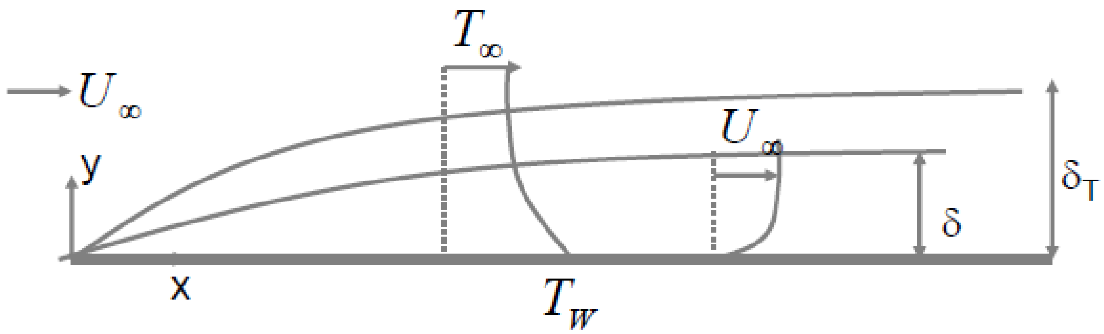
Lecture Twelve

Convection Heat Transfer

1-

Boundary layer equations (laminar flow)



- Equations for 2D, laminar, steady boundary layer flow

Conservation of mass: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

Conservation of x-momentum: $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)$

Conservation of energy: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)$

- Note: for a flat plate, U_∞ is constant, hence $\frac{dU_\infty}{dx}=0$

Exact solutions: Blasius

Boundary layer thickness $\frac{\delta}{x}=\frac{4.99}{\sqrt{\mathrm{Re}_x}}$

Skin friction coefficient $C_f=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}=\frac{0.664}{\sqrt{\mathrm{Re}_x}}$

$$\left(\mathrm{Re}_x = \frac{U_\infty x}{\nu}, \quad \tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \right)$$

Average drag coefficient $C_D = \frac{1}{L}\int_0^L C_f\,dx = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \quad \left(\mathrm{Re}_L = \frac{U_\infty L}{\nu} \right)$

Local Nusselt number $\quad Nu_x = 0.339\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$

Average Nusselt number $\quad \overline{Nu} = 0.678\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$

2-

Heat transfer coefficient

- Local heat transfer coefficient:

$$h_x = \frac{Nu_x k}{x} = \frac{0.339\,k\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{x}$$

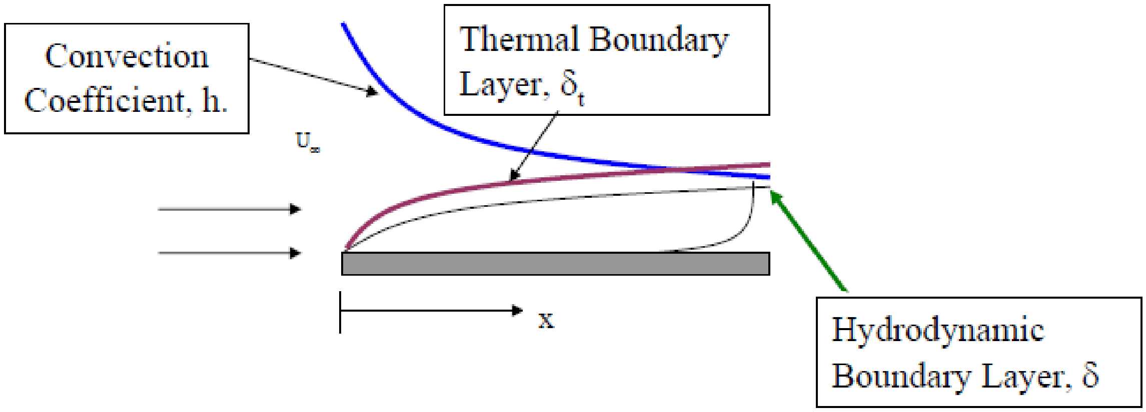
- Average heat transfer coefficient:

$$\bar{h} = \frac{\overline{Nu}\;k}{L} = \frac{0.678\,k\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}}{L}$$

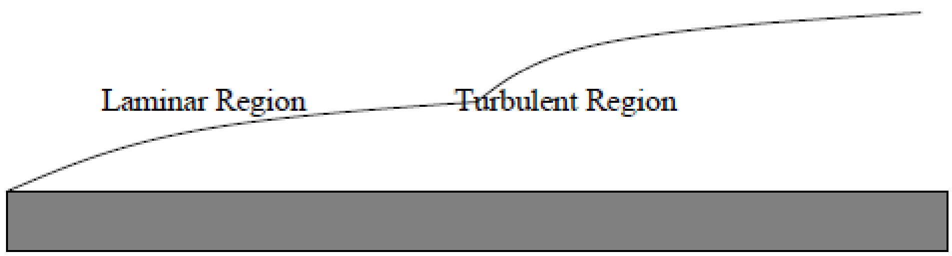
- Recall: $q_w = \bar{h}A(T_w - T_\infty)$, heat flow rate from wall

- Film temperature, T_{film}

For heated or cooled surfaces, the thermophysical properties within the boundary layer should be selected based on the average temperature of the wall and the free stream; $T_{film} = \frac{1}{2}(T_w + T_\infty)$



Laminar and turbulent b.l.



3-

Turbulent boundary layer

* Re_x increases with x. Beyond a critical value of Reynolds number ($Re_x = Re_{xc}$), the flow becomes transitional and eventually turbulent.

$$Re_{xc} = \frac{U_\infty x_c}{\nu} \quad \text{(For flow over flat plate, } x_c \approx 5\times10^5\text{)}$$

* Turbulent b.l. equations are similar to laminar ones, but infinitely more difficult to solve.

* We will mainly use correlations based on experimental data :

$$C_f = 0.059\,\mathrm{Re}_x^{-0.2} \qquad (\mathrm{Re}_x > 5\times10^5)$$

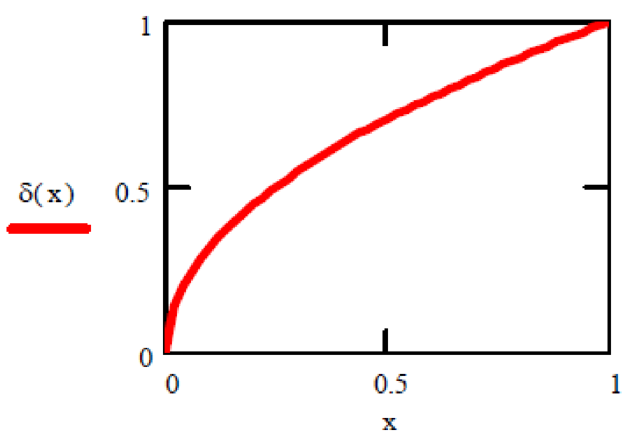
$$C_D = 0.072\,\mathrm{Re}_L - \frac{1}{\mathrm{Re}_L}\left(0.072\,\mathrm{Re}_{xc}^{0.8} - 1.328\,\mathrm{Re}_{xc}^{0.5}\right)$$

$$Nu_x = 0.029\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

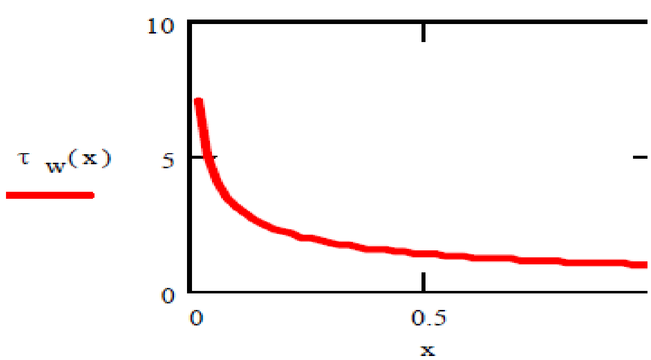
$$\overline{Nu} = 0.036\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} - \mathrm{Pr}^{1/3}\left(0.036\,\mathrm{Re}_{xc}^{0.8} - 0.664\,\mathrm{Re}_{xc}^{0.5}\right)$$

* Calculate heat transfer coefficient in usual way : $h = \dfrac{Nu\,k}{x}$ etc.

Laminar Boundary Layer Developmen



- Boundary layer growth: $\delta \propto \sqrt{x}$
- Initial growth is fast
- Growth rate $d\delta/dx \propto 1/\sqrt{x}$, decreasing downstream.



- Wall shear stress: $\tau_w \propto 1/\sqrt{x}$
- As the boundary layer grows, the wall shear stress decreases as the velocity gradient at the wall becomes less steep.

4-

Forced convection over exterior bodies

- Much more complicated.
- Some boundary layer may exist, but it is likely to be curved and U∞ will not be constant.
- Boundary layer may also separate from the wall.
- Correlations based on experimental data can be used for flow and heat transfer calculations
- Reynolds number should now be based on a characteristic diameter.

$$\mathrm{Re}_D = \frac{\rho U_\infty D}{\mu}$$

- If body is not circular, the equivalent diameter Dh is used

$$D_h = \frac{4 \times \text{Area}}{\text{Perimeter}}$$

$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U_\infty^2 A_{normal}} \quad ; \quad \overline{N}_u = \frac{\overline{h} D}{k} \quad ; \quad \overline{h} = \frac{\overline{N}_u k}{D}$$

Flow over circular cylinders

$$\overline{\mathrm{Nu}} = C\,\mathrm{Re}_D^m \frac{\mathrm{Pr}^{.62}}{\mathrm{Pr}_s^{.25}}$$

Re_D	C	m
1 – 40	0.75	0.4
40 - 10^3	0.51	0.5
10^3 - 2×10^5	0.26	0.6
2×10^5 - 10^6	0.08	0.7

All properties at free stream temperature, Pr_s at cylinder surface temperature