CHAPTER 6 BENDING

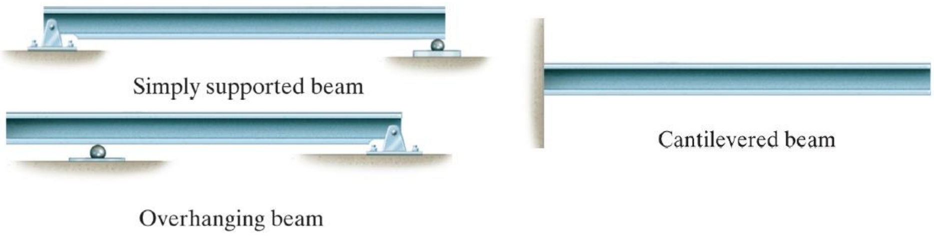
1. SHEAR AND MOMENT DIAGRAM

A ***Beam*** is a member that is slender and support loadings that are applied to its longitudinal axis.

Beams are considered as one of the most important structural elements.

• Beams support: Floors, bridges, wing of an aircraft, axle of a car, boom of a crane

• A beam is a structure that carries transverse loads. This means that a cross section will carry a ***shear force*** and a ***bending moment***.



Why do we need to know shear force and bending moment in a beam?

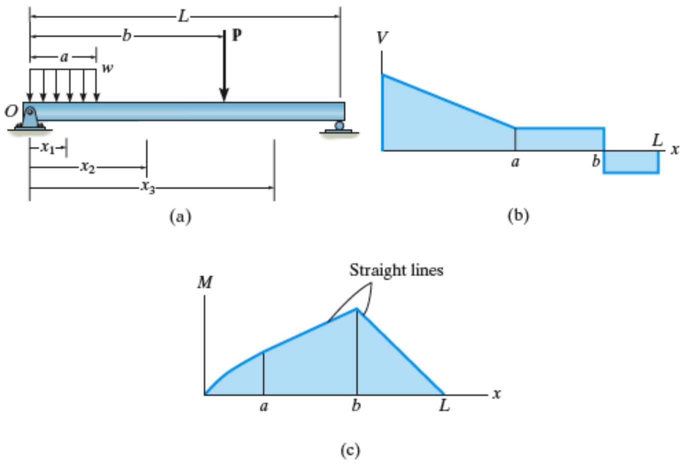
• The design of a beam requires a detailed knowledge of the variation of the internal shear force *V* and bending moment *M* acting at each point along the axis of the beam.

• After the shear force and bending-moment analysis is done, the theory of mechanics of materials and an appropriate engineering design code can be used to determine the beam's required cross-sectional area.

How to obtain a relation between V and M in terms of x?

• The variations of *V* and *M* in terms of the position *x* along the beam's axis are obtained using the method of sections. To apply this method we section the beam at an arbitrary distance *x* from one end rather than at a specified point.

• The plots will show graphical variations of *V* and *M* as functions of *x* and are named ***shear diagram*** and ***bending-moment diagram***.



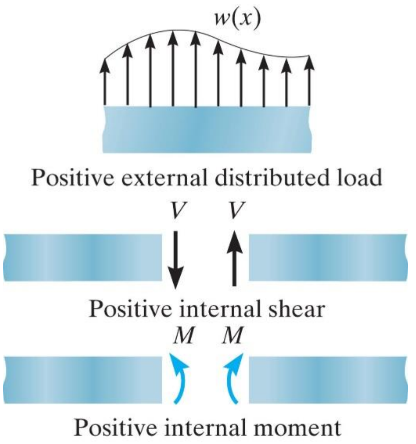
Sign Convention

The choice of a sign convention **is arbitrary**. However, we will adopt the following convention:

• ***Normal force (N)*** will be considered positive when generates tension.

• ***Shear force (V)*** will be considered positive when causes the beam segment where it acts to rotate clockwise.

• ***Bending moment (M)*** will be considered positive when the segment on which it acts tends to bend in a concave upward manner. *A positive bending moment creates compression at the top of the beam and tension at the bottom of the beam.*



Beam sign convention

One way to plot these quantities is by using the method of sections to find V and M as functions of length.

Procedures for Constructing Shear and Moment Diagram:

• First, draw FBD for the entire beam and the equilibrium equations are used to calculate the reaction forces.

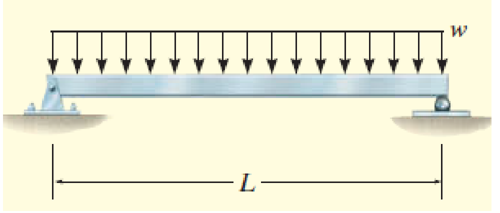
• It is necessary to recognize that V and M may not be able to be represented as a single function over the entire length. In this case, the beam must first be divided into sections where V and M can be represented by single functions. This occurs ***between points of concentrated force or moment***, ***supports, or a change in the functional representation of a distributed load***.

• Then, using the method of sections, interior cuts are made for each section at an arbitrary distance x measured from the left hand end of the beam.

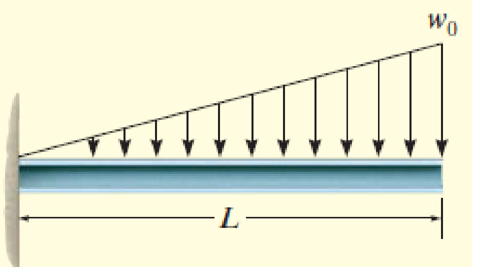
• Next, the FBD of each part of the beam is drawn assuming V and M in their positive directions and apply equilibrium equations to determine functions for V and M in terms of x.

• Then the functions obtained are plotted.

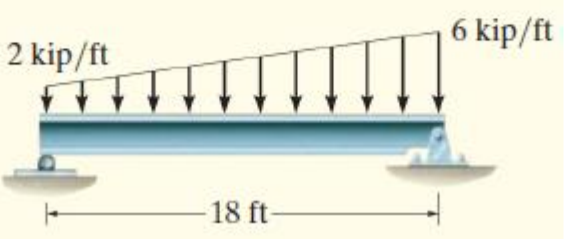
Example 1: Draw the shear and moment diagrams for the beam shown in Figure.



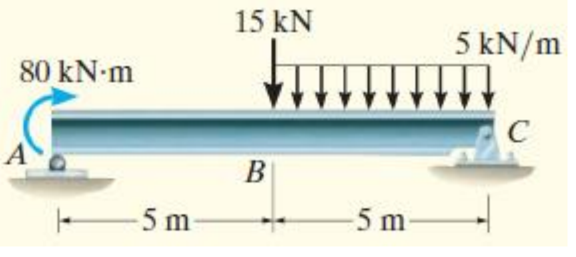
Example 2: Draw the shear and moment diagrams for the beam shown in Figure.



Example 3: Draw the shear and moment diagrams for the beam shown in Figure.



Example 4: Draw the shear and moment diagrams for the beam shown in Figure.

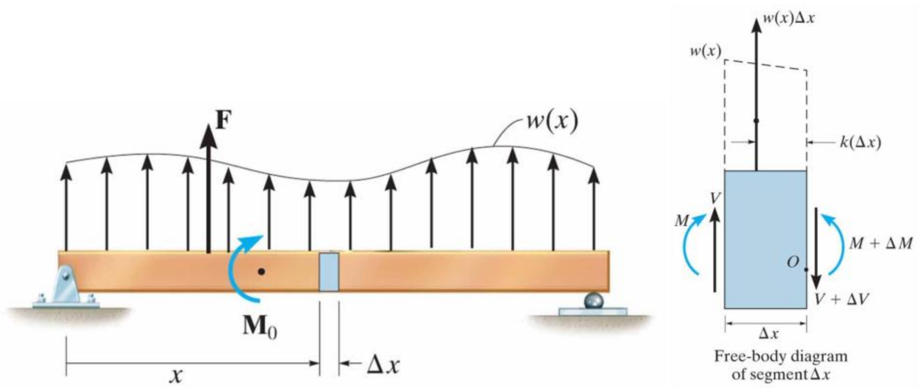


2. GRAPHICAL METHOD FOR CONSTRUCTING MOMENT AND SHEAR DIAGRAM

When a beam is subjected to several different loadings, the construction of the shear force and bending moment diagrams can be quite involving.

2.1. Regions of Distributed Load

By using the differential relation that exists between distributed load and shear, and between shear and moment, the construction of the shear and moment diagrams can be simplified.



$$+\uparrow \Sigma F_y = 0; \qquad V + w(x)\,\Delta x - (V + \Delta V) = 0$$

$$\Delta V = w(x)\,\Delta x$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad -V\,\Delta x - M - w(x)\,\Delta x\left[\tfrac{1}{2}(\Delta x)\right] + (M + \Delta M) = 0$$

$$\Delta M = V\,\Delta x + w(x)\,\tfrac{1}{2}(\Delta x)^2$$

If Δx is very small such that the term containing Δx^2 can be neglected when compared to the other terms: $V\,\Delta x = \Delta M$

$$\frac{dV}{dx} = w(x)$$

slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V(x)$$

slope of moment diagram at each point = shear at each point

$$\Delta V = \int w(x)dx$$

change in shear = area under distributed loading

$$\Delta M = \int V(x)dx$$

change in moment = area under shear diagram

These equations are valid in ***regions of distributed load*** (between concentrated forces and moments).

By integrating:

$$\frac{dV}{dx} = w(x) \quad \text{and} \quad \frac{dM}{dx} = V$$

The following expressions are obtained:

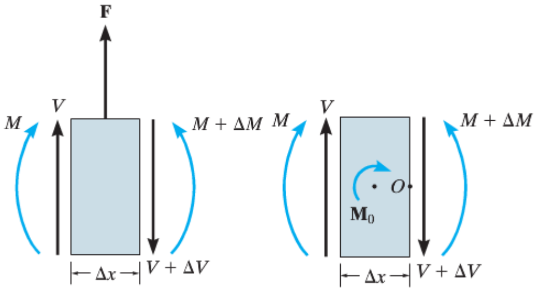
$$V_2 - V_1 = \int w(x)dx \qquad or \qquad V_2 = V_1 + \int w(x)dx$$

That is, the shear at some point x_2 equals the shear at the starting point x_1 plus the area under the distributed load curve between the two points.

$$M_2 - M_1 = \int V(x)dx \qquad or \qquad M_2 = M_1 + \int w(x)dx$$

That is, the moment at a point x_2 equals the moment at the starting point x_1 plus the area under the shear diagram between the two points.

2.2. Regions of Concentrated Force and Moment



$$+\uparrow \Sigma F_y = 0; \qquad V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

• At points of concentrated forces, a discontinuity in the shear diagram in ***the same direction*** as the force occurs, the magnitude of this discontinuity is equal to the concentrated force.

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \Delta M - M_0 - V\,\Delta x - M = 0$$

Letting $\Delta x \to 0$, we get

$$\Delta M = M_0$$

• At points of concentrated moments, a discontinuity in the moment diagram occurs, the magnitude of the discontinuity is equal to the magnitude of the concentrated moment.

• The direction of the jump is ***up*** when the moment is ***clockwise*** and ***down*** when ***counterclockwise***.

Thus, the procedure for graphically constructing shear and moment diagrams can be summarized as follows:

1. Draw a free body diagram of the beam and use equilibrium to determine the reaction forces.

2. Draw the shear diagram using the following rules:

a. Start at the left end ($x = 0$)

b. When a concentrated force appears, follow the force. That is jump in the direction of the force with a magnitude equal to the magnitude of the force.

c. Between concentrated forces, apply: $V_2 = V_1 + \int w(x)\, dx$ to find the shear at the end of the section

d. Then apply $dV/dx = w(x)$ to draw the line over the section.

3. Draw the moment diagram by following these steps:

a. Start at the left end.

b. When a concentrated moment appears, a jump up is drawn if the moment is clockwise or down if the moment is counterclockwise. The magnitude of the discontinuity is equal to the magnitude of the moment.

c. Between concentrated moments, apply: $M_2 = M_1 + \int V(x)\, dx$ to find the moment at the end of a section.

d. Then apply $dM/dx = V(x)$ to draw the line over the section.

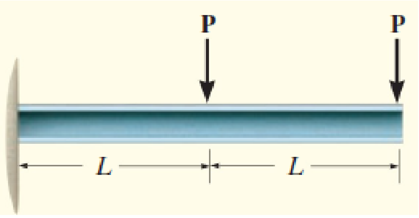
Notes:

1. Both diagrams must return to zero at the right end of the beam. If one of them does not, then a mistake has been made. For example, an area may have been calculated incorrectly or the reactions may be wrong.

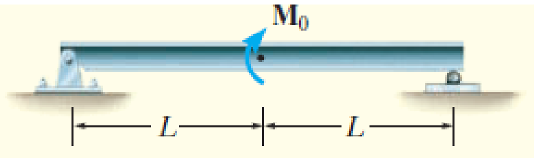
2. The maximum moment will occur at an end or where $dM/dx = V = 0$. Therefore, it is necessary to determine M at all places where $V = 0$.

3. If $w(x)$ is a curve of degree n, $V(x)$ will be a curve of degree $n + 1$ and $M(x)$ will be a curve of degree $n + 2$. For example, if $w(x)$ is uniform, $V(x)$ will be linear and $M(x)$ will be parabolic.

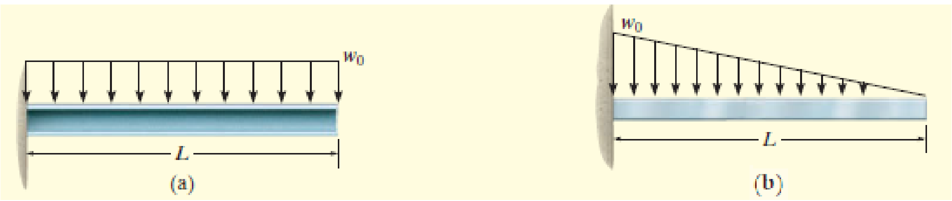
Example 5: Draw the shear and moment diagrams for the beam shown in Figure.



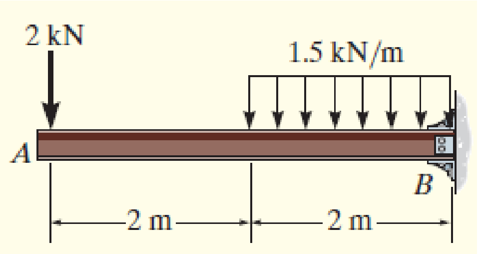
Example 6: Draw the shear and moment diagrams for the beam shown in Figure.



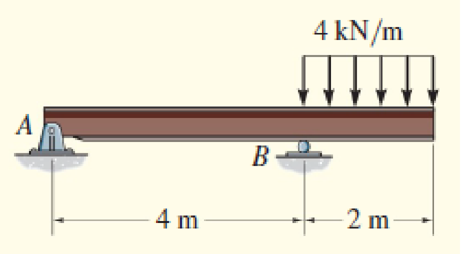
Example 7: Draw the shear and moment diagrams for each of beams shown in Figures.



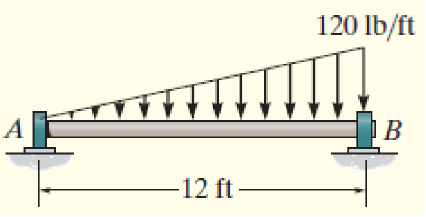
Example 8: Draw the shear and moment diagrams for the cantilever beam shown in Figure.



Example 9: Draw the shear and moment diagrams for the cantilever beam shown in Figure.

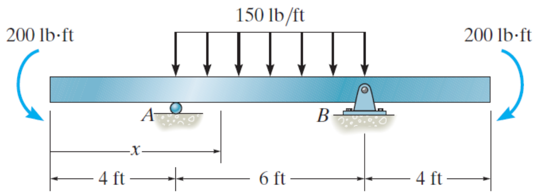


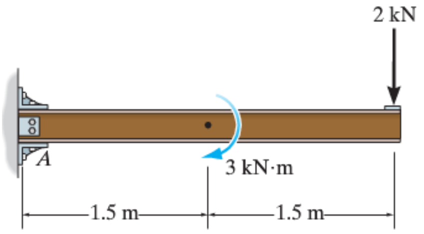
Example 10: The shaft in Figure is supported by a thrust bearing at A and a journal bearing at B. Draw the shear and moment diagrams.



Sheet No. 1

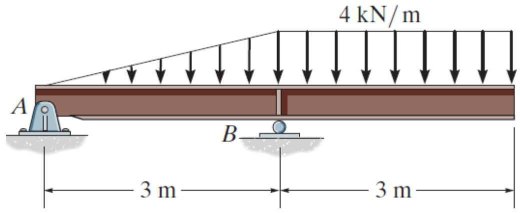
Q 1: Draw the shear and moment diagrams for the beam.

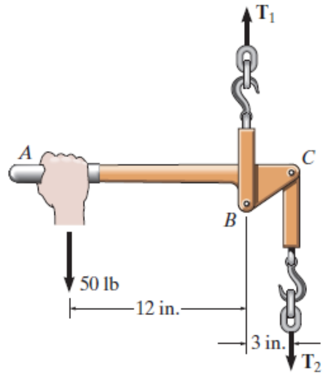




Q 2: Draw the shear and moment diagrams for the cantilever beam.

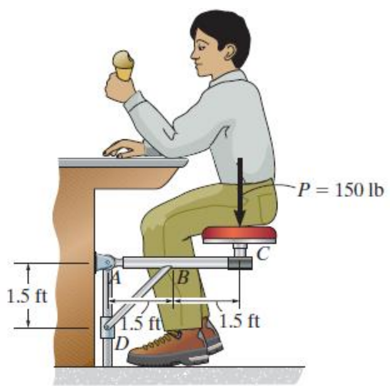
Q 3: Draw the shear and moment diagrams for the overhang beam.

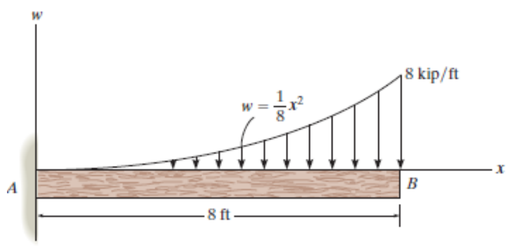




Q 4: If the force applied to the handle of the load binder is 50 lb, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.

Q 5: Members ABC and BD of the counter chair are rigidly connected at B and the smooth collar at D is allowed to move freely along the vertical post. Draw the shear and moment diagrams for member ABC.





Q 6: Draw the shear and moment diagrams for the figures below.