CHAPTER 5 TORSION

3. POWER TRANSMISSION

• Torsion members are often used to transfer power produced by a machine.

• When used for this purpose, torsion members are subjected to a torque that depends on the power generated by the machine and the angular speed of the shaft.

• ***Power*** is the rate of doing work and is related to the torque carried and the angular velocity of the shaft.

$$P = T\omega \qquad P = 2\pi f T \qquad \omega = 2\pi f$$

f is frequency which is a measure of the number of revolutions or cycles per second and is expressed in hertz (1Hz = 1 cycle/s).

1 hp = 550 lb-ft/s

	System of Units	
	S I	*English*
P	watts (W)	lb–ft /s
T	N – m	lb–ft
ω	rad/s	rad/s

Shaft Design

• If the shear stress allowable (τ_{allow}) by the shaft material is known:

$$\tau_{allow} = \left.\frac{T\,c}{J}\right|_{max}$$

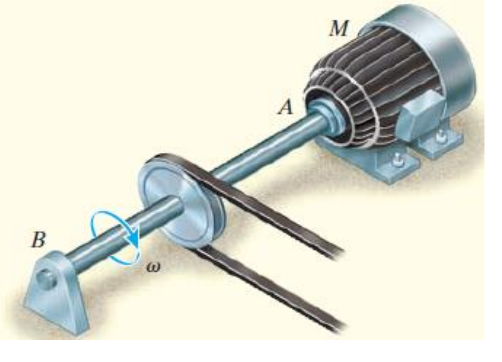
• Which results in the following relation

$$\frac{J_{req}}{c_{req}} = \frac{T}{\tau_{allow}}$$

• For a solid cross section:

$$\frac{\frac{\pi c^4}{2}}{c} = \frac{T}{\tau_{allow}} \qquad \text{or,} \qquad c = \left(\frac{2\,T}{\pi \tau_{allow}}\right)^{1/3}$$

Example 4: A solid steel shaft AB, shown in Figure is to be used to transmit 5 hp from the motor M to which it is attached. If the shaft rotates at ω = 175 rpm and the steel has an allowable shear stress of τ_{allow} = 14.5 ksi, determine the required diameter of the shaft to the nearest 1/8 in.



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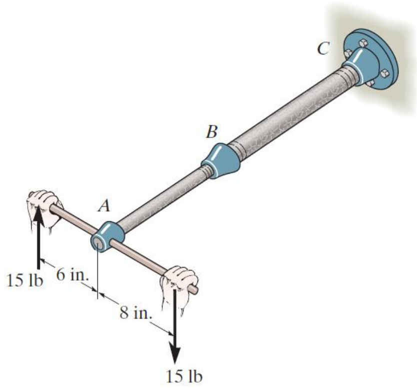
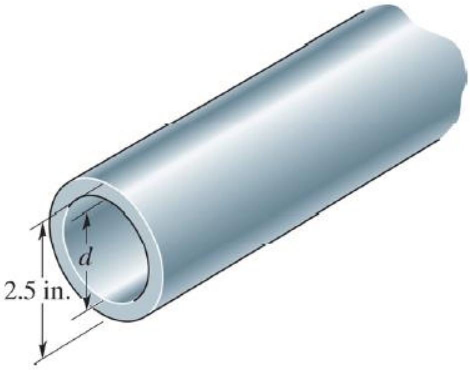
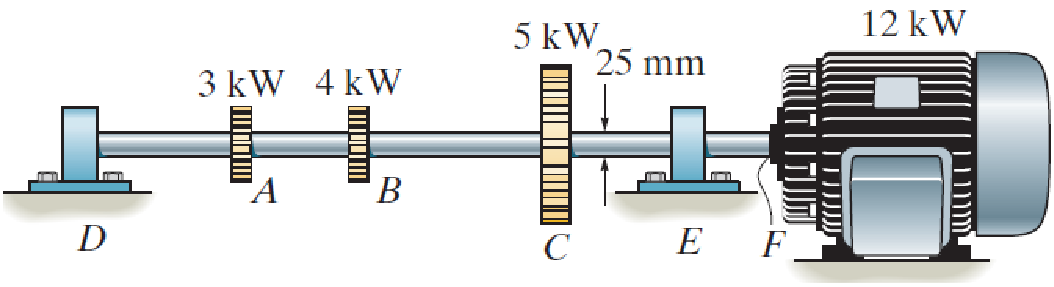
Q 1: The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



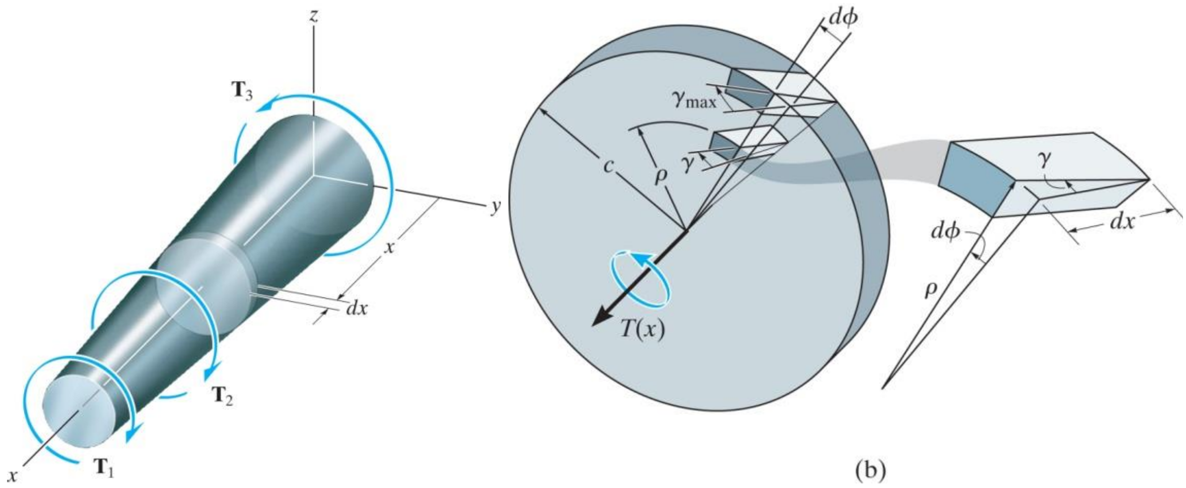


Q 2: A steel tube having an outer diameter of 2.5 in. is used to transmit 9 hp when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest 1/8 in. if the allowable shear stress is τ_{allow} = 10 ksi.

Q 3: The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



4. ANGLE OF TWIST



Sometimes the design of a shaft depends on restricting the amount of rotation or twist. Furthermore, it is necessary to calculate the angle of twist in a shaft when the reactions of a statically indeterminate shaft are to be determined.

Objective: Determine a formula to calculate the ***angle of twist*** $\boldsymbol{\phi(x)}$ of one end of a shaft with respect to the other end.

Assume:

1) A circular shaft with variable cross section.

2) Shaft made of homogeneous material with linear-elastic behavior

As previously determined:

$$d\phi = \gamma \frac{dx}{\rho}$$

Since it is assumed that the behavior of the material is linear-elastic, it is possible to apply Hooke's law, $\gamma = \tau/G$, and the relation between shear stress and torque, $\tau = T(x)\,\rho/J(x)$, such that the shear strain can be expressed as

$$\gamma = T(x)\rho/J(x)G(x)$$

Therefore, the angle of twist of the differential disk $d\phi$ is:

$$d\phi = \frac{T(x)}{J(x)G(x)}\,dx$$

By integrating over the entire shaft:

$$\phi = \int_0^L \frac{T(x)\,dx}{J(x)G(x)}$$

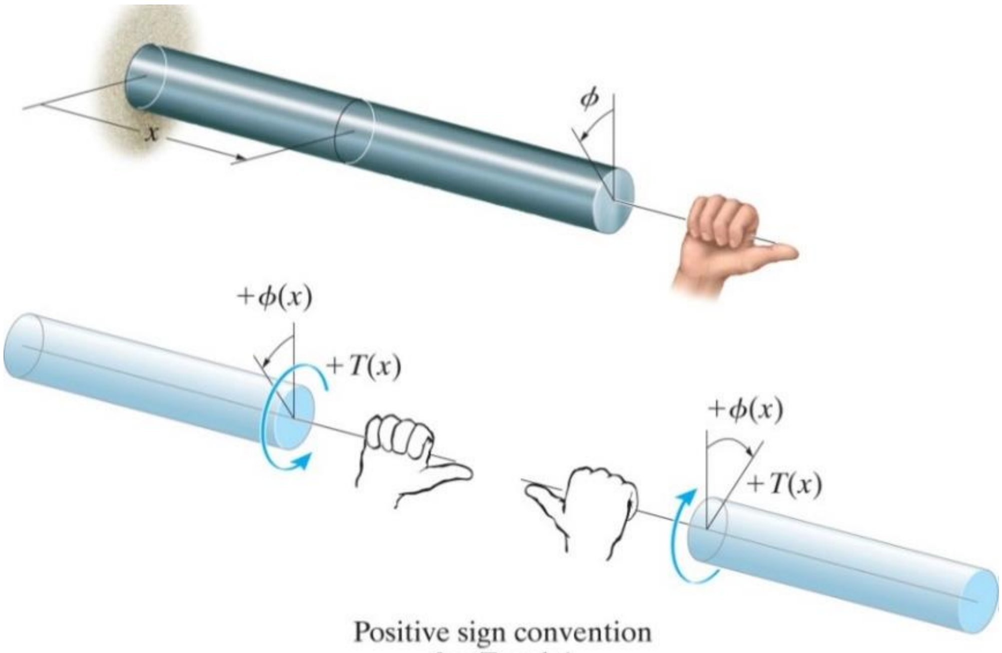
For the particular case in which the torque along the shaft and the cross section of the shaft are constant, then:

$$\phi = \frac{TL}{JG}$$

Multiple Torques: If the shaft is subjected to several different torques, or there are abrupt changes in the cross section area or shear modulus, then:

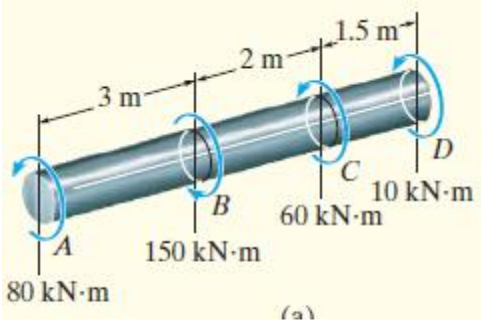
$$\phi = \sum \frac{TL}{JG}$$

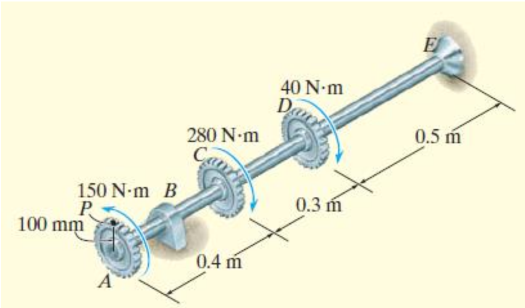
Sign Convention: The right hand rule is used to establish the sign of the torque and angle of twist: torque and angle of twist are positive if the thumb is directed outwards from the shaft when the fingers wrap around the shaft following the tendency of rotation.



Positive sign convention for T and ϕ.

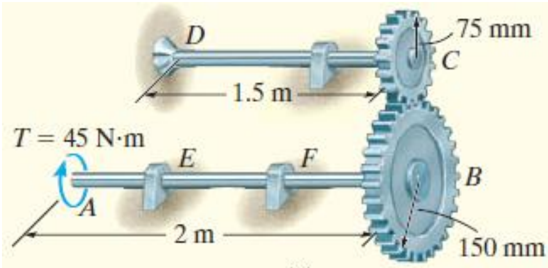
Example 5: Determine the angle of twist of the end A of the A-36 steel shaft shown in Figure. Also, what is the angle of twist of A relative to C? The shaft has a diameter of 200 mm. Take G = 75 GPa.

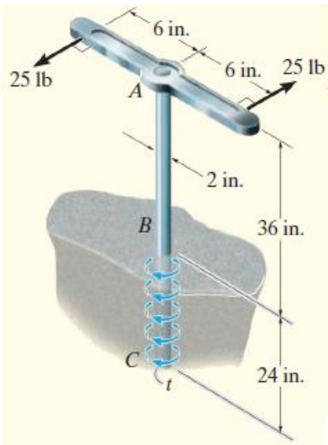




Example 6: The gears attached to the fixed-end steel shaft are subjected to the torques shown in Figure. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth P on gear A. The shaft turns freely within the bearing at B.

Example 7: The two solid steel shafts shown in Figure are coupled together using the meshed gears. Determine the angle of twist of end A of shaft AB when the torque T = 45 N. m is applied. Shaft DC is fixed at D. Each shaft has a diameter of 20 mm. G = 80 GPa.





Example 8: The 2-in.-diameter solid cast-iron post shown in Figure is buried 24 in. in soil. If a torque is applied to its top using a rigid wrench, determine the maximum shear stress in the post and the angle of twist of the wrench. Assume that the torque is about to turn the bottom of the post, and the soil exerts a uniform torsional resistance of t lb. in. /in. along its 24-in. buried length. G = $5.5(10^3)$ ksi.

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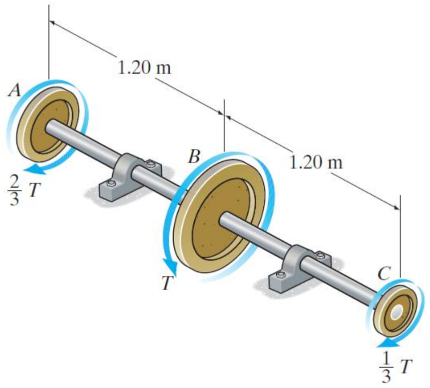
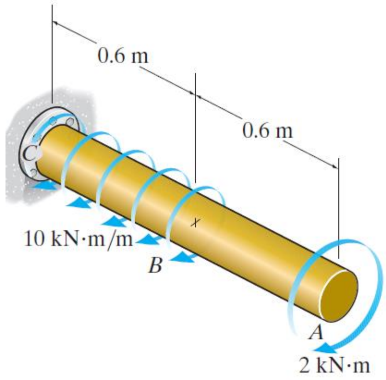
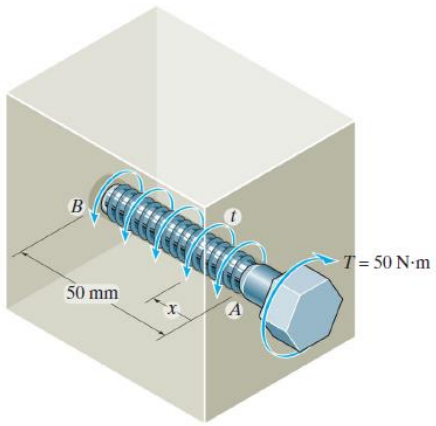


Q 1: The 60-mm-diameter shaft is made of 6061-T6 aluminum (G = 26 GPa). If the allowable shear stress is τ_{allow} = 80 MPa, and the angle of twist of disk A relative to disk C is limited so that it does not exceed 0.06 rad, determine the maximum allowable torque T.

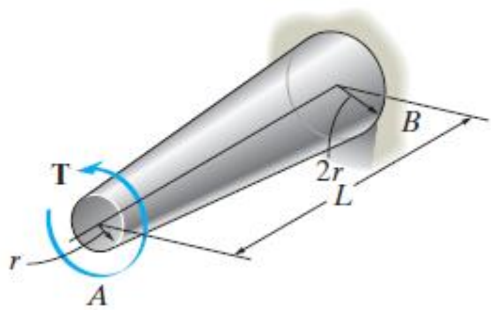
Q 2: The 80-mm diameter shaft is made of 6061-T6 aluminum alloy (G = 26 GPa) and subjected to the torsional loading shown. Determine the angle of twist at end A.





Q 3: The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation t = (kx^2) N . m/m, where x is in meters. If a torque of T = 50 N . m is applied to the bolt head, determine the constant *k* and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.

Q 4: The tapered shaft has a length L and a radius r at end A and 2r at end B. If it is fixed at end B and is subjected to a torque T, determine the angle of twist of end A. The shear modulus is G.



5. STATICALLY INDETERMINATE TORQUE-LOADED MEMBERS

A shaft is said to be ***statically indeterminate*** if the moment equation of equilibrium is not sufficient to determine the unknown torques acting on the element.

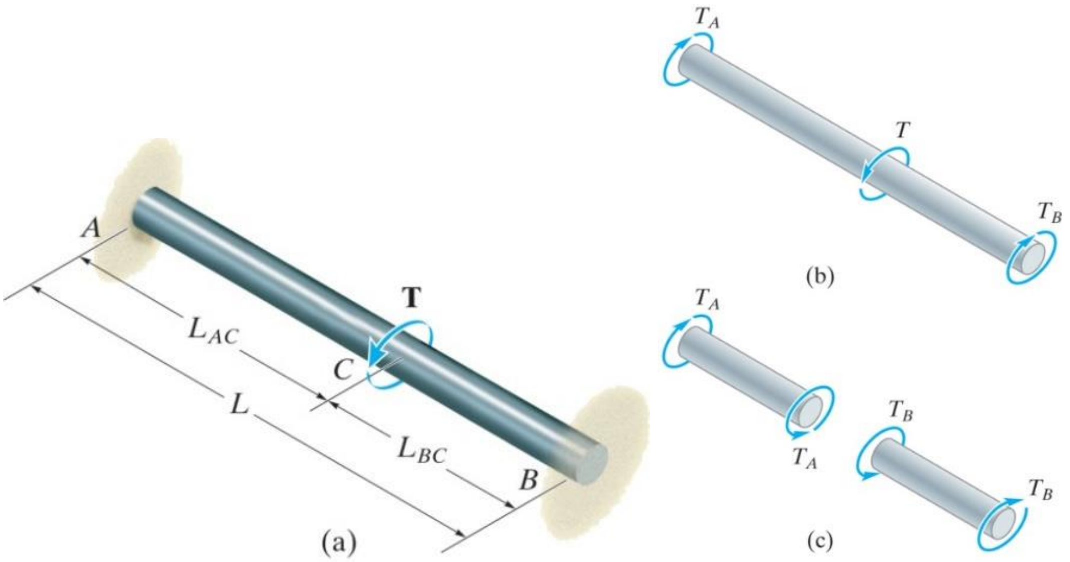
In order to determine the unknown reactions, a procedure analogous to the axial load problem is followed.

Thus, when there are more reactions than equations the problem is statically indeterminate.

To solve the problem first it is necessary to present a FBD of the shaft, along with the equilibrium equation.

$$\sum T = 0$$

Then, the compatibility equations are presented as needed involving the angle of twist based on the way the shaft is constrained. For example, if the shaft is fixed at both ends, then $\phi = 0$.



$T - T_A - T_B = 0$

$$\phi_{A/B} = 0$$

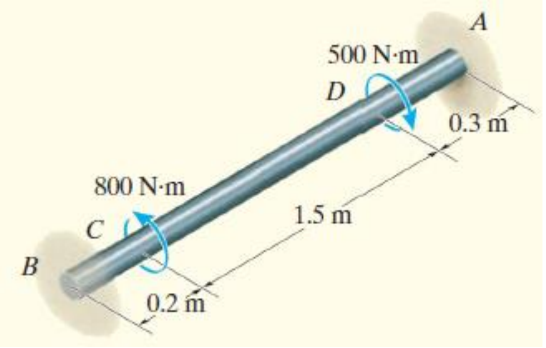
$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0$$

Solve

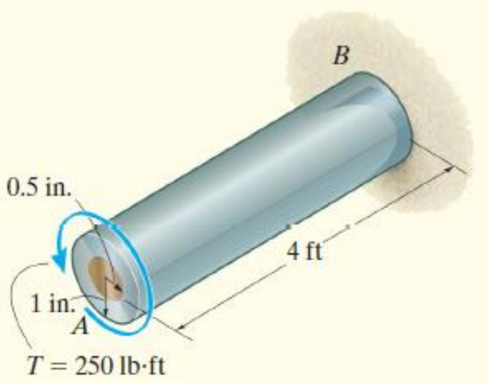
$$T_A = T\left(\frac{L_{BC}}{L}\right)$$

$$T_B = T\left(\frac{L_{AC}}{L}\right)$$

Example 9: The solid steel shaft shown in Figure has a diameter of 20 mm. If it is subjected to the two torques, determine the reactions at the fixed supports A and B.



Example 10: The shaft shown in Figure is made from a steel tube, which is bonded to a brass core. If a torque of T = 250 lb . ft is applied at its end, plot the shear-stress distribution along a radial line on its cross section. Take $G_{st} = 11.4(10^3)$ ksi, $G_{br} = 5.20(10^3)$ ksi.



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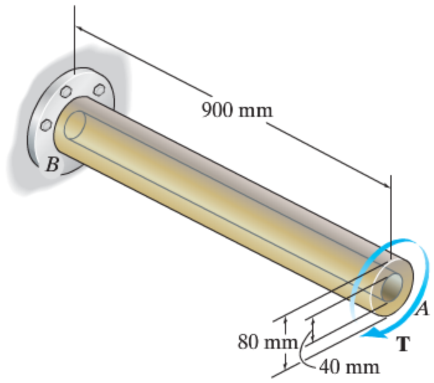
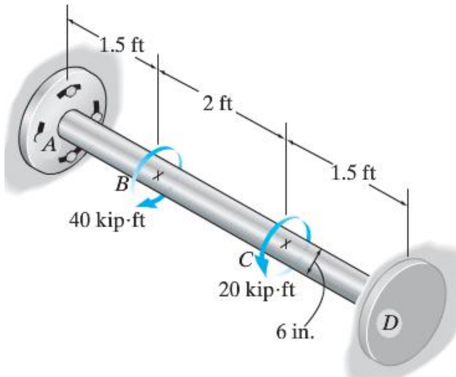
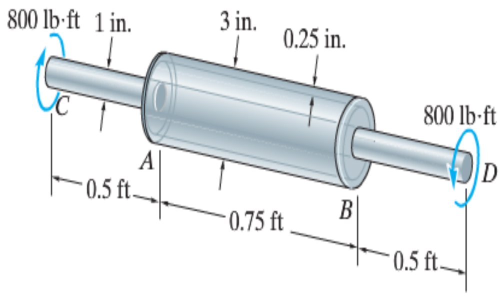


Q 1: The Am1004-T61 magnesium (G = 18 GPa) tube is bonded to the A-36 steel (G = 75 GPa) rod. If a torque of T = 5 kN·m is applied to end A, determine the maximum shear stress in each material. Sketch the shear stress distribution.

Q 2: The shaft is made of A-36 steel and is fixed at end D, while end A is allowed to rotate 0.005 rad when the torque is applied. Determine the torsional reactions at these supports.





Q 3: The composite shaft consists of a mid-section that includes the 1-in. diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 lb·ft. The material is A-36 steel.

Q 4: The motor A develops a torque at gear B of which is applied along the axis of the 2-in.-diameter steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft and do not resist torque. $G_{st} = 12(10^3)$ ksi.