CHAPTER 2 STRAIN

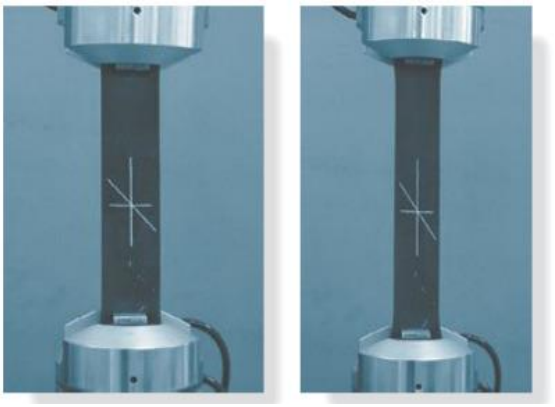
1. DEFORMATION

When a force is applied on a body it tends to change the shape of the body. These changes in the shape of the body are called **deformation**.

When a force acts on a body, depending on the material of the body, the force can produce a large or an unnoticeable deformation.

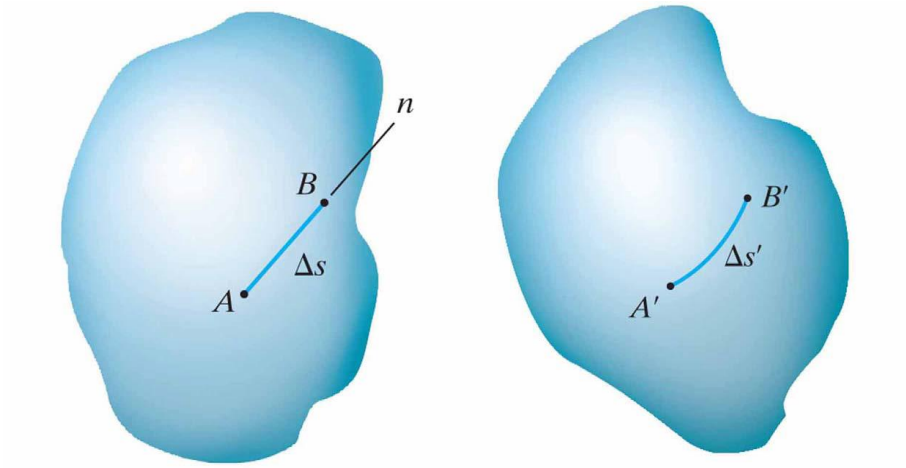
In general, the deformation of a body is not uniform across its volume.

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.



2. EQUILIBRIUM OF A DEFORMED BODY

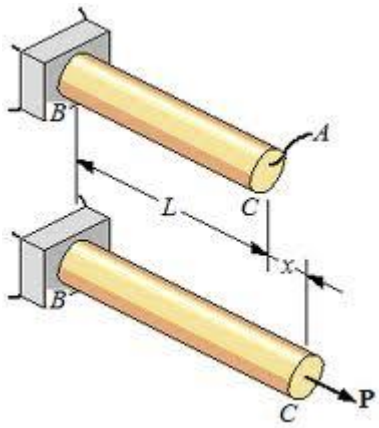
The changes in the size of a body that is subjected to a certain load are due to **normal strain**, ε.



$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$

Normal strain is defined as a change in length per unit length. The assumption here is that if the material is homogeneous, the change in length will be equally distributed along the length.

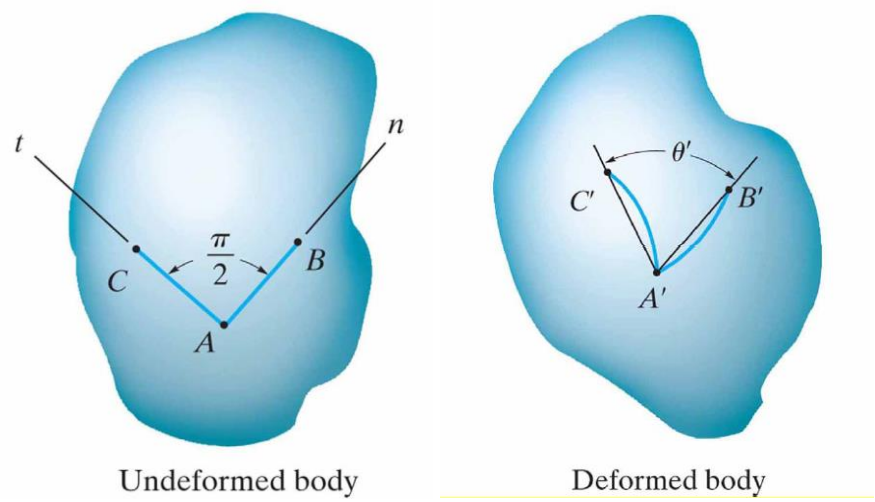


$$\varepsilon = \frac{l_{final} - l_{initial}}{l_{initial}} = \frac{\Delta l}{l_0}$$

Note that normal strain is a dimensionless quantity, since it is a ratio of two lengths. If the SI system is used, ordinarily, for most engineering applications, ε will be very small, so measurements of strain are in micrometers per meter (μm/m), where 1 μm = 10^{-6} m. In the Foot-Pound-Second system, strain is often stated in units of inches per inch (in./in.). Sometimes for experimental work, strain is expressed as a percent (e.g., 0.001 m/m = 0.1%).

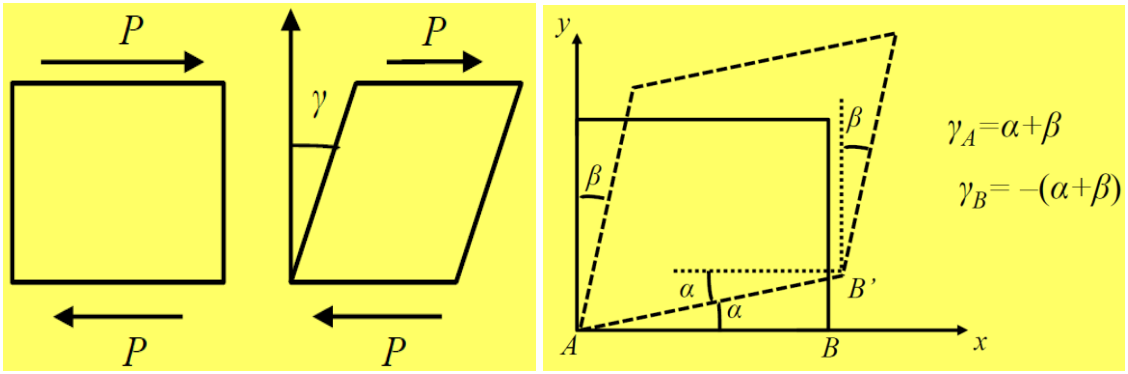
3. SHEAR STRAIN

If two perpendicular lines are selected before any load is applied to a body, the **shear strain** is the change in the angle that occurs between these two lines after the load is applied to the body, **γ (gamma).**



$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

If two perpendicular lines are selected before any load is applied to a body, the **shear strain** is the change in the angle that occurs between these two lines after the load is applied to the body.



Strain is dimensionless; therefore **γ** must be expressed in radians. Most engineering design use small strain analysis, in which the strain is very small compared to 1. It can be used to approximate sin θ = θ, tan θ = θ, provided θ is very small.

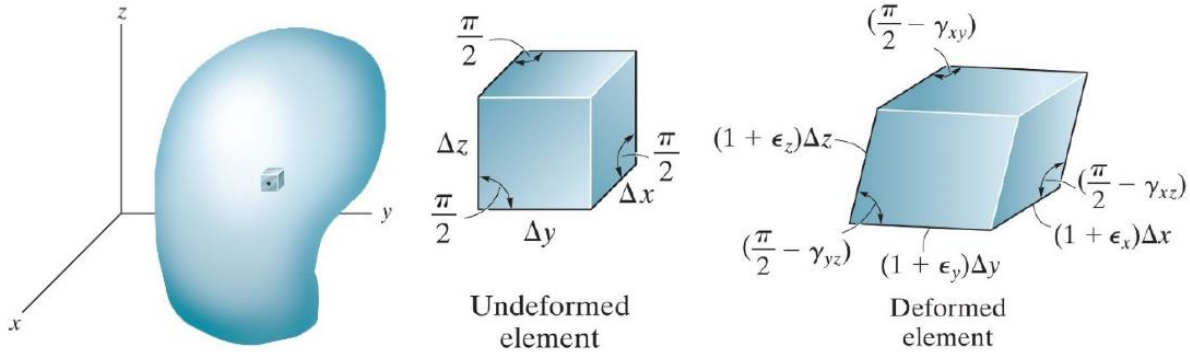
4. CARTESIAN STRAIN COMPONENTS

There are **six** components of strain:

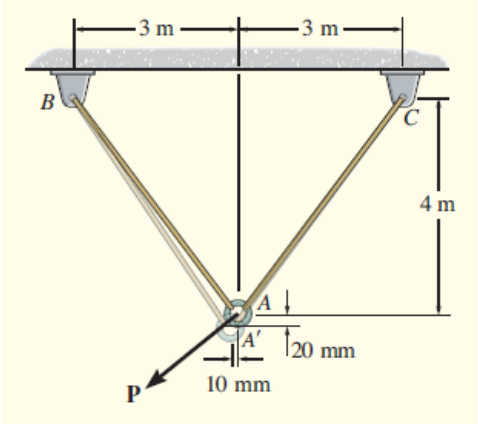
Normal strain: ε_x, ε_y, ε_z.

Shear strain: γ_{xy}, γ_{xz}, γ_{yz}.

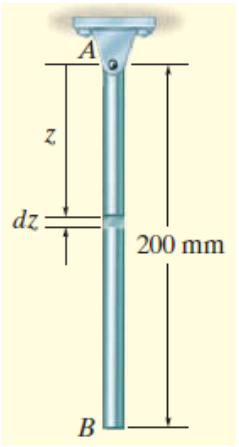
In general strains are small: ε, γ << 1.



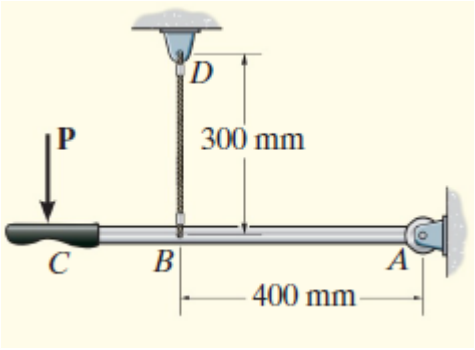
Example 1: Determine the average normal strains in the two wires in Figure if the ring at A moves to A'.



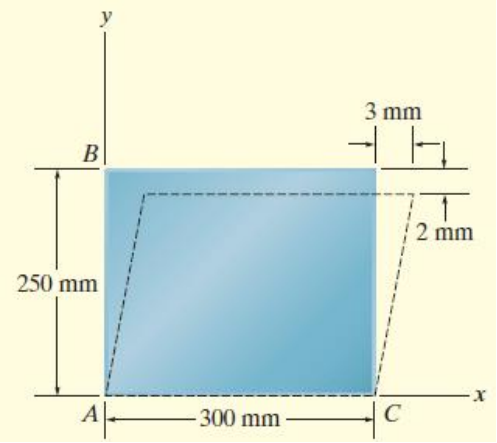
Example 2: The slender rod shown in Figure is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\varepsilon_z = 40(10^{-3})\, z^{1/2}$, where z is measured in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.



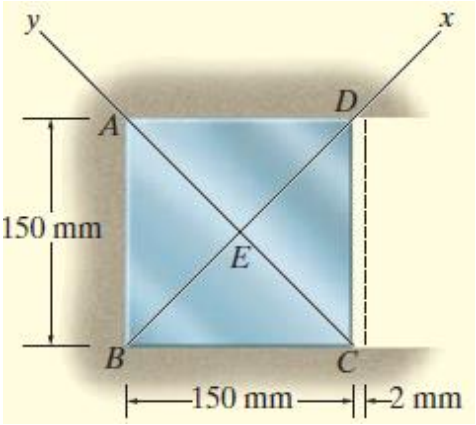
Example 3: When force P is applied to the rigid lever arm ABC in Figure, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain in wire BD.



Example 4: Due to a loading, the plate is deformed into the dashed shape shown in Fig. Determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate at A relative to the x and y axes.

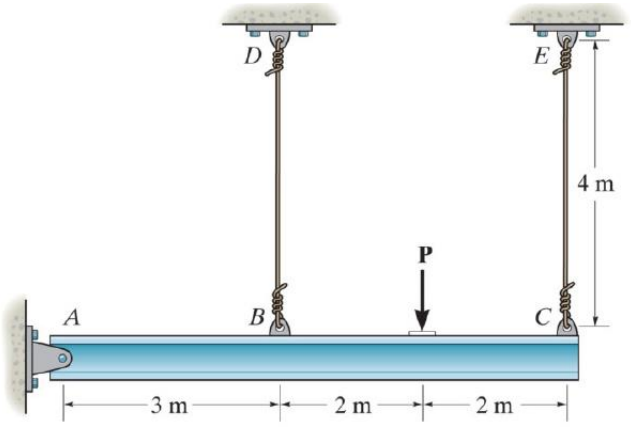


Example 5: The plate shown in Figure is fixed connected along AB and held in the horizontal guides at its top and bottom, AD and BC. If its right side CD is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal AC, and (b) the shear strain at E relative to the x, y axes.

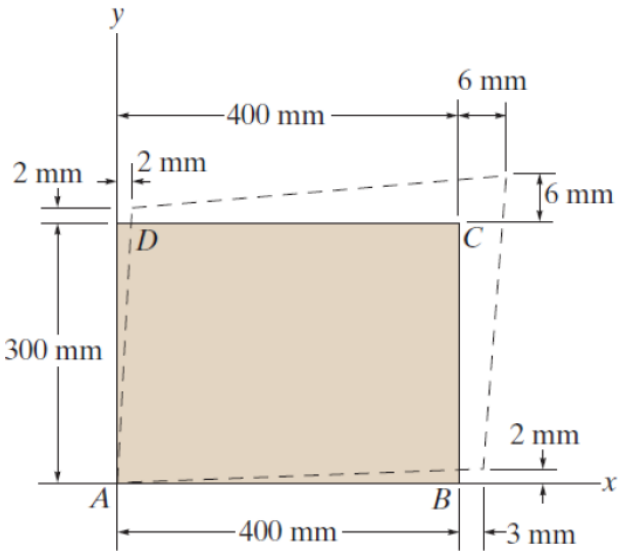


Sheet No. 1

Q 1: The rigid beam is supported by a pin at A and wires BD and CE. If the load P on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



Q 2: The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal AC and BD, and the average shear strain at corner A and B relative to the x, y axes.



Q 3: The force applied to the handle of the rigid lever arm causes the arm to rotate clockwise through an angle of 3° about pin A. Determine the average normal strain developed in the wire. Originally, the wire is unstretched.