Experiment No.14

The Parallel RLC Resonance Circuit

Object

To perform be familiar with The Parallel RLC Resonance Circuit and their laws.

Theory

the analysis of a *parallel RLC circuits* can be a little more mathematically difficult than for series RLC circuits so in this tutorial about parallel RLC circuits only pure components are assumed in this tutorial to keep things simple.

This time instead of the current being common to the circuit components, the applied voltage is now common to all so we need to find the individual branch currents through each element. The total impedance, Z of a parallel RLC circuit is calculated using the current of the circuit similar to that for a DC parallel circuit, the difference this time is that admittance is used instead of impedance. Consider the parallel RLC circuit illustrated in Figure 1. The AC voltage source is

$$V(t) = V_0 sin\omega t$$

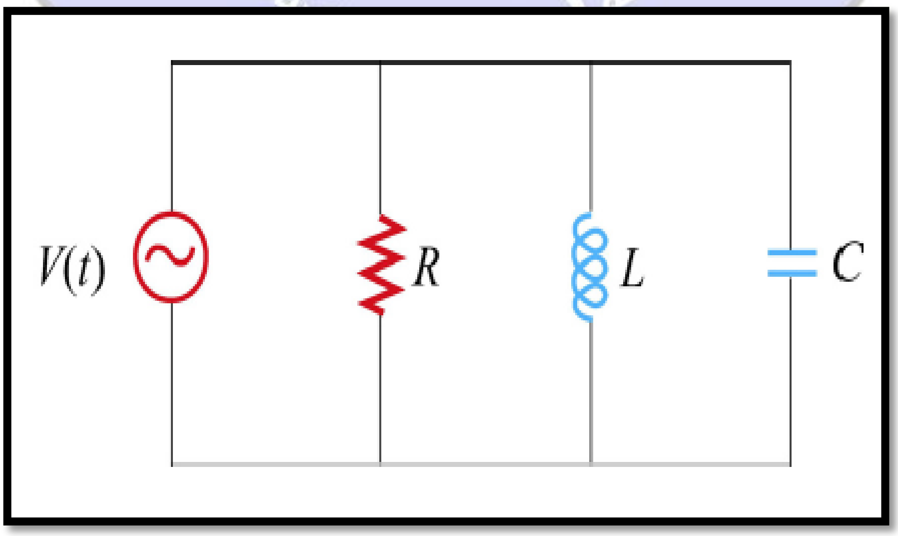

Figure 1. Parallel RLC circuit.

Unlike the series RLC circuit, the instantaneous voltages across all three circuit elements R, L, and C are the same, and each voltage is in phase with the current through the resistor. However, the currents through each element will be different.

The current in the resistor is

$$I_R(t) = \frac{V(t)}{R} = \frac{V_0}{R}\sin\omega t = I_{R0}\sin\omega t \quad \text{.......................(1)}$$

where $I_{R0} = V_0/R$The voltage across the inductor is

$$V_L(t) = V(t) = V_0 \sin\omega t = L\frac{dI_L}{dt} \quad \text{...........................(2)}$$

which gives

$$I_L(t) = \int_0^t \frac{V_0}{L}\sin\omega t' dt' = -\frac{V_0}{\omega L}\cos\omega t = \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...(3)}$$

where $I_{L0} = V_0/X_L$ and $X_L = \omega L$ is the inductive reactance.

Similarly, the voltage across the capacitor is $V_C(t) = V_0 sin\omega t = Q(t)/C$, which implies

$$I_C(t) = \frac{dQ}{dt} = \omega C V_0 \cos\omega t = \frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{....(4)}$$

where $I_{C0} = V_0/X_C$ and $X_C = 1/\omega C$ is the capacitive reactance.

Using Kirchhoff's junction rule, the total current in the circuit is simply the sum of all three currents.

$$I(t) = I_R(t) + I_L(t) + I_C(t)$$

$$= I_{R0}\sin\omega t + I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right) + I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad ....(5)$$

The currents can be represented with the phasor diagram shown in Figure .2

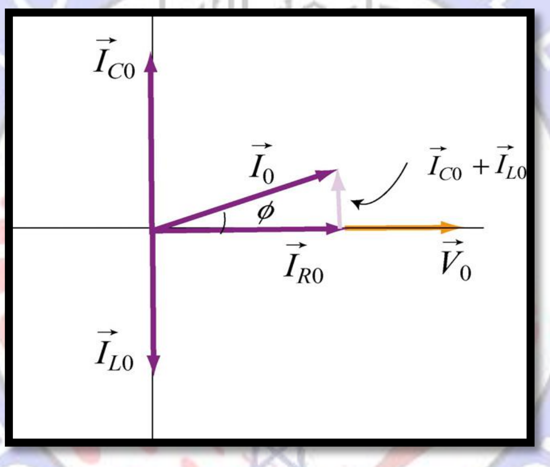

Figure .2 Phasor diagram for the parallel RLC circuit

From the phasor diagram, we see that

$$\vec{I}_0 = \vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0} \quad ....(6)$$

and the maximum amplitude of the total current, I_0, can be obtained as

$$I_0 = |\vec{I}_0| = |\vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0}| = \sqrt{I_{R0}^2 + (I_{C0} - I_{L0})^2}$$
$$= V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = V_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{.......(7)}$$

Note however, since $I_R(t)$,$I_L(t)$ and $I_C(t)$are not in phase with one another, I_0 is not equal to the sum of the maximum amplitudes of the three currents:

$$I_0 \neq I_{R0} + I_{L0} + I_{C0} \quad \text{......................................(8)}$$

With $I_0 = V_0/Z$ the (inverse) impedance of the circuit is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{.............(9)}$$

The relationship between Z, R, X_L and X_C is shown in Figure .3

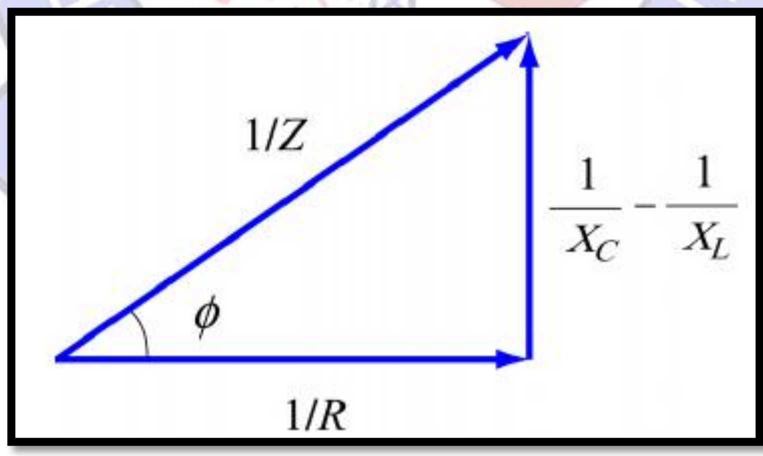

Figure .3 Relationship between Z, R, X_L *and* X_C in a parallel *RLC* circuit.

From the figure or the phasor diagram shown in Figure .2, we see that the phase can be obtained as

$$\tan\phi = \left(\frac{I_{C0} - I_{L0}}{I_{R0}}\right) = \frac{\frac{V_0}{X_C} - \frac{V_0}{X_L}}{\frac{V_0}{R}} = R\left(\frac{1}{X_C} - \frac{1}{X_L}\right) = R\left(\omega C - \frac{1}{\omega L}\right)$$

.....(10)

The resonance condition for the parallel RLC circuit is given by $\emptyset = 0$, which implies

$$\frac{1}{X_C} = \frac{1}{X_L}$$

....(11)

The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

....(12)

which is the same as for the series RLC circuit. From Eq. (9), we readily see that 1/Z is minimum (or Z is maximum) at resonance. The current in the inductor exactly cancels out the current in the capacitor, so that the total current in the circuit reaches a minimum, and is equal to the current in the resistor:

$$I_0 = \frac{V_0}{R}$$

....(13)

As in the series RLC circuit, power is dissipated only through the resistor. The average power is

$$\langle P(t)\rangle = \langle I_R(t)V(t)\rangle = \langle I_R^2(t)R\rangle = \frac{V_0^2}{R}\langle \sin^2 \omega t\rangle = \frac{V_0^2}{2R} = \frac{V_0^2}{2Z}\left(\frac{Z}{R}\right)$$

.....(14)

Thus, the power factor in this case is

$$\text{power factor} = \frac{\langle P(t) \rangle}{V_0^2 / 2Z} = \frac{Z}{R} = \frac{1}{\sqrt{1+\left(R\omega C - \frac{R}{\omega L}\right)^2}} = \cos\phi \quad ..(15)$$

Procedure

1. Connect the resistance and inductance in series as shown in Fig.6

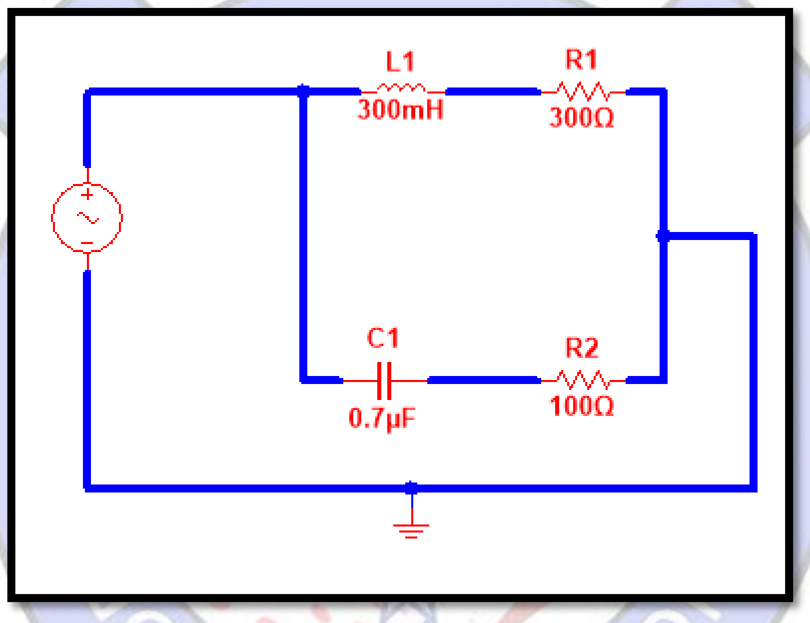

Fig.4

2. Adjust the function generator to 200 Hz, 10 V and 50% duty cycle.
3. Select sine, waveform, turn the circuit ON for about 50 ms then turn it OFF.
4. Measure the phase shift between the current I and the input voltage V using oscilloscope.
5. Draw the result of the display graph.
6. Draw the Phasor diagram

Resonance

1. Connect the circuit as shown in Fig. (5).

2. Set the voltmeter to 6 Vrms.
3. Select sine waveform; vary the oscillator from 14 kHz to 17 KHz in steps of 0.5 kHz.
4. Record the reading of the voltmeter at each step as in table (1)
5. Evaluate the impedance of the circuit at each step.
6. Plot a graph of X_L, and X_c w.r.t. of frequency
7. Plot a graph of impedance w.r.t. of frequency
8. Determine the value of the impedance at the resonant frequoncy.
9. Compare the value of the resonant frequency to the theoretical value.

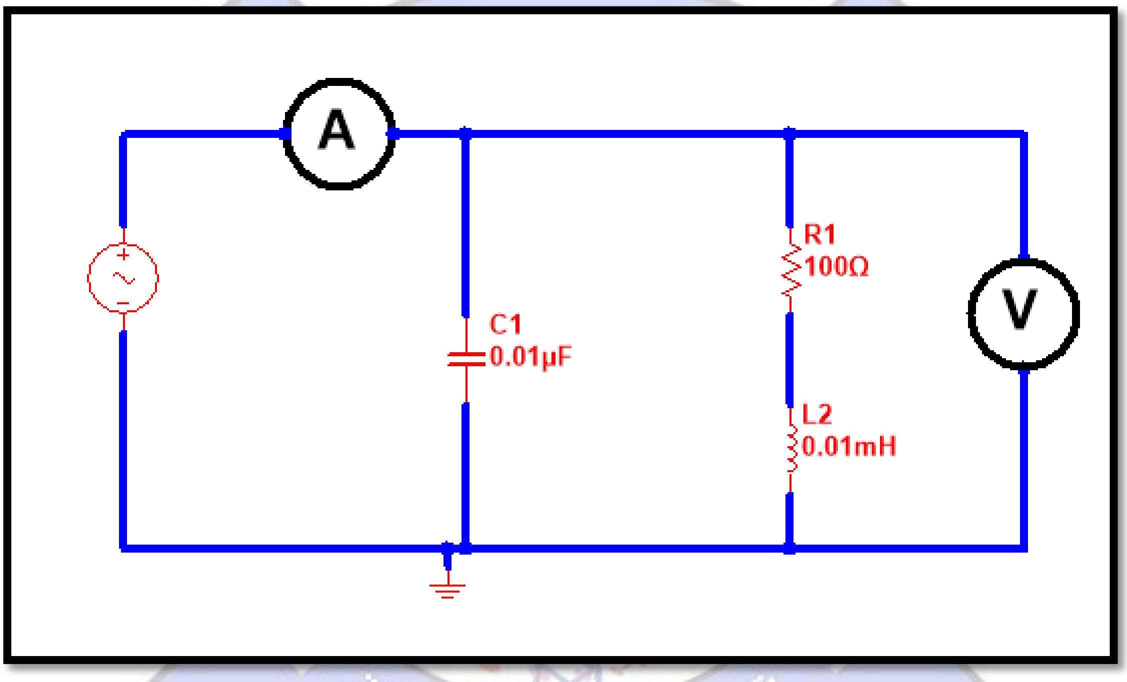

Fig. (5).

Frequency(kHz)	$I_{r.m.s}$(A)

table (1)

Discussion

1. Can we obtain a plot of X_L, against frequency f experimentally?
2. Explain why Phasor and impedance diagrams have the same angles.
3. What is the value of Phasor shift if R= 300Ω L= 400mH with f=50Hz. Discuss the increase or decrease in the phase shift.