CHAPTER 1 STRESS

1. INTRODUCTION

Mechanics of materials is a branch of mechanics that studies the internal effects of strain and stress in a solid body that is subjected to an external loading.

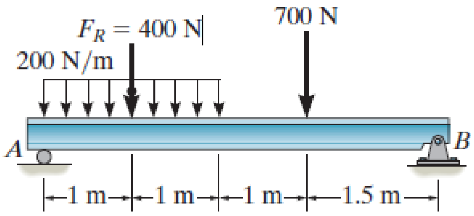
Stress is associated with the strength of the material from which the body is made.

Strain is a measure of the deformation of the body.

Mechanics of materials also studies the **stability** of a body such as a column when is subjected to compression.

2. EQUILIBRIUM OF A DEFORMED BODY

Loads. A body can be subjected to both surface loads and body forces. ***Surface loads*** that act on a small area of contact are reported by ***concentrated forces***, while ***distributed loadings*** act over a larger surface area of the body. When the loading is coplanar, as in Fig. 1–1*a*, then a resultant force $\mathbf{F}_R$ of a distributed loading is equal to the area under the distributed loading diagram, and this resultant acts through the geometric center or centroid of this area.



(a)

Fig. 1–1

A body force is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth’s gravitation or its electromagnetic field. Although these forces affect all the particles composing the body, they are normally represented by a single concentrated force acting on the body. In the case of gravitation, this force is called the ***weight W*** of the body and acts through the body’s center of gravity.

Support Reactions. For bodies subjected to coplanar force systems, the supports most commonly encountered are shown in Table 1–1. As a general rule, ***if the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member.*** For example, the roller support only prevents translation perpendicular or normal to the surface. Hence, the roller exerts a normal force F on the member at its point of contact. Since the member can freely rotate about the roller, a couple moment cannot be developed on the member.

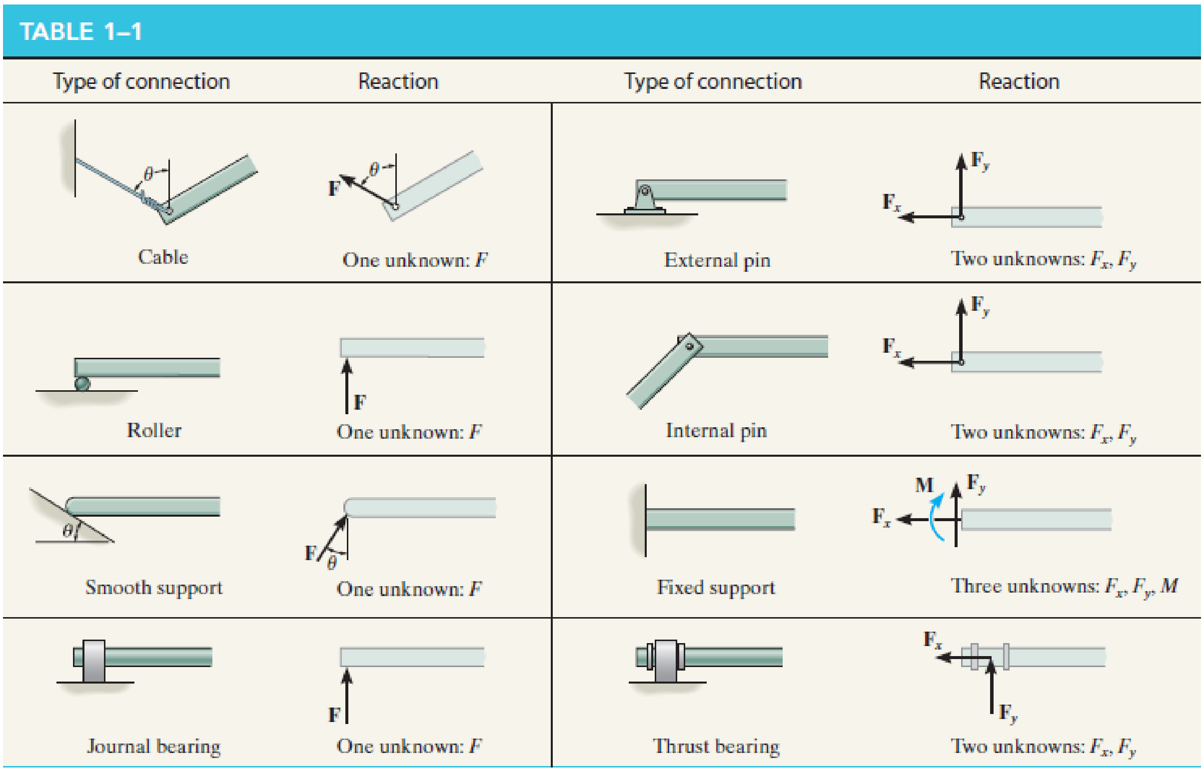


TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
Cable	One unknown: F	External pin	Two unknowns: F_x, F_y
Roller	One unknown: F	Internal pin	Two unknowns: F_x, F_y
Smooth support	One unknown: F	Fixed support	Three unknowns: F_x, F_y, M
Journal bearing	One unknown: F	Thrust bearing	Two unknowns: F_x, F_y

Equations of Equilibrium. Equilibrium of a body requires both a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments***, to prevent the body from rotating.

$$\Sigma \mathbf{F} = \mathbf{0} \qquad \Sigma \mathbf{M}_O = \mathbf{0}$$

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$
$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_O = 0$$

Internal Resultant Loadings. In mechanics of materials, statics is primarily used to determine the resultant loadings that act within a body. This is done using the ***method of sections.*** For example, consider the body shown in Fig. 1–2*a*, which is held in equilibrium by the four external forces. The body's weight is not shown, since it is assumed to be quite small, and therefore negligible compared with the other loads. In order to obtain the internal loadings acting on a specific region within the body, it is necessary to pass an imaginary section or "cut" through the region where the internal loadings are to be determined. The two parts of the body are then separated, and a free-body diagram of one of the parts is drawn. When this is done, there will be a distribution of internal force acting on the "exposed" area of the section, Fig. 1–2*b*. These forces actually represent the effects of the material of the top section of the body acting on the bottom section.

Although the exact distribution of this internal loading may be *unknown*, its resultants $\mathbf{F}_R$ and $\mathbf{M}_{RO}$, Fig. 1–2*c*, are determined by applying the equations of equilibrium to the segment shown in Fig. 1–2*c*. Here these loadings act at point *O*; however, this point is often chosen at the centroid of the sectioned area.

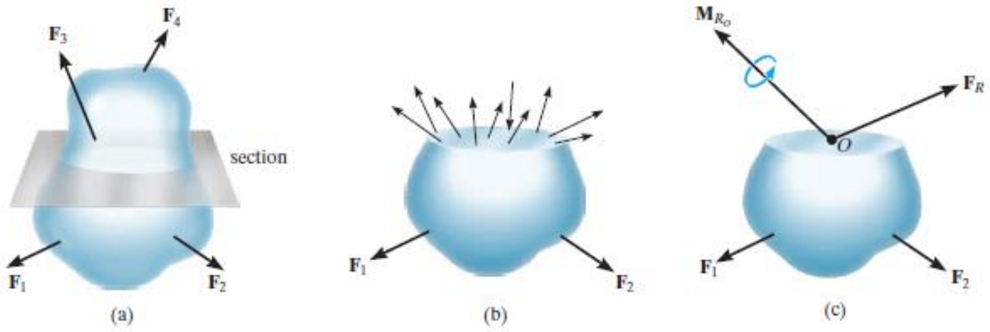


Fig. 1–2

For a **three dimensional** problem, there are six reactions on any cross section (3 forces & 3 moments). For **two dimensional** (*coplanar loadings*) problem, there are 2 force reactions and 1 bending moment.

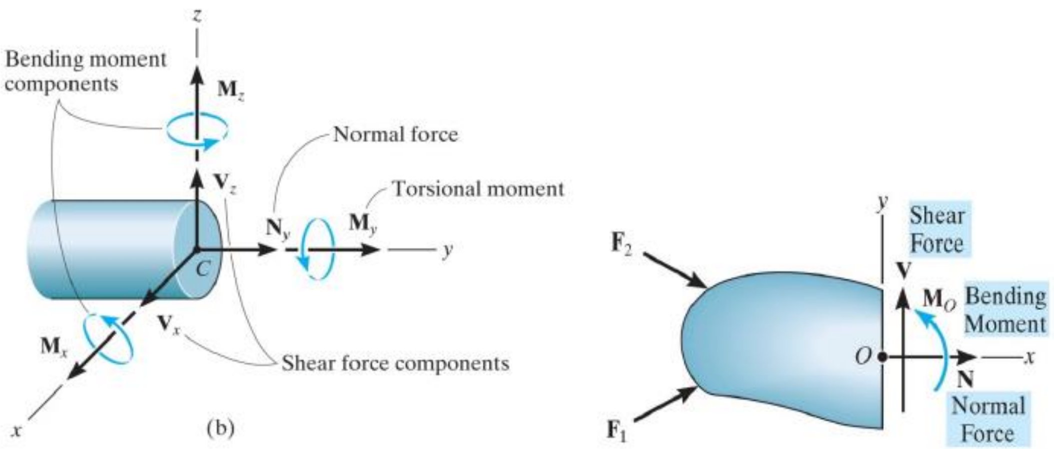


Fig. 1–3

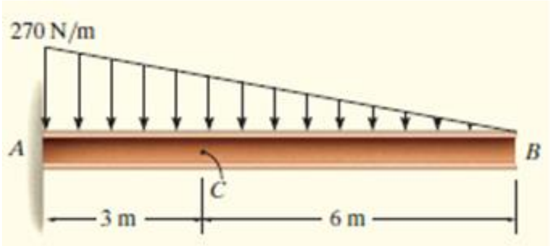
Normal force, N. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.

Shear force, V. The shear force lies in the plane of the area, and it is the body to slide over one another.

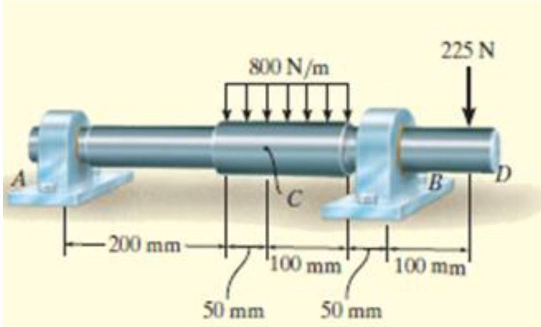
Torsional moment or torque, T. This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

Bending moment, M. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

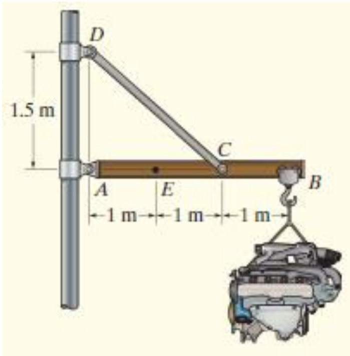
Example 1: Determine the resultant internal loadings acting on the cross section at *C* of the cantilevered beam shown in Fig.



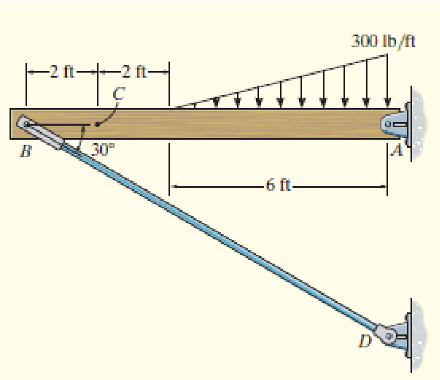
Example 2: Determine the resultant internal loadings acting on the cross section at C of the machine shaft shown in Fig. The shaft is supported by journal bearings at A and B, which only exert vertical forces on the shaft.



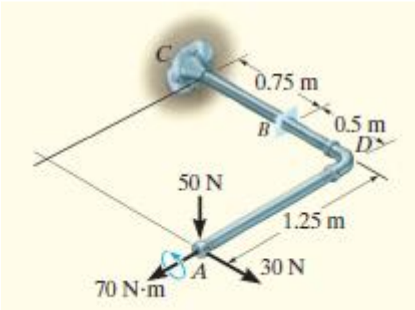
Example 3: The 500-kg engine is suspended from the crane boom in Fig. Determine the resultant internal loadings acting on the cross section of the boom at point E.



Example 4: Determine the resultant internal loadings acting on the cross section at C of the beam shown in Fig.



Example 5: Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. End A is subjected to a vertical force of 50 N, a horizontal force of 30 N, and a couple moment of 70 N # m. Neglect the pipe's mass.



Sheet No. 1

Q 1: Determine the internal normal forces, shear forces, and bending moment at point C in the beam. **Fig. 1.**

Q 2: The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC. Also, determine the internal resultant loadings acting on the cross sections passing through the handle arm at D and E. **Fig. 2.**

Q 3: The pipe assembly is subjected to a force of 600 N at B. Determine the resultant internal loadings acting on the cross section at C. **Fig. 3.**

Q 4: Determine the resultant internal loadings acting on the cross section at a-a. **Fig. 4.**

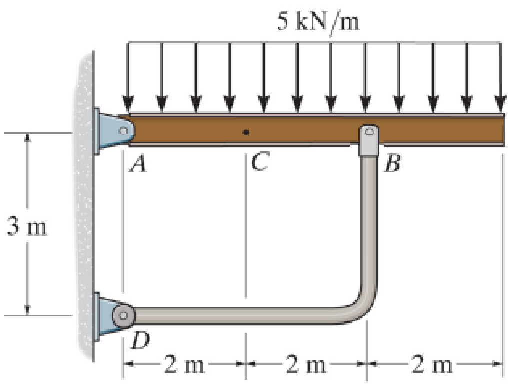


Fig. 1

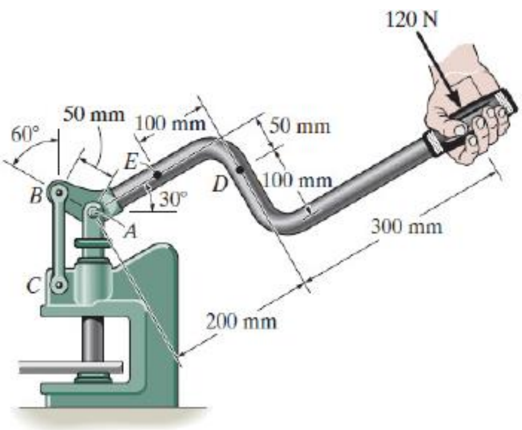


Fig. 2

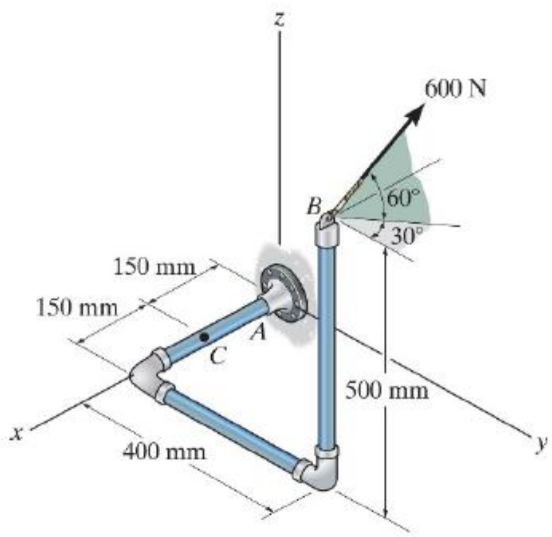


Fig. 3

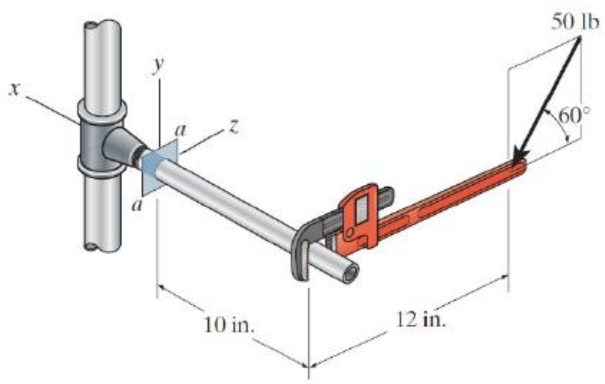
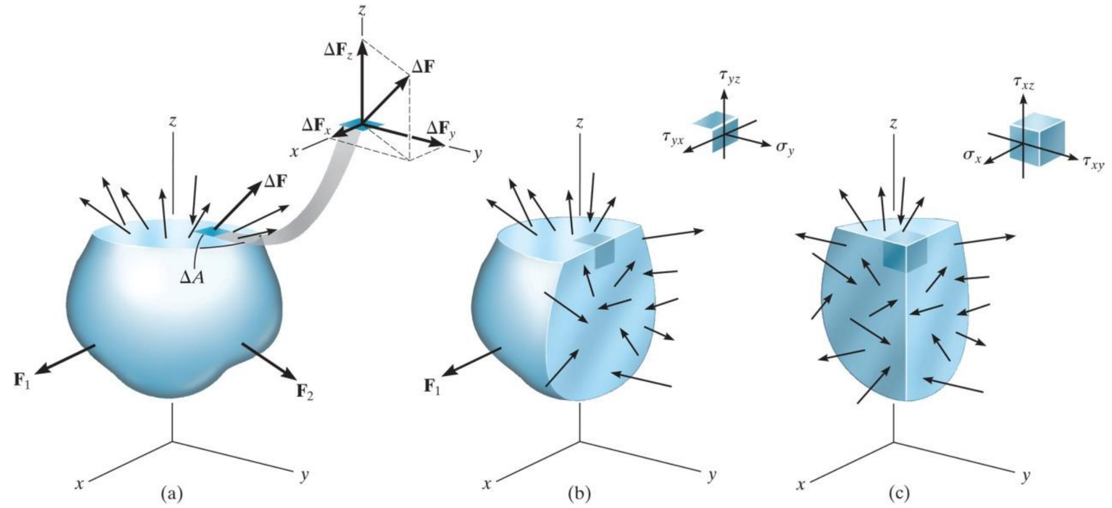


Fig..4

3. STRESS

The force and moment acting at *O* are the resultant effects of the actual *distribution of loading* acting over the sectioned area. By dividing the sectioned area into smaller areas, it is seen that these smaller areas are subjected to a general force $\Delta \mathbf{F}$. $\Delta \mathbf{F}$ can be expressed in terms of its components: ΔFx, ΔFy, and ΔFz.



As ΔA approaches to zero, $\Delta \mathbf{F}$ also tends to zero. But the ratio of the force to the area in general approaches to a finite limit. This ratio is called ***stress.***

Stress is the *intensity of the internal force* acting on a specific *area* passing through a particular point of interest.

The assumptions we are holding are that the material is **continuous** and **cohesive**.

3.1 Normal stress

Normal Stress (σ) is the intensity of the force acting normal (perpendicular) to ΔA.

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

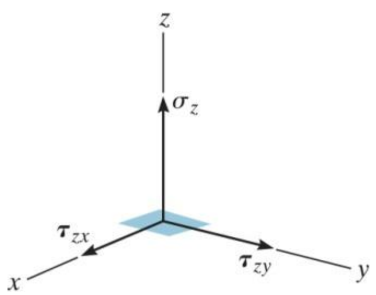
If the normal stress "pulls" on ΔA, it is referred to as tensile stress, whereas if it "pushes" on ΔA it is called compressive stress.

3.2 Shear stress

Shear Stress (τ) is the intensity of the force acting tangent to ΔA.

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

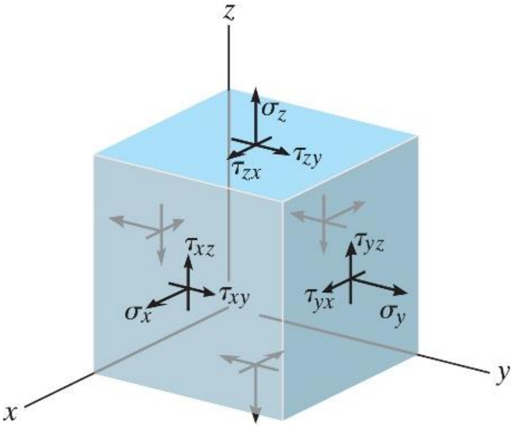
$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

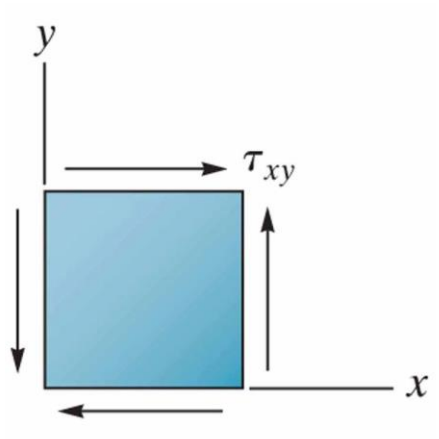


3.3 General state of stress

The state of stress of a point is represented by an infinitesimal element at the point. We cut out a cubic volume element of the material, the state of stress is then characterized by three components acting on each face of the element.

A normal stress (σ) may be tensile (T) or compressive (C).

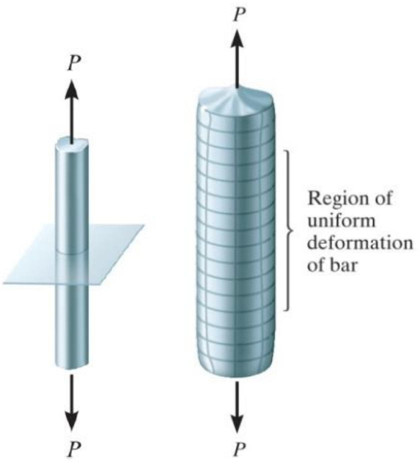


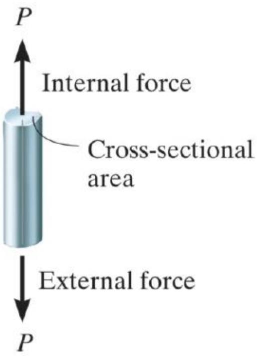


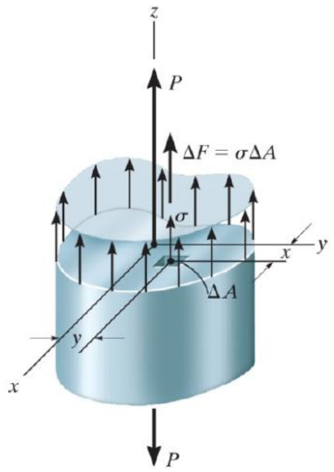
The units of stress are N/m^2 (Pascal, Pa), MPa, lb/in^2 (psi), ksi (1 ksi = 1000 psi).

4. AVERAGE NORMAL STRESS IN AN AXIALLY LOADED BAR

Consider an axially loaded bar shown here. The bar is prismatic and the material is both **homogeneous** and **isotropic**. When the load *P* is applied to the bar through the centroid of its cross-sectional area, the bar will deform uniformly throughout the central region of its length.







If the bar is cut through a plane perpendicular to the axis of the bar, the cross section experiences a normal force equal to P. This force is not concentrated at a point, but instead it is distributed over the cross section area of the bar. The average normal stress can be calculated by:

$$\sigma_{Avg} = \frac{P}{A}$$

If the bar is homogeneous and isotropic, and if the resultant force acting on the cross section passes through the centroid of the cross section, then the actual stress is uniformly distributed and equal to the average stress

$$\sigma = \frac{P}{A}$$

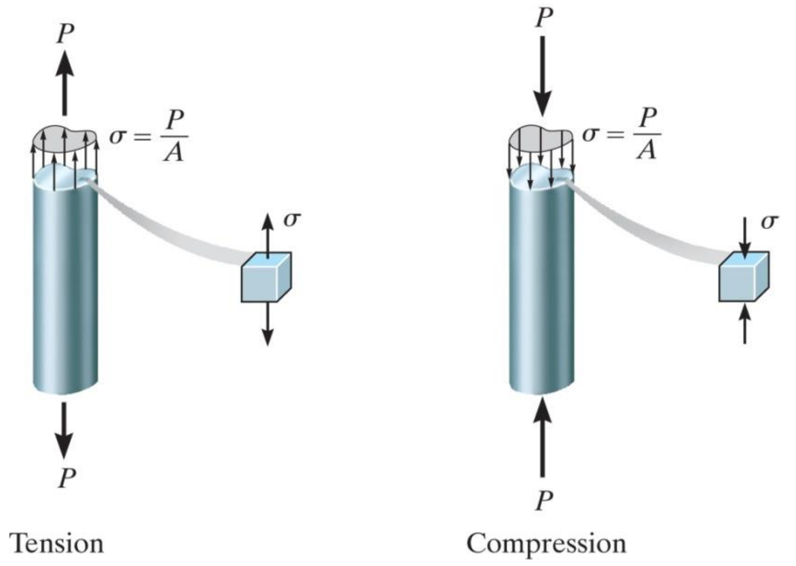
Where,

σ = average normal stress at any point on the cross section

P = internal resultant normal force, passing centroid of the cross section area.

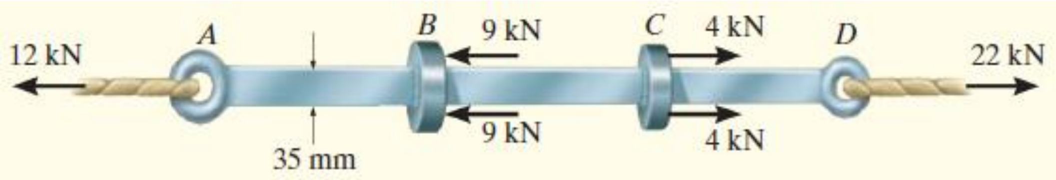
A = cross-sectional area of the bar

The equation applies to members subjected either tension or compression.

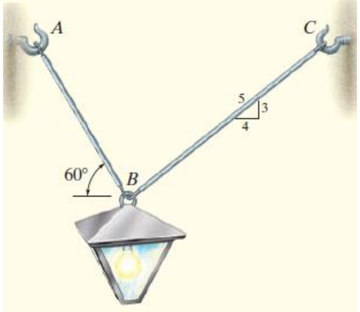


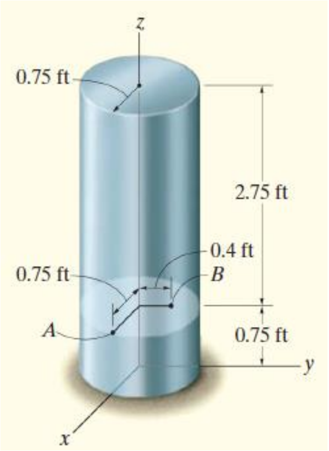
Occasionally, the bar may be subjected to several external loads along its axis, or change in its cross-sectional may occur. As a result, the normal stress could vary from one section to the next. In this case it is necessary to determine the internal force P at various sections along the bar.

Example 6: The bar in Fig. has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



Example 7: The 80-kg lamp is supported by two rods AB and BC as shown in Fig. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.





Example 8: The cylinder shown in Fig. is made of steel having a specific weight of $\gamma_{st} = 490$ lb/ft^3. Determine the average compressive stress acting at points A and B.

Example 9: Member AC shown in Figure is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.