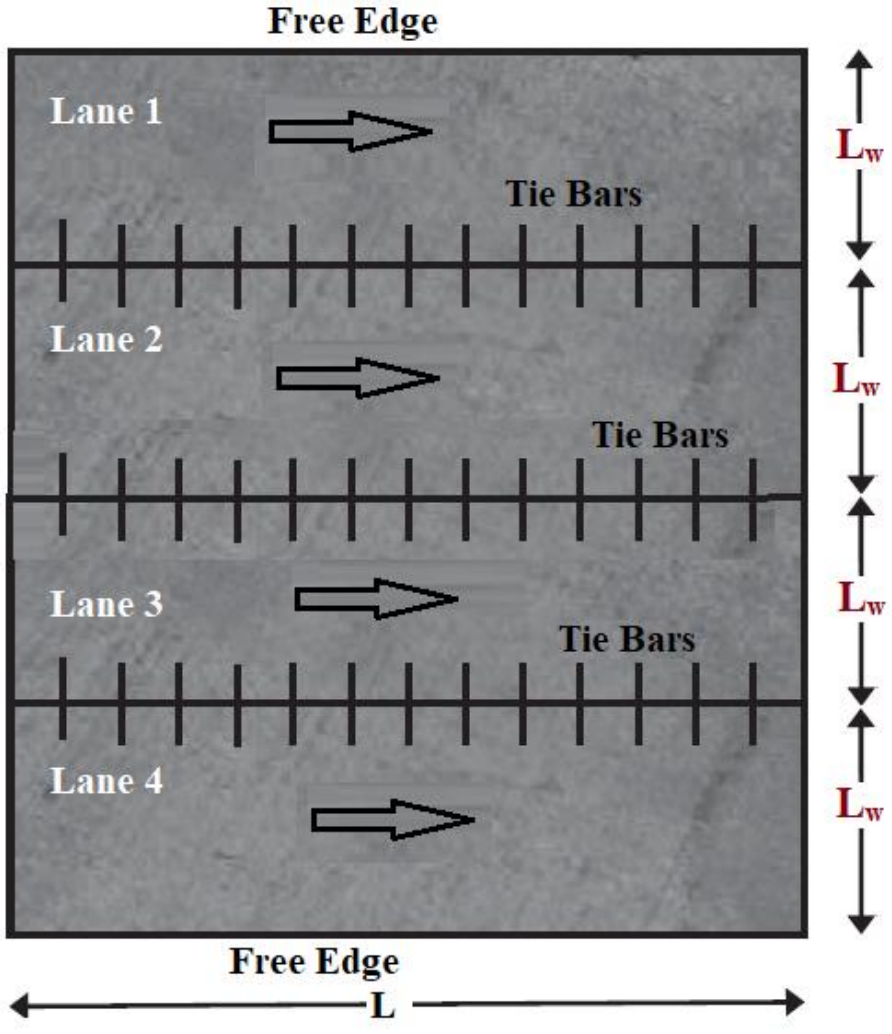

Figure 3.17. Four lanes highway.

$$t = \frac{1}{2}\left(\frac{f_s d}{v}\right) \quad \ldots\ldots\ldots\ 3.36$$

Note:

- The length **t** should be increased by **3 in. (76 mm)** for misalignment.
- It should be noted that many agencies use a standard tie-bar design to simplify the construction. Tie bars 0.5 in. (12.5 mm) in diameter by 36 in . (100 cm) long spaced at intervals of 30 to 40 in. (75 to 100 cm) are most commonly used.

Example 7:

Same pavement as Example 6. Determine the diameter, spacing, and length of the tie bars required, as shown in Figure 3.18.

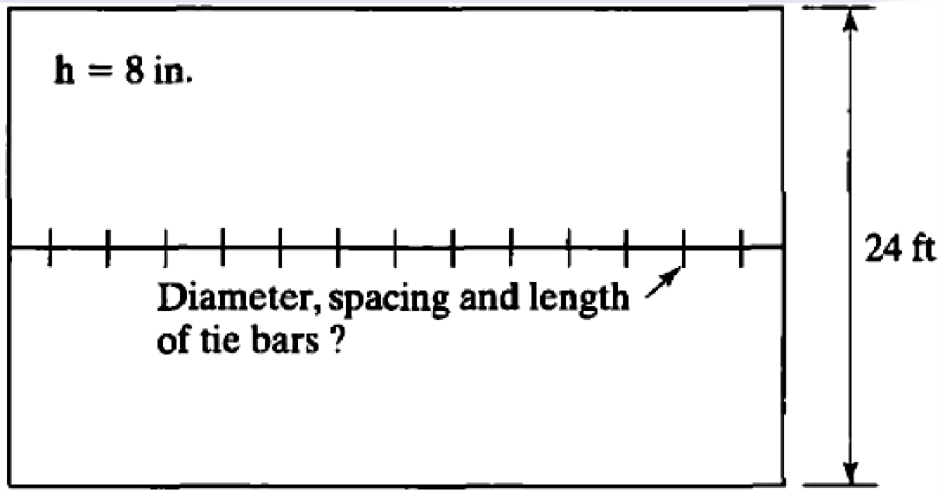

Figure 3.18. Example 7.

Solution:

Assume f_s = 27,000 psi (186 MPa).

With L' = 12 ft = 144 in. (3 .66 m), from Eq . 3.34,

A_S = 0 .0868 × 8 × 144 × 1 .5/27,000 = 0.00556 in.^{2}/in .

If No.4 (0 .5 in) bars are used, from Table 3.1., the cross-sectional area of one bar is 0.2 in.^{2}.

The spacing of the bar = 0.2/0.00556 = 36 in. (914 mm).

Assume that v = 350 psi , from Eq. 3.35, t = 0.5 × 27,000 × 0 .5/350 = 19 .3 in. (353 mm).

After adding 3 in. (76 mm), t = 19 .3 + 3 = 22.3 in . (use 24 in. or 610 mm) . The design selected is No. 4 deformed bars, 24 in . (610 mm) long and 3 ft (0 .9 m) on centers.

3.3. Design of Dowel-Bars

- The design of dowels and joints is mostly based on experience, although some theoretical methods on the design of dowels are available. The size of dowels to be used depends on the thickness of slab as shown in Table 3.2.
- It can be seen that the diameter of dowels is equal to one-eighth of the slab thickness (**d_{dowle} = $h_{thickness}$ /8**).
- It is recommended to use of **1.25 in . (32 mm) diameter dowels for highway pavements less than 10 in. (25.4 cm) thick** and **1 .5 in. (38 mm) diameter dowels for pavements 10 in. (25.4 cm) thick or greater.**
- **All dowels spaced at 12 in. (center to center).**
- Dowel bars are usually used across a transverse joint to transfer the loads to the adjoining slab. The stress and deflection at the joint are much smaller when the loads are carried by two slabs, instead of by one slab alone. The use of dowels can minimize faulting and pumping.

Table 3.2. Recommended Dowel Size and Length

Slab thickness (in.)	Dowel diameter (in.)	Dowel length (in.)
5	$\frac{5}{8}$	12
6	$\frac{3}{4}$	14
7	$\frac{7}{8}$	14
8	1	14
9	$1\frac{1}{8}$	16
10	$1\frac{1}{4}$	18
11	$1\frac{3}{8}$	18
12	$1\frac{1}{2}$	20

Allowable Bearing Stress (f_b)

Because concrete is much weaker than steel, the size and spacing of dowels required are **governed by the bearing stress between dowel and concrete**. The allowable bearing stress can be determined by Equation 3.37.

$$f_b = \left(\frac{4-d}{3}\right)f'_c \quad \text{… … … … … 3.37}$$

Where: f_b is the allowable bearing stress in psi,

d is the dowel diameter in inches, and f'_c is the ultimate compressive strength of concrete psi.

3.3.1. Bearing Stress on One Dowel

✓ If the load applied to one dowel is known, the maximum bearing stress can be determined theoretically by assuming the dowel to be a beam and the concrete to be a Winkler foundation.

✓ Using the original solution by Timoshenko (1940) indicated that the maximum deformation of concrete under the dowel, as shown in Figure 3.19. can be expressed by Equation 3.38.

$$y_o = \frac{P_t(2+\beta z)}{4\beta^3 E_d I_d} \quad \ldots\ldots\ldots\ldots\ldots 3.38$$

<u>Where</u>: y_o is the deformation of the dowel at the face of the joint, P_t is the load on one dowel, z is the joint width, E_d is Young's modulus of the dowel, I_d is the moment of inertia for dowel steel bar and β is the relative stiffness of a dowel embedded in concrete .

$$I_d = \frac{1}{64}\pi d^4 \ldots\ldots\ldots 3.39 \qquad \beta = \sqrt[4]{\frac{Kd}{4E_d I_d}} \quad \ldots\ldots\ldots 3.40$$

Also, **K** is the modulus of dowel support, which ranges from 300,000 to 1,500,000 pci. The bearing stress σ_b is proportional to the deformation, Equation 3.41 :

$$\sigma_b = Ky_0 = \frac{KP_t(2+\beta z)}{4\beta^3 E_d I_d} \quad \ldots\ldots\ldots 3.41$$

The bearing stress obtained from Eq. 3.41 (***σ_b for one dowel bar***) should be compared with the allowable bearing stress ***(f_b computed by Eq 3.37)***. **If the actual bearing stress is greater than allowable, then larger dowel bars or smaller dowel spacing should be used**.

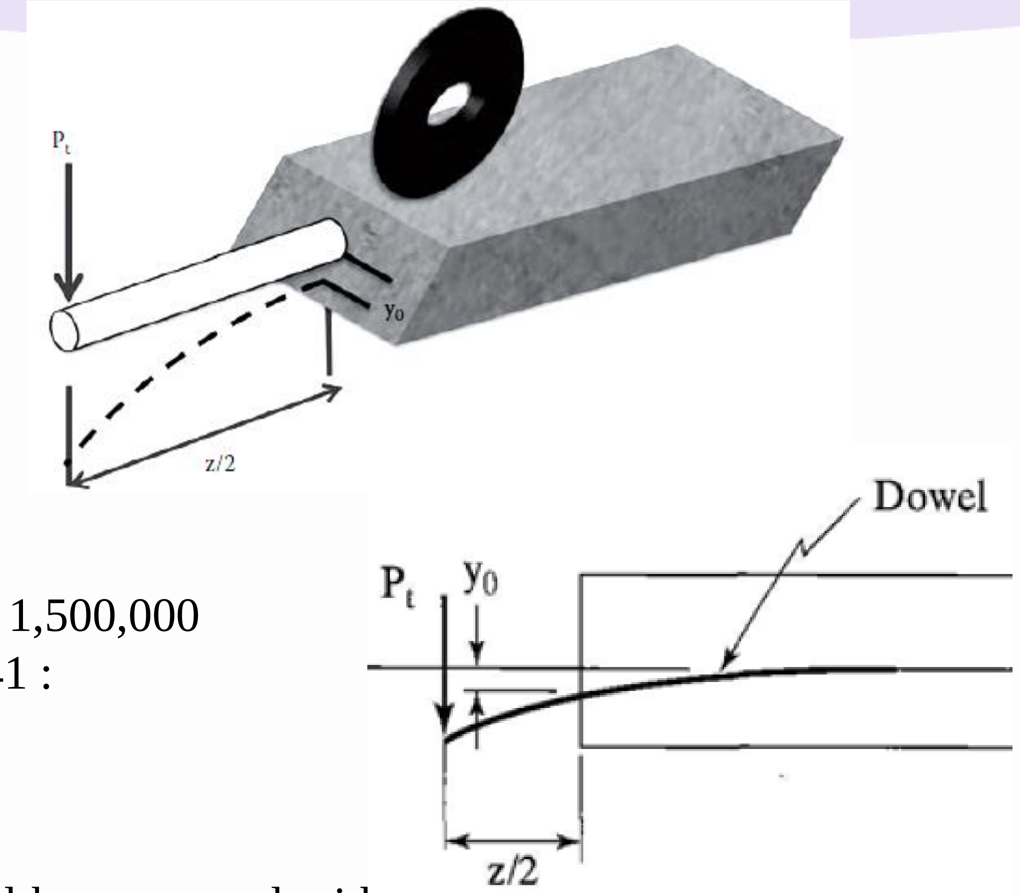

Figure 3.19. Dowel deformation under wheel load.

3.3.2. Dowel Group Action

- When a load W is applied on one slab near the joint, as shown in Figure 3.20. **part of the load will be transferred to the adjacent slab through the dowel group**. If the dowels are **100% efficient**, both slabs will **deflect the same amount and there active forces under both slabs will be the same, each equal to *0.5W***, which is also the total shear force transferred by the dowel group. If the dowels are **less than 100% efficient**, **as in the case of old pavements where some dowels become loose,** the reactive forces **under the loaded slab will be greater than *0.5W*, while those under the unloaded slab will be smaller than 0.5W.** As a result, the total shear force on the dowels is smaller than 0.5W. Therefore , the use of 0.5W for the design of dowels is **more conservative**.

- Based on Westergard' solutions, it was found that the maximum negative moment for both interior and edge loadings occurs at a distance of 1.8ℓ from the load, where ℓ is the radius of relative stiffness defined by Equation 3.11. When the moment is maximum, the shear force is equal to zero. It is therefore reasonable to assume that the shear in each dowel decreases inversely with the distance of the dowel from the point of loading, being maximum for the dowel under or nearest to the point of loading and zero at a distance of 1.8ℓ .

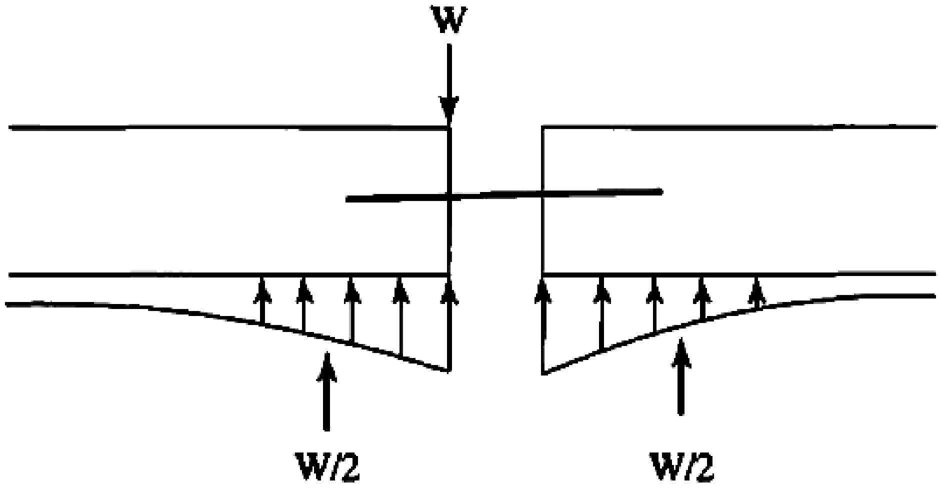

Figure 3.20. Load transfer through dowel group.

Example 8:

Figure 3.21. shows a concrete pavement 8 in. (203 mm) thick having a joint width of 0.2 in. (5.1 mm), a modulus of subgrade reaction of 100 pci (27 kN/m^3), and a modulus of dowel support of 1.5 x 10^6 pci (407 GN/m3). A load of 9000 lb (40 kN) is applied over the outermost dowel at a distance of 6 in. (152 mm) from the edge. The dowels are 3/4 in. (19 mm) in diameter and 12 in. (30 5 mm) on centers. Determine the maximum bearing stress between dowel and concrete , assume f'_c is 3000 psi.

Solution:

From Eq. 3.11.

$$\ell = [4 \times 10^6 \times 512/(12 \times 0.9775 \times 100)]^{0.25} = 36.35 \text{ in.}$$

If the dowel directly under the load is subjected to a shear force P_t, the forces on the dowels within a distance of 1.8ℓ , or 66 in. can be determined by assuming a straight-line variation, as shown in Figure 3.21.

The sum of the forces on all dowels is $3.27P_t$, which must be equal to one-half of the applied load based on 100% joint efficiency, or P t = 4500/3 .27 = 1376 lb (6.1 kN).

From Eq. 3.39. $I_d = \pi(0.75)^4/64 = 0.0155$ *in.*

From Eq. 3.40. $\beta = [1.5 \times 10^6 \times 0.75/(4 \times 29 \times 10^6 \times 0.0155)]^{0.25}$

$= 0.889$ in.

From Eq. 3.41, $\sigma_b = 1.5 \times 10^6 \times 1376 \times (2 + 0.889 \times 0.2)1(4 \times 0.703$ $29 \times 10^6 \times 0.0155) = 3556$ psi.

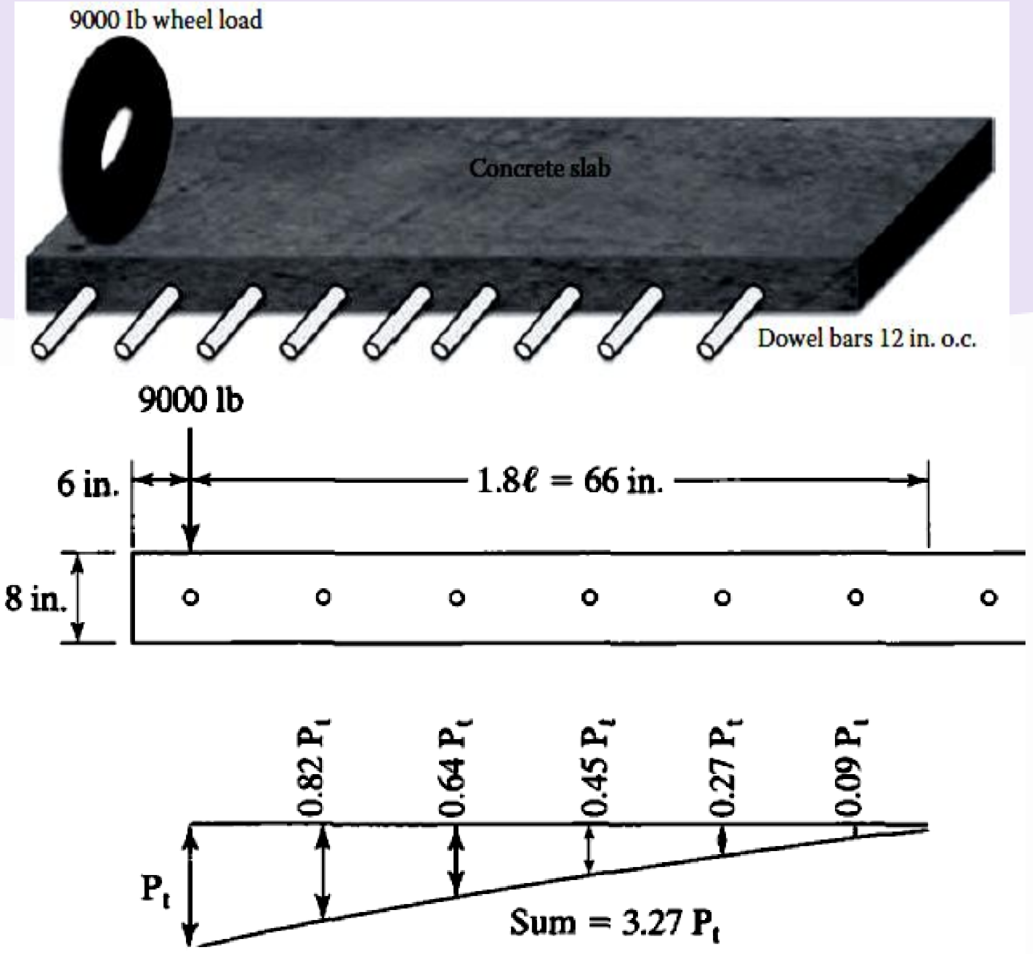

Figure 3.21. Example 8.

For a 3000 psi concrete, the allowable bearing stress obtained from Eq. 3.37 is:

$f_b = (4 - 0.75) \times 3000/3 = 3250$ psi .

Because the actual bearing stress is about 10% greater than the allowable, the design is not considered satisfactory .

Example 4.13 p. 175 is an important example in the text book of Yang.

3.4. Design of Joints

✓ Joints are typically used in common pavements such as JPCP, JRCP, and CRCP to allow for controlled concrete slab movement and cracking. **Transverse contraction joints are typically used in JPCP and JRCP**, which are usually doweled for heavily trafficked pavements.

✓ **Construction transverse or longitudinal joints** are joints between slabs that result when concrete is placed at different times, for example, at the end of the daily pavement construction operation, due to equipment breakdown, or during long delays.

✓ **Transverse expansion joints** are placed at specific locations to allow the pavement to expand without damaging adjacent structures such as bridges, drainage structures, or the pavement itself .

3.4.1. Contraction Joints

Figure 3.22 shows a conventional dummy-groove contraction joint. For this joint, a groove is cut or formed at the pavement surface to make certain that cracking will occur at this location. In Figure **3.22** the dummy-groove joint contains no dowel bar; load transfer is accomplished by grain interlock of the cracked lower portion of the slab.

Contraction joints are intended to relieve only tensile stresses resulting from contraction and warping of the concrete. The joint will not relieve expansion stresses. In some cases, load transfer from grain interlock may be questioned, and thus a dowel bar is placed across the joint. Dowel bars are generally of a standard design that varies from locale to locale. They are generally spaced at middepth of the slab. The dowel bar is intended to transfer the load across the joint, and, since the slabs move in relation to one another (in a longitudinal direction), it is necessary to lubricate at least one-half of the dowel bar to permit freedom of sliding.

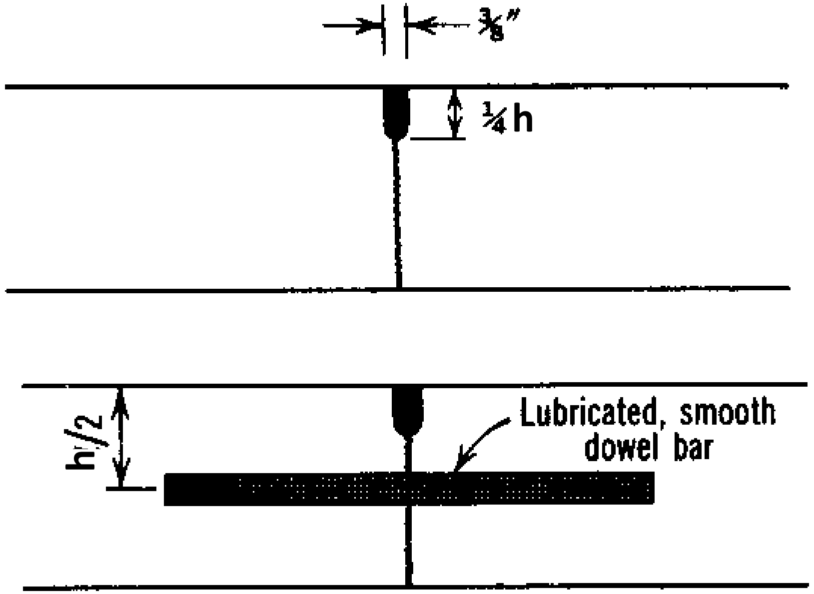

Figure 3.22. Contraction Joints

Note: the groove depth of contraction joints are generally between 1/4 – 1/3 the thickness of the slab and typically spaced every 3.1 – 15 m (12 – 50 ft.)

3.4.2. Expansion joints

An expansion joint is a joint placed at a specific location to allow the pavement to expand without damaging adjacent structures such as bridges, drainage, and utility structures or the pavement itself. As the pavement expands due to temperature and moisture changes, the expansion joints will tend to close over a period of several years. Typically, the width of an expansion joint is approximately 3/4 in. or more. Sealing material are used to fill the joint's opening. A special type of dowel assembly is used to transfer load across expansion joints. The special joint dowel system is fabricated with a cap on one end of each dowel to create a void in the slab to accommodate the dowel as the adjacent slab closes the expansion joint as shown in Figure 3.23.

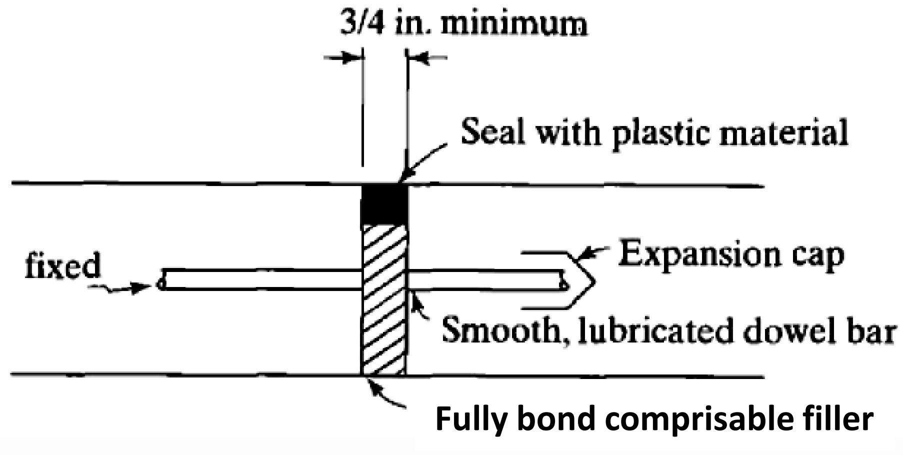

Figure 3.23. Expansion Joint

3.4.3. Construction joints

Construction joints are either transverse or longitudinal joints that result when concrete is placed at different times. A good practice is that transverse construction joints should be placed where a planned contraction joint should be located. Construction joints should not be skewed due to the difficulty in construction. Transverse construction joints should be doweled but not keyed.

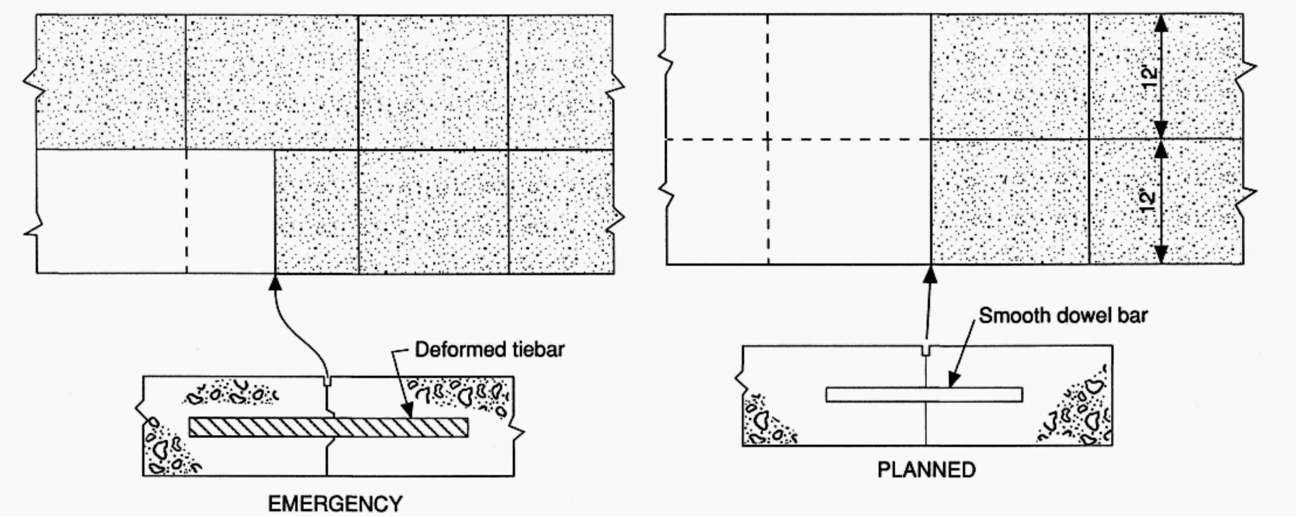

Figure 3.24. Planned and emergency construction joints for lane-at-a-time paving

3.4.4. Longitudinal Joints

Longitudinal joints are used in highway pavements to relieve curling and warping stresses. Different types of longitudinal joints are used, depending on whether the construction is full width or lane-at-a-time. In the full-width construction, as shown in Figure 3.25, the most convenient type is the dummy groove joint, in which tie bars are used to make certain that aggregate interlock is maintained.

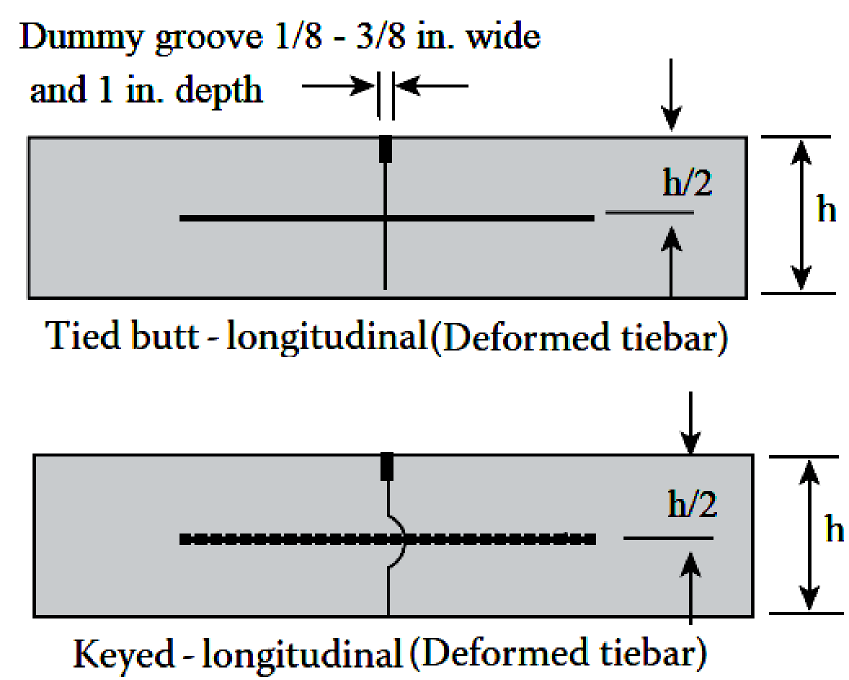

Figure 3.25. Longitudinal Joints.

Thank you for your attention