Lecture Fourteen

Energy Balance Equation (Internal Flow)

1- Example.

Example: We would like to design a solar water heater that can heat up the water temperature from 20° C to 50° C at a water flow rate of 0.15 kg/s. The water is flowing through a 5 cm diameter pipe and is receiving a net solar radiation flux of 200 W per unit length (meter). Determine the total pipe length required to achieve the goal.

a- **How do we determine the heat transfer coefficient, h?**

b- How can we determine the required pipe length?

In our example, we need to first calculate the Reynolds number: water at 35°C, Cp=4.18(kJ/kg.K), $\mu=7\times10^{-4}$ (N.s/m^2), k_f=0.626 (W/m.K), Pr=4.8.

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{\dot{m}/_A D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.15)}{\pi(0.05)(7\times 10^{-4})} = 5460$$

Re > 4000, it is turbulent pipe flow.

Use the Gnielinski correlation, from the Moody chart, f = 0.036, Pr = 4.8

$$\mathrm{Nu_D} = \frac{(f/8)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1+12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3}-1)} = \frac{(0.036/8)(5460-1000)(4.8)}{1+12.7(0.036/8)^{1/2}(4.8^{2/3}-1)} = 37.4$$

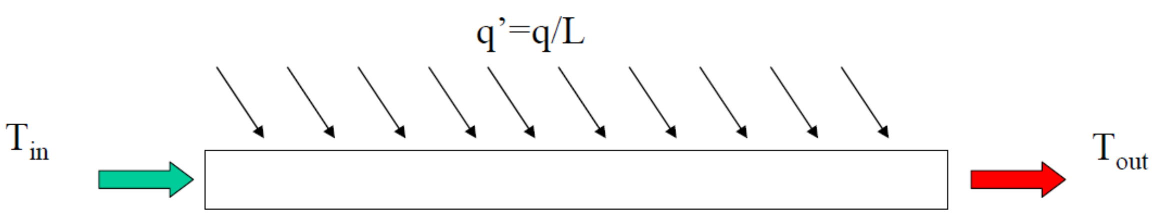
$$h = \frac{k_f}{D} Nu_D = \frac{0.626}{0.05}(37.4) = 469(W/m^2.K)$$

How can we determine the required pipe length?

Use energy balance concept: (energy storage) = (energy in) minus (energy out)
energy in = energy received during a steady state operation (assume no loss)

$$q'(L) = \dot{m}C_P(T_{out} - T_{in}),$$

$$L = \frac{\dot{m}C_P(T_{in} - T_{out})}{q'} = \frac{(0.15)(4180)(50-20)}{200} = 94(m)$$



2- Temperature Distribution.

a-

Can we determine the water temperature variation along the pipe?

Recognize the fact that the energy balance equation is valid for any pipe length x:

$$q'(x) = \dot{m}C_P(T(x) - T_{in})$$

$$T(x) = T_{in} + \frac{q'}{\dot{m}C_P}x = 20 + \frac{200}{(0.15)(4180)}x = 20 + 0.319x$$

It is a linear distribution along the pipe

b-

How about the surface temperature distribution?

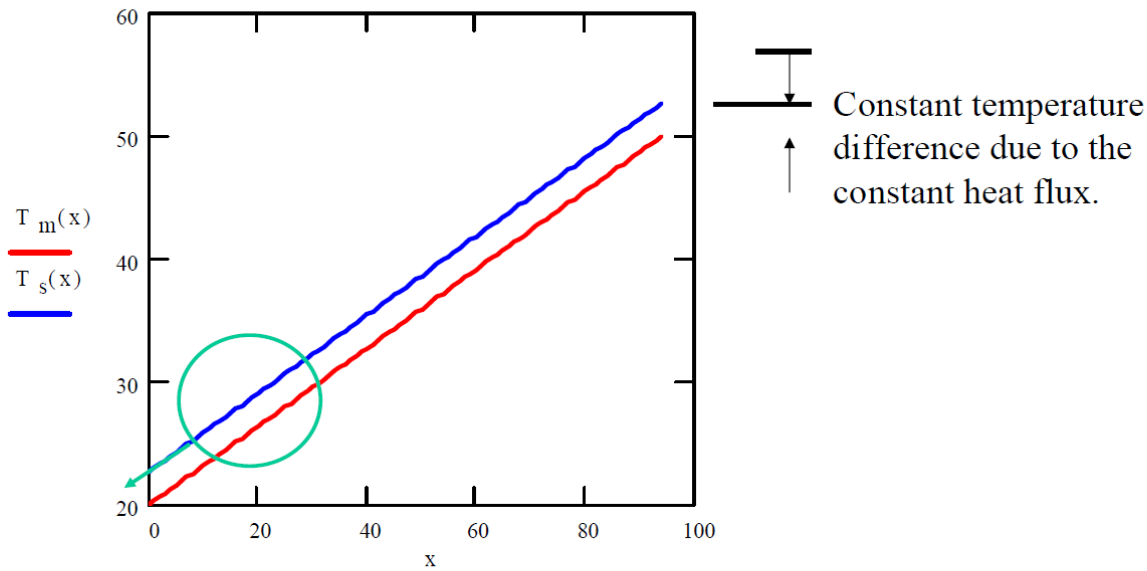
From local Newton's cooling law:

$$q = hA(T_s - T_m) \Rightarrow q'\Delta x = h(\pi D \Delta x)(T_s(x) - T_m(x))$$

$$T_s(x) = \frac{q'}{\pi D h} + T_m(x) = \frac{200}{\pi(0.05)(469)} + 20 + 0.319x = 22.7 + 0.319x \quad (°C)$$

At the end of the pipe, $T_s(x = 94) = 52.7(°C)$

Temperature variation for constant heat flux

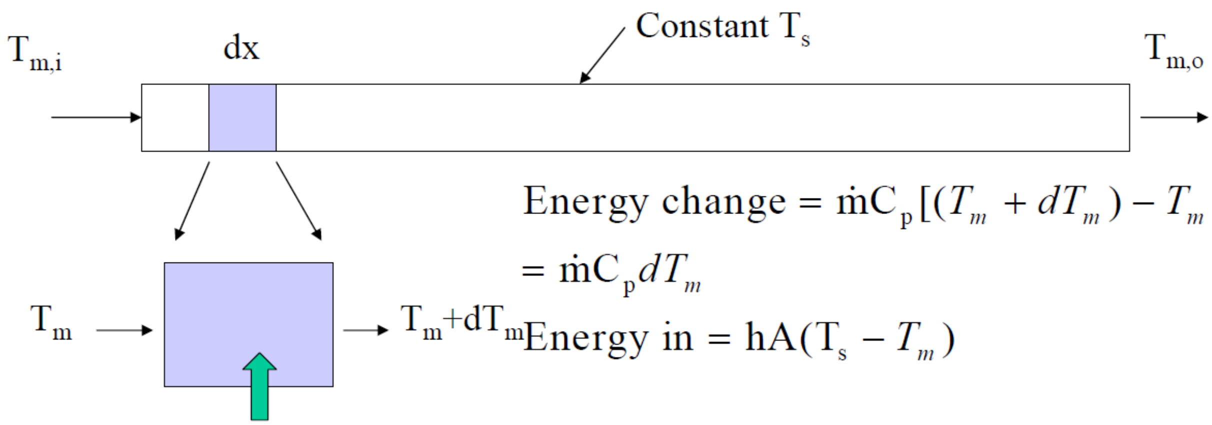


Note: These distributions are valid only in the fully developed region. In the entrance region, the convection condition should be different. In general, the entrance length x/D≈10 for a turbulent pipe flow and is usually negligible as compared to the total pipe length.

3-

Internal Flow Convection
-constant surface temperature case

Another commonly encountered internal convection condition is when the surface temperature of the pipe is a constant. The temperature distribution in this case is drastically different from that of a constant heat flux case. Consider the following pipe flow configuration:



$$\text{Energy change} = \dot{m}C_p[(T_m + dT_m) - T_m]$$

$$= \dot{m}C_p dT_m$$

$$\text{Energy in} = hA(T_s - T_m)$$

$$q_s = hA(T_s - T_m)$$

$$\text{Energy change} = \text{energy in}$$

$$\dot{m}C_p dT_m = hA(T_s - T_m)$$

a-

Temperature distribution

$$\dot{m}C_p dT_m = hA(T_s - T_m),$$

Note: $q = hA(T_s - T_m)$ is valid locally only, since T_m is not a constant

$$\frac{dT_m}{(T_m - T_s)} = -\frac{hA}{\dot{m}C_P},$$ where $A = Pdx$, and P is the perimeter of the pipe

Integrate from the inlet to a diatance x downstream:

$$\int_{T_{m,i}}^{T_m(x)} \frac{dT_m}{(T_m - T_s)} = -\int_0^x \frac{hP}{\dot{m}C_P} dx = -\frac{P}{\dot{m}C_P}\int_0^x h dx$$

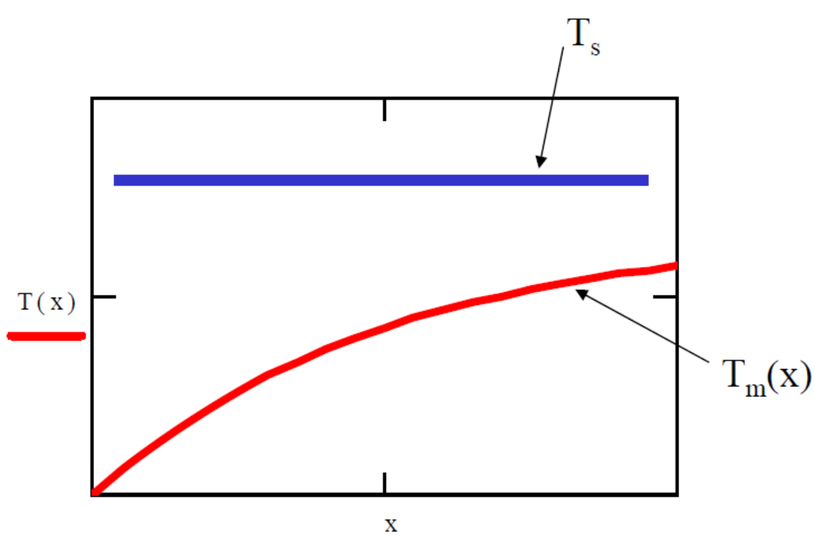
$\ln(T_m - T_s)\Big|_{T_{m,i}}^{T_m(x)} = -\frac{P\bar{h}}{\dot{m}C_P}x,$ where L is the total pipe length

and $\bar{h}$ is the averaged convection coefficient of the pipe between 0 & x.

$$\bar{h} = \frac{1}{x}\int_0^x h dx, \qquad \text{or} \int_0^x h dx = \bar{h}x$$

$$\frac{T_m(x) - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{P\bar{h}}{\dot{m}C_P}x\right),$$ for constant surface temperature

Constant surface temperature



The difference between the averaged fluid temperature and the surface temperature decreases exponentially further downstream along the pipe.

4-

Log-Mean Temperature Difference

For the entire pipe:

$$\frac{T_{m,o}-T_s}{T_{m,i}-T_s}=\frac{\Delta T_o}{\Delta T_i}=\exp\left(-\frac{\bar{h}(PL)}{\dot{m}C_P}\right) \quad \text{or } \dot{m}C_P=-\frac{\bar{h}A_s}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$q=\dot{m}C_P(T_{m,o}-T_{m,i})=\dot{m}C_P((T_s-T_{m,i})-(T_s-T_{m,o}))$$

$$=\dot{m}C_P(\Delta T_i-\Delta T_o)=\bar{h}A_s\frac{\Delta T_o-\Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}=\bar{h}A_s\Delta T_{lm}$$

where $\Delta T_{lm}=\dfrac{\Delta T_o-\Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$ is called the log mean temperature difference.

This relation is valid for the entire pipe.