CHAPTER 3 MECHANICAL PROPERTIES OF MATERIAL

5. STRAIN ENERGY

• A material that is deformed by an external loading stores energy *internally* throughout its volume.

• This energy is known as **strain energy**: $\Delta U = ½\ \sigma.\varepsilon.\Delta V$

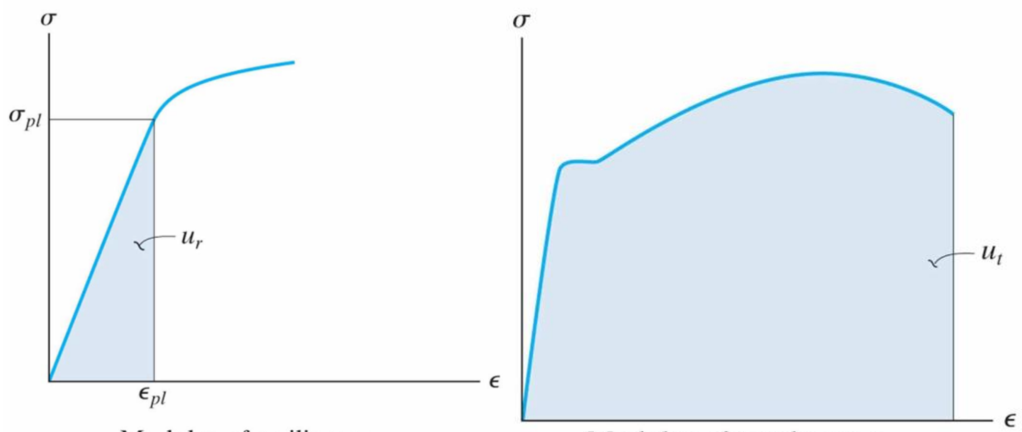
• For applications it is more convenient to specify the *strain energy per unit volume* known as ***strain energy density***: $u = ½\ \sigma.\varepsilon.$

• For linearly elastic materials ($\sigma = E\varepsilon$): $u = ½\ \sigma^2 /E$.

• The **Modulus of Resilience, u_r,** is defined as the strain energy density at the moment the stress, σ, reaches the proportional limit: $u_r = ½\ \sigma_{pl}\ \varepsilon_{pl} = ½\ \sigma^2_{pl} /E$.

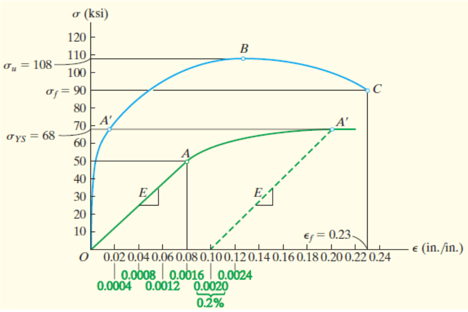
• u_r represents the ability of the material to absorb energy without any permanent damage.

• The **Modulus of Toughness, u_t,** indicates the strain energy density of the material before ir fractures.

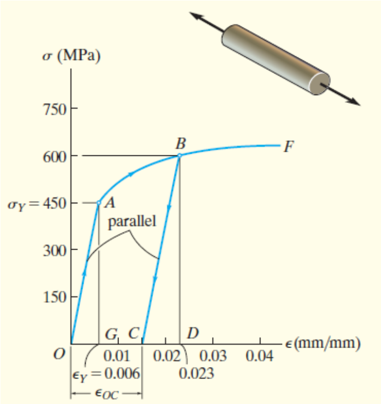


Modulus of resilience u_r Modulus of toughness u_t

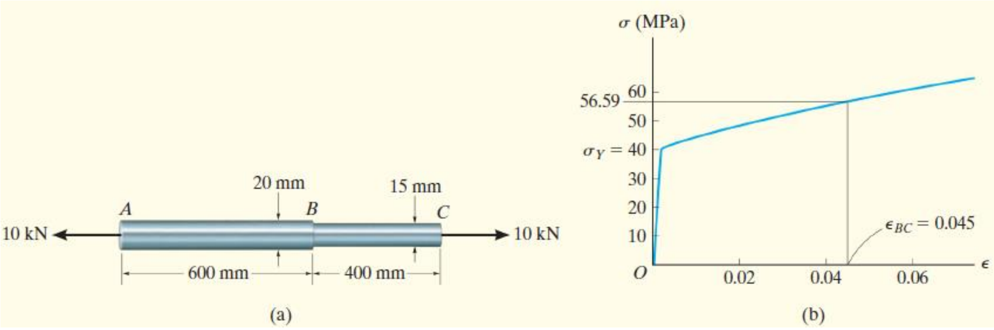
Example 1: A tension test for a steel alloy results in the stress–strain diagram shown in Fig. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.



Example 2: The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. If a specimen of this material is stressed to σ = 600 MPa, determine the permanent set that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.

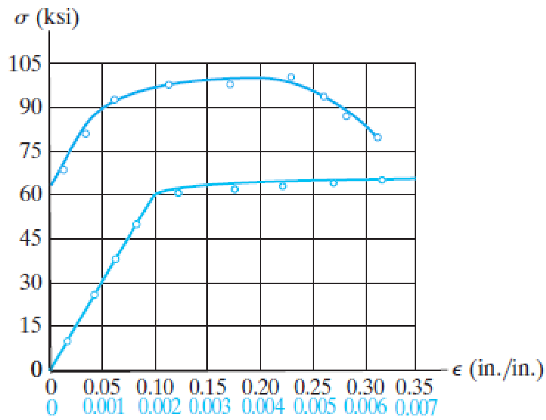


Example 3: The aluminum rod, shown in Figure (a), has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram is shown in Figure (b), determine the approximate elongation of the rod when the load is applied. Take E_{al} = 70 GPa.

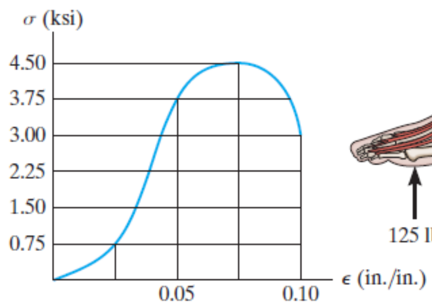


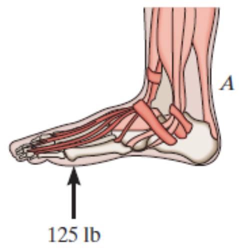
Sheet No. 1

Q 1: The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gage length of 2 in. is given in the figure. (a) Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support. (b) If the specimen is loaded until it is stressed to 90 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.

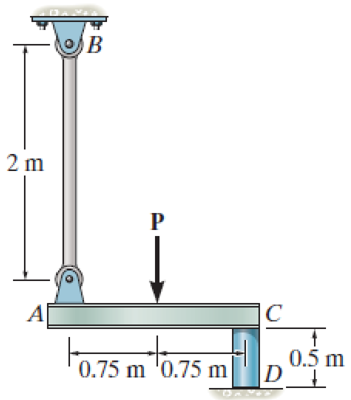


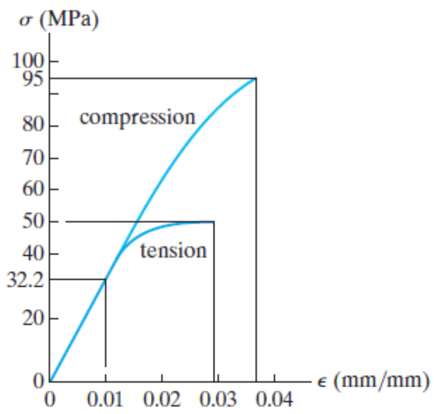
Q 2: The σ-ε diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 6.5 in. and an approximate cross-sectional area of 0.229 in^2, determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.75 lb.





Q 3: The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of P = 80 KN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





6. POISSON'S RATIO

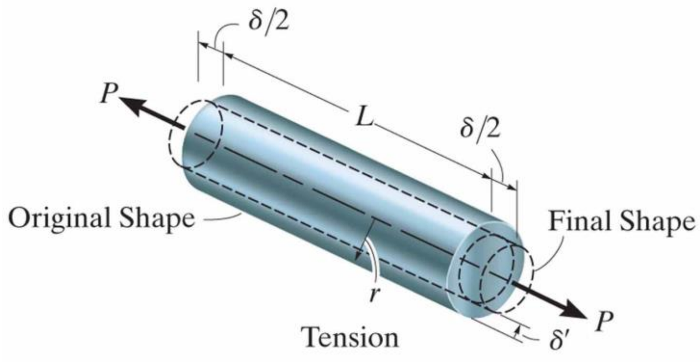
When a deformable body is subjected to an axial tensile force, it not only elongates longitudinally but also contracts laterally. This is called the ***Poisson effect***.

• Consider a bar subjected to an axial load *P* and as a result it elongates axially by δ and contracts radially by δ*'*.

• Then Poisson's ratio is defined as:

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

$$\epsilon_{\text{long}} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{\text{lat}} = \frac{\delta'}{r}$$



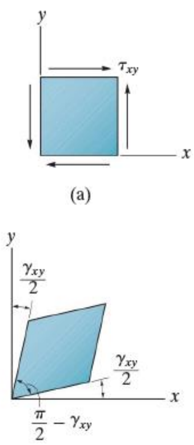
7. THE SHEAR STRESS – STRAIN DIAGRAM

• The behavior of a material subjected to pure shear is studied by applying a torsional load to specimens in the shape of thin tubes.

• Measurements of the torsional load applied to the specimen and the angle of twist experienced are collected to create a shear stress–strain diagram.

• The shear stress–strain and normal stress–strain diagrams are different but have analogous attributes.

• There is a proportional region for which the shear form of Hooke's Law can be written as



$$\tau = G\gamma$$

• ***G*** is the **shear modulus of elasticity** (also called **modulus of rigidity**) or slope of the proportional region, represents the slope of the line on the τ–γ diagram.

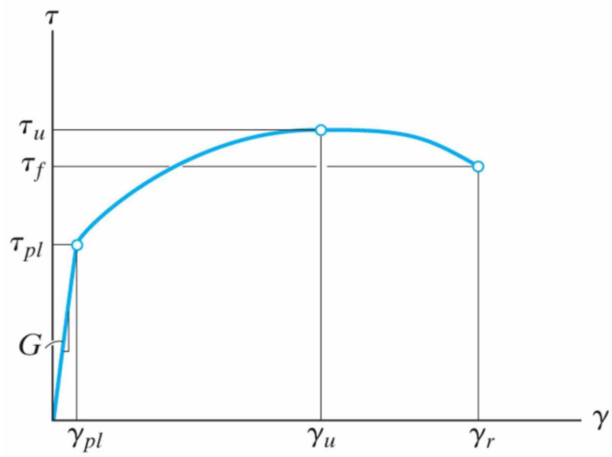
• τ_{pl} is the proportional limit.

• τ_u is the ultimate shear stress.

• τ_f is the shear stress at the point of fracture.

• The modulus of elasticity, the modulus of rigidity and the Poisson's ratio are related by.

$$G = \frac{E}{2(1+\nu)}$$



8. FAILURE OF MATERIALS DUE TO CREEP AND FATIGUE

The mechanical properties of a material have up to this point been discussed only for a static or slowly applied load at constant temperature. In some cases, however, a member may have to be used in an environment for which loadings must be sustained over long periods of time at elevated temperatures, or in other cases, the loading may be repeated or cycled. We will not consider these effects in this book, although we will briefly mention how one determines a material's strength for these conditions, since they are given special treatment in design.

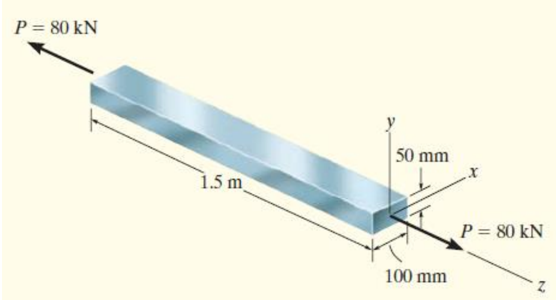
Creep. When a material has to support a load for a very long period of time, it may continue to deform until a sudden fracture occurs or its usefulness is impaired. This time-dependent permanent deformation is known as ***creep***. Normally creep is considered when metals and ceramics are used for structural members or mechanical parts that are subjected to high temperatures. As a typical example, consider the fact that a rubber band will not return to its original shape after being released from a stretched position in which it was held for a very long period of time. In the general sense, therefore, both *stress and/or temperature* play a significant role in the *rate* of creep.

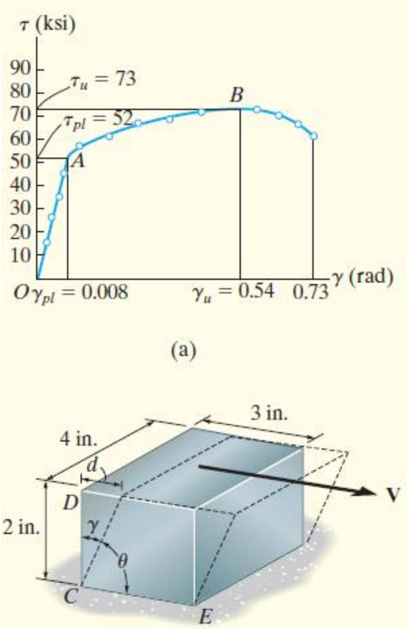
For practical purposes, when creep becomes important, a member is usually designed to resist a specified creep strain for a given period of time. An important mechanical property that is used in this regard is called the *creep strength*. This value represents the highest stress the material can withstand during a specified time without exceeding an allowable creep strain. The creep strength will vary with temperature, and for design, a given temperature, duration of loading, and allowable creep strain must all be specified.

Fatigue. When a metal is subjected to repeat cycles of stress or strain, it causes its structure to break down, ultimately leading to fracture. This behavior is called fatigue, and it is usually responsible for a large percentage of failures in connecting rods and crankshafts of engines; steam or gas turbine blades; connections or supports for bridges, railroad wheels, and axles; and other parts subjected to cyclic loading. In all these cases, fracture will occur at a stress that is less than the material's yield stress.

In order to specify a safe strength for a metallic material under repeated loading, it is necessary to determine a limit below which no evidence of failure can be detected after applying a load for a specified number of cycles. This limiting stress is called the ***endurance or fatigue limit***. Using a testing machine for this purpose, a series of specimens are each subjected to a specified stress and cycled to failure.

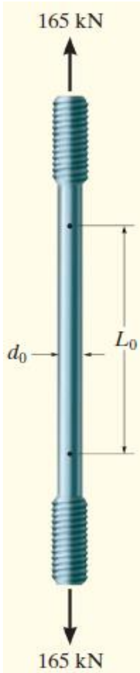
Example 1: A bar made of A-36 steel has the dimensions shown in Fig. If an axial force of P = 80 kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically.





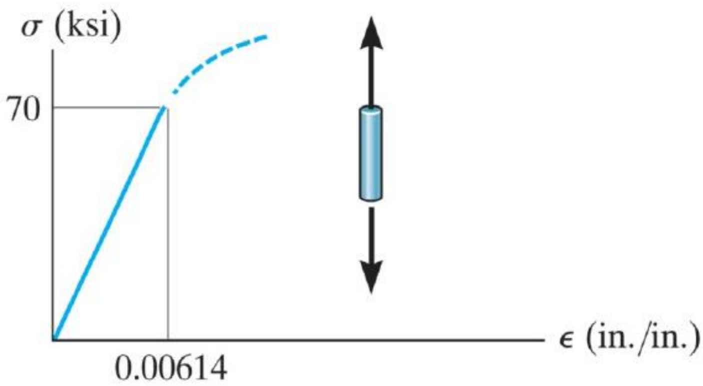
Example 2: A specimen of titanium alloy is tested in torsion and the shear stress–strain diagram is shown in Fig. a. Determine the shear modulus G, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance d that the top of a block of this material, shown in Fig. b, could be displaced horizontally if the material behaves elastically when acted upon by a shear force **V**. What is the magnitude of **V** necessary to cause this displacement?

Example 3: An aluminum specimen shown in Figure has a diameter of d_0 = 25 mm and a gage length of L_0 = 250 mm. If a force of 165 kN elongates the gage length 1.20 mm, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take G_{al} = 26 GPa and σ_Y = 440 MPa.

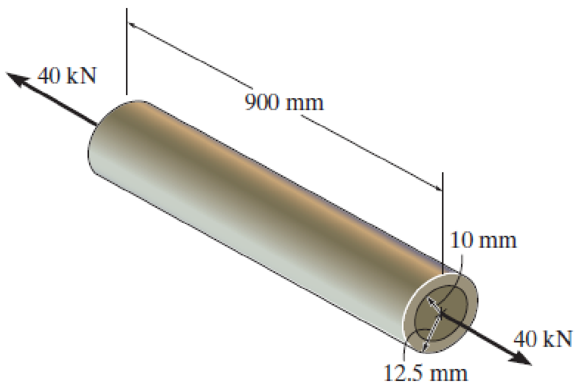


Sheet No. 2

Q 1: The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



Q 2: The thin-walled tube is subjected to an axial force of 40 kN. If the tube elongates 3 mm and its circumference decreases 0.09 mm, determine the modulus of elasticity, Poisson's ratio, and the shear modulus of the tube's material. The material behaves elastically.



Q 3: The wires each have a diameter of 12 in., length of 2 ft, and are made from 304 stainless steel. (a) If P = 6 kip, determine the angle of tilt of the rigid beam AB. (b) Determine the magnitude of force P so that the rigid beam tilts 0.015°.