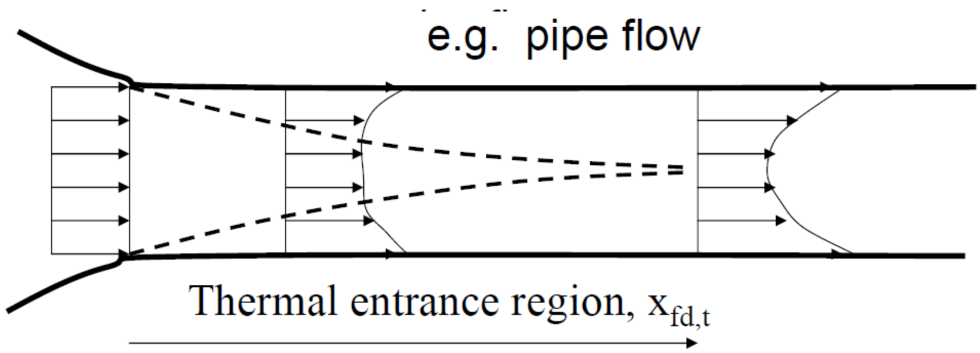
Lecture Thirteen

Forced Convection (Internal Flow)

1- Thermal Conditions.

Laminar or turbulent
entrance flow and fully developed thermal condition

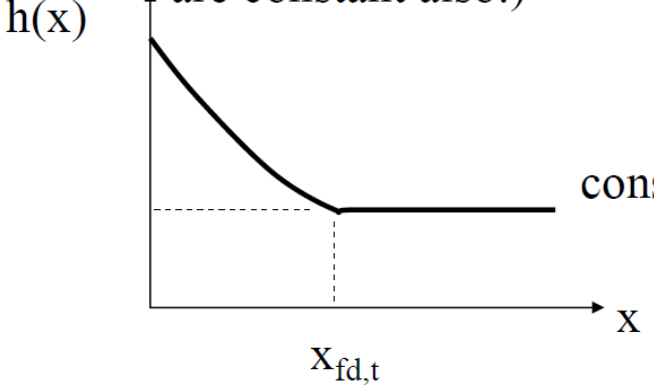


For laminar flows the thermal entrance length is a function of the Reynolds number and the Prandtl number: $x_{fd,t}/D \approx 0.05Re_D Pr$, where the Prandtl number is defined as $Pr = \nu/\alpha$ and α is the thermal diffusivity.

For turbulent flow, $x_{fd,t} \approx 10D$.

2- The convection coefficient.

- For a fully developed pipe flow, the convection coefficient is a constant and is not varied along the pipe length. (as long as all thermal and flow properties are constant also.)



3- Energy Transfer.

• Newton's law of cooling: $q''_S = hA(T_S - T_m)$
Question: since the temperature inside a pipe flow is not constant, what temperature we should use. A mean temperature T_m is defined.

Consider the total thermal energy carried by the fluid as

$$\int_A \rho V C_v T dA = \text{(mass flux) (internal energy)}$$

Now image this same amount of energy is carried by a body of fluid with the same mass flow rate but at a uniform mean temperature T_m. Therefore T_m can be defined as

$$T_m = \frac{\int_A \rho V C_v T dA}{\dot{m} C_v}$$

Consider T_m as the reference temperature of the fluid so that the total heat transfer between the pipe and the fluid is governed by the Newton's cooling law as: $q_s'' = h(T_s - T_m)$, where h is the local convection coefficient, and T_s is the local surface temperature. Note: usually T_m is not a constant and it varies along the pipe depending on the condition of the heat transfer.

4- Energy Balance Equation.

Example: We would like to design a solar water heater that can heat up the water temperature from 20° C to 50° C at a water flow rate of 0.15 kg/s. The water is flowing through a 5 cm diameter pipe and is receiving a net solar radiation flux of 200 W per unit length (meter). Determine the total pipe length required to achieve the goal.

a- How do we determine the heat transfer coefficient, h?

b- How can we determine the required pipe length?

There are a total of six parameters involving in this problem: h, V, D, ν, k_f, c_p. The last two variables are thermal conductivity and the specific heat of the water. The temperature dependence is implicit and is only through the variation of thermal properties. Density ρ is included in the kinematic viscosity, $\nu=\mu/\rho$. According to the Buckingham theorem, it is possible for us to reduce the number of parameters by three. Therefore, the convection coefficient relationship can be reduced to a function of only three variables:

$Nu=hD/k_f$, Nusselt number, $Re=VD/\nu$, Reynolds number, and $Pr=\nu/\alpha$, Prandtl number.

This conclusion is consistent with empirical observation, that is $Nu=f(Re, Pr)$. If we can determine the Reynolds and the Prandtl numbers, we can find the Nusselt number, hence, the heat transfer coefficient, h.

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5- Convection Correlations.

a- Laminar, fully developed circular pipe flow:

$$Nu_D = \frac{hD}{k_f} = 4.36, \quad \text{when } q_s'' = \text{constant},$$

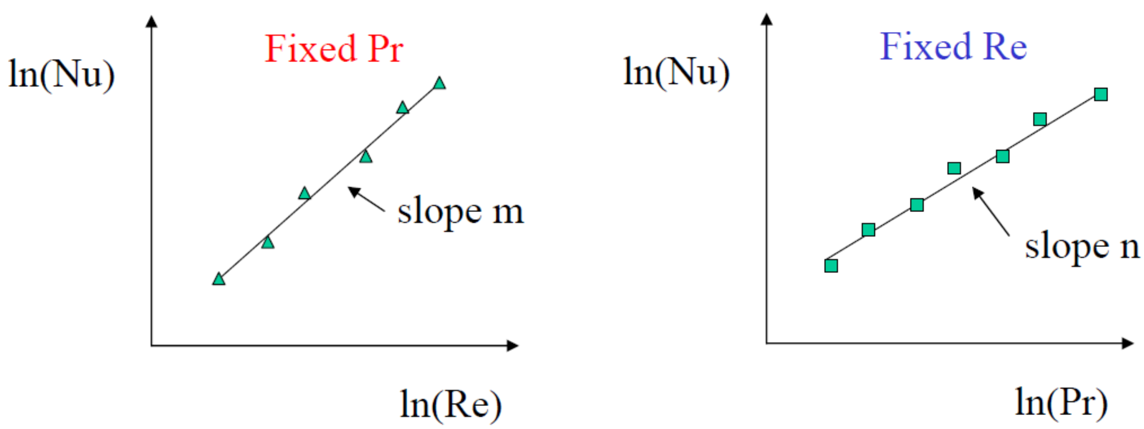
$$Nu_D = 3.66, \quad \text{when } T_s = \text{constant}$$

Note: the therma conductivity should be calculated at T_m.

b-

⇒ Fully developed, turbulent pipe flow: Nu = f(Re, Pr),

Nu can be related to Re & Pr experimentally, as shown.



6- Empirical Correlations.

Dittus-Boelter equation: $Nu_D = 0.023 Re^{4/5} Pr^n$,

where n = 0.4 for heating ($T_s > T_m$), n = 0.3 for cooling ($T_s < T_m$).

The range of validity: $0.7 \le Pr \le 160$, $Re_D \ge 10,000$, $L/D \ge 10$.

Note: This equation can be used only for moderate temperature difference with all the properties evaluated at T_m.

Other more accurate correlation equations can be found in other references.
Caution: The ranges of application for these correlations can be quite different.

For example, the Gnielinski correlation is the most accurate among all these equations:

$$Nu_D = \frac{(f/8)(Re_D - 1000)\,Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

It is valid for $0.5 < Pr < 2000$ and $3000 < Re_D < 5 \times 10^6$.
All properties are calculated at T_m.