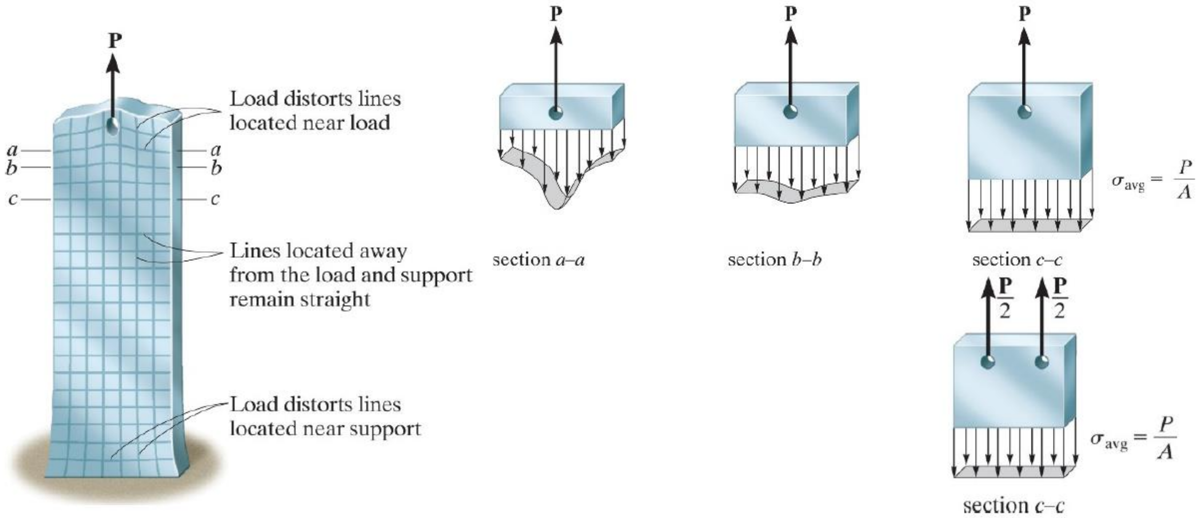
CHAPTER 4 AXIAL LOAD

1. SAINT-VENANT'S PRINCIPLE



The ***localized deformation*** caused by ***P*** vanishes at a location sufficiently away (at least equal to the ***largest dimension*** of the loaded cross section) from the point of application of the load.

Saint–Venant's Principle states that:

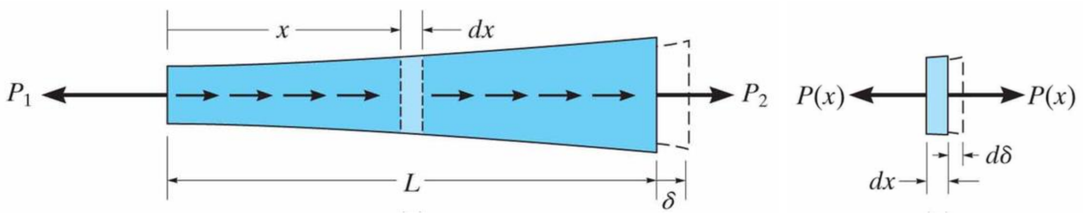
Stress and strain produced at points in a body sufficiently removed from the region of load application will be the same as the stress and strain produced by any applied loadings that have the same statically equivalent resultant, and are applied to the body within the same region.

2. ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

Objective. To determine the elastic displacement of a member subjected to axial loads.

• Consider a bar with a cross section area that varies gradually over the length of the bar.

• The bar is subjected to concentrated loads at its ends and a variable load along its length.



$$\sigma = \frac{P(x)}{A(x)} \text{ and } \epsilon = \frac{d\delta}{dx}$$

• Assuming that $\sigma < \sigma_{pl}$: $\sigma = E\,\varepsilon$.

$$\sigma = E(x)\epsilon$$

$$\frac{P(x)}{A(x)} = E(x)\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)dx}{A(x)E(x)}$$

Then, for the entire length of the bar, the displacement is found as:

$$\boxed{\delta = \int_0^L \frac{P(x)dx}{A(x)E(x)}}$$

Where:

δ = displacement of one point on the bar relative to the other point

L = original length of bar

P(x) = internal axial force at the section, located a distance x from one end

A(x) = cross-sectional area of the bar, expressed as a function of x

E = modulus of elasticity for the material

• If the bar has a constant cross section area, the material is homogeneous (E constant) and the external load, P, applied to the bar is constant, we have

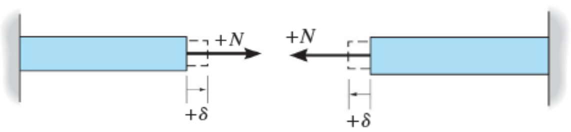
$$\boxed{\delta = \frac{PL}{AE}}$$

• For a bar that is subjected to different axial forces along its length, or the cross section area or modulus of elasticity changes suddenly from one region of the bar to the next, the total displacement on the bar can be calculated as:

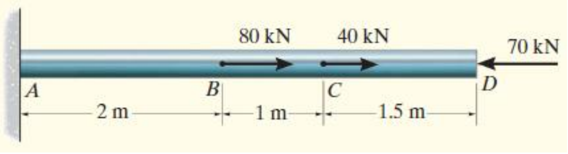
$$\boxed{\delta = \sum \frac{PL}{AE}}$$

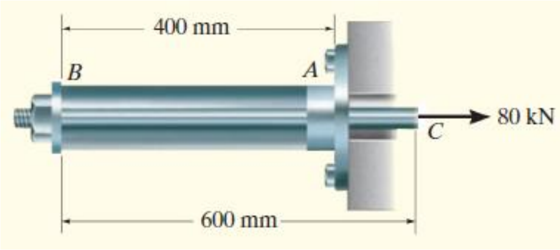
• If a force and displacement cause tension, they are considered positive

• If a force and displacement cause contraction and compression, they are considered negative.



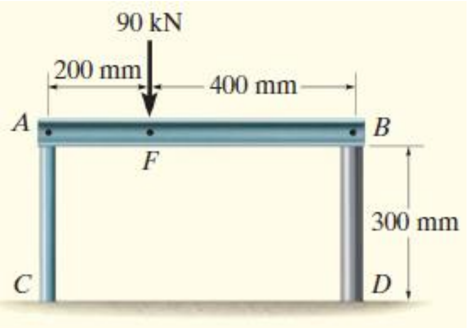
Example 1: The uniform A-36 steel bar in Figure has a diameter of 50 mm and is subjected to the loading shown. Determine the displacement at D, and the displacement of point B relative to C.

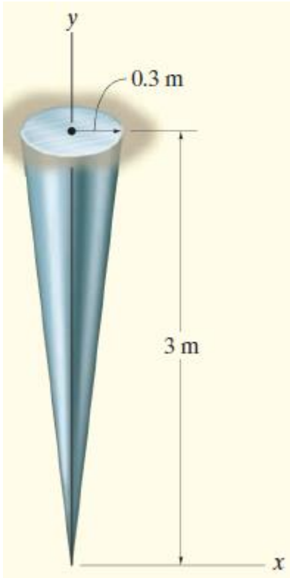




Example 2: The assembly shown in Figure consists of an aluminum tube AB having a cross-sectional area of 400 mm^2. A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end C of the rod. Take E_{st} = 200 GPa, E_{al} = 70 GPa.

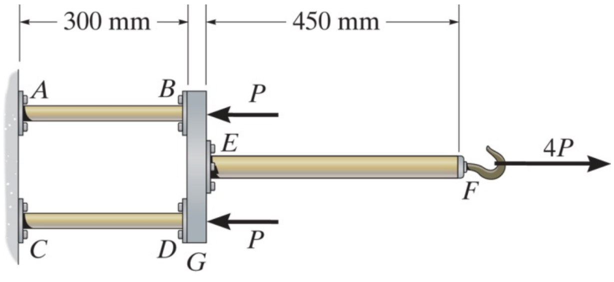
Example 3: Rigid beam AB rests on the two short posts shown in Figure. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take E_{st} = 200 GPa, E_{al} = 70 GPa.





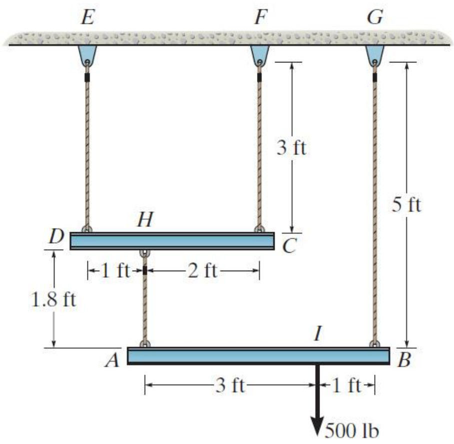
Example 4: A member is made of a material that has a specific weight of γ = 6 kN/m^3 and modulus of elasticity of 9 GPa. If it is in the form of a *cone* having the dimensions shown in Figure, determine how far its end is displaced due to gravity when it is suspended in the vertical position.

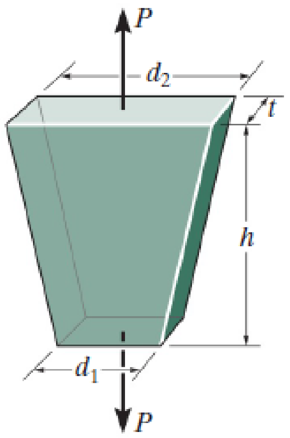
Sheet No. 1



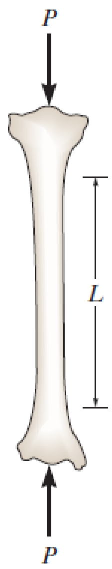
Q 1: The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If the horizontal displacement of end F of rod EF is 0.45 mm, determine the magnitude of P.

Q 2: The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the 500-lb load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 0.025 in^2.





Q 3: Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



Q 4: Bone material has a stress–strain diagram that can be defined by the relation $\sigma = E[\varepsilon>(1 + kE\varepsilon)]$, where k and E are constants. Determine the compression within the length L of the bone, where it is assumed the cross-sectional area A of the bone is constant.