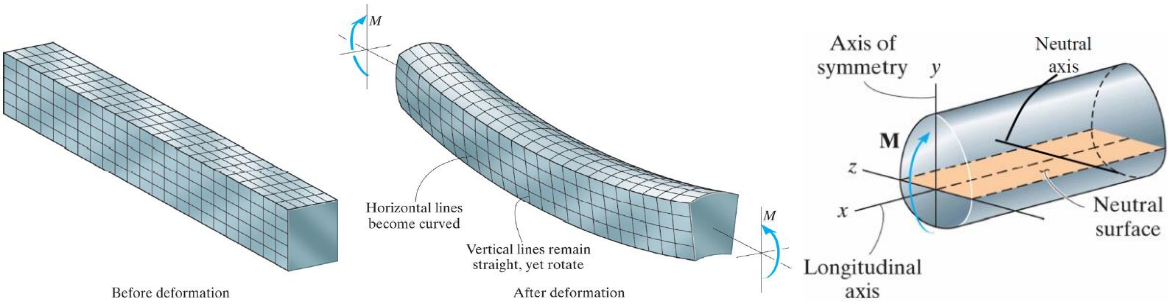
CHAPTER 6 BENDING

3. BENDING DEFORMATION OF A STRAIGHT MEMBER



Consider a beam with a cross section area symmetric to an axis and upon which a bending moment is applied perpendicular to the axis of symmetry of the cross section area.

• After the moment is applied, the beam takes a curved shape.

• The lines along the length become arcs and the vertical lines become radial, intersecting at a common center of curvature.

• The upper part of the beam is in ***compression*** while the lower one is in ***tension***.

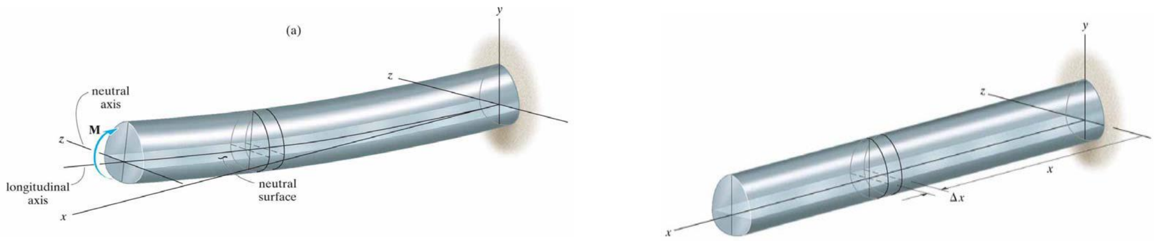
• Somewhere in between, there must be a point that has no stress or strain.

• If there were a line along in the longitudinal direction through the center of the beam, it would not have changed its length.

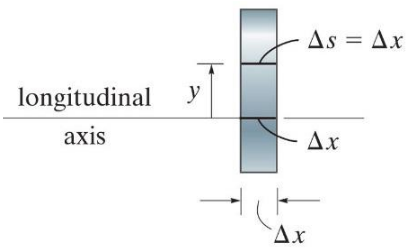
• The surface formed by the points of the beam that do not experience any stress is called ***neutral surface.***

• Since vertical lines remain straight then the ***cross section remains planar***.

• The intersection of the neutral surface with the cross section is called the ***neutral axis.***



• To determine the relation between the distortion of the beam and the strain of the material a differential element of the beam is to be considered.



Undeformed element

Δx is contained in the neutral surface and therefore its length remains constant. Δs which is located some distance y above the neutral surface will contract to become $\Delta s'$. Thus, the strain in the element located at y is:

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

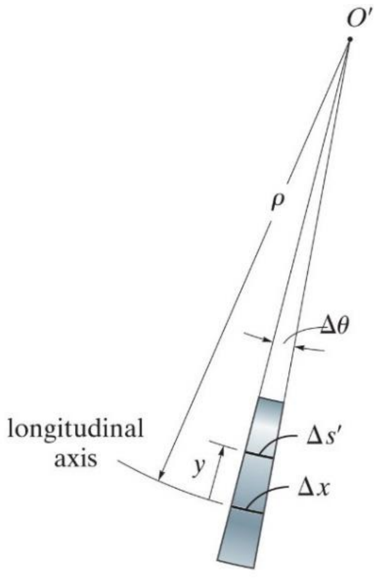
To determine ε in terms of y, observe that:

$$\Delta x = \Delta s = \rho \Delta \theta$$

$$\Delta s' = (\rho - y)\Delta \theta$$

$$\epsilon = \lim_{\Delta \theta \to 0} \frac{(\rho - y)\Delta \theta - \rho \Delta \theta}{\rho \Delta \theta}$$

$$\epsilon = -\frac{y}{\rho}$$

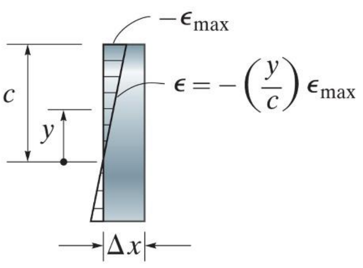


Deformed element

This shows that the longitudinal **normal strain** of any element within the beam depends on its location y on the cross section and on the radius of curvature of the beam's longitudinal axis.

Thus, the longitudinal normal strain is linearly related to y from the neutral axis.

The maximum strain occurs at the outermost fiber of the beam.



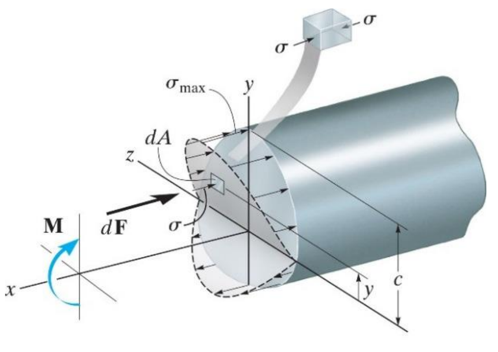
Normal strain distribution

$$\frac{\varepsilon}{\varepsilon_{max}} = -\left(\frac{\frac{y}{\rho}}{\frac{c}{\rho}}\right)$$

$$\varepsilon = -\left(\frac{y}{c}\right)\varepsilon_{max}$$

4. THE FLEXURE FORMULA

Since the longitudinal normal strain varies linearly with y measured from the neutral surface ($\varepsilon = -y/\rho$), it is possible to relate the stress distribution in a beam to the bending moment acting on the beam's cross section area assuming a linear relation between normal strain and **normal stress**.



Bending stress variation

Thus, assume that the material stays within the linear, elastic region of its behavior, then

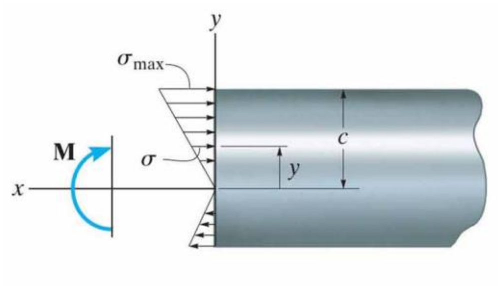
$$\sigma = E\varepsilon = -E\frac{y}{\rho}$$

The relation states that

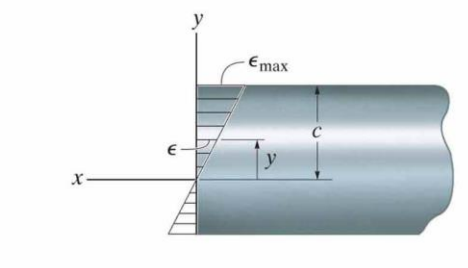
• The stress varies ***linearly*** along the cross section area of the beam.

• ***The maximum stress or strain occurs at the point farthest from the neutral surface.***

$$\sigma_{max} = -E\frac{c}{\rho}$$



Bending stress variation



Normal strain variation
(profile view)

Which results in:

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{\max}$$

As observed from this equation: Under a positive moment which acts in the +z–direction, positive values of y produce negative values of σ, that is compressive stress, and negative values of y produce positive values of σ, that is tensile stress.

Since the ***resultant force*** produced by the stress distribution over the cross section area of the beam must be equal to zero. That is the stress distribution must be statically equivalent to the forces on the beam cross section:

$$\int_A dF = F_R = \int_A \sigma dA = 0$$

$$\int_A -\left(\frac{y}{c}\right)\sigma_{max} dA = 0$$

$$-\frac{\sigma_{max}}{c}\int_A y dA = 0$$

$$\int_A y dA = 0$$

Recall the formula to find the centroid of an area:

$$\bar{y} = \frac{\int_A y dA}{\int_A dA} = 0$$

This means that **the *neutral axis* is also the horizontal *centroidal axis* for the cross section**. Thus, once the centroid of the beam's cross section area is determined, the neutral axis is known.

Similarly, the moment produced by the stress distribution about the neutral axis must be equal to the resultant internal moment *M*:

$$(M_R)_z = \Sigma M_z; \qquad M = \int_A y\, dF = \int_A y(\sigma\, dA) = \int_A y\left(\frac{y}{c}\sigma_{max}\right) dA$$

$$M = \frac{\sigma_{max}}{c}\int_A y^2\, dA$$

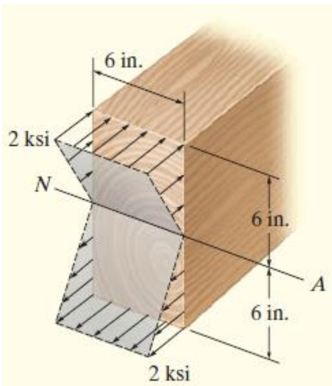
$$\sigma_{max} = \frac{Mc}{I} \qquad\qquad \sigma = -\frac{My}{I}$$

Notice that y is measured upward from the neutral axis and that M is pointed toward the top of the cross section.

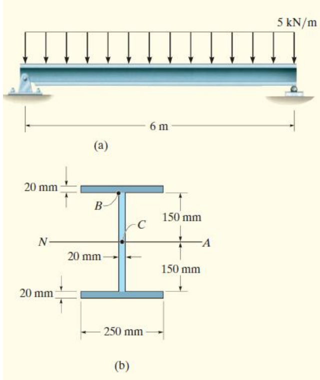
Here, I is the moment of inertia of the cross-sectional area about the neutral axis. It is possible to eliminate the need for a sign by noticing that σ is positive when the stress is tensile (+) and σ is negative when the stress is compressive (-). Thus, the ***flexure formula*** can be presented as:

$$\boldsymbol{\sigma = \frac{My}{I}}$$

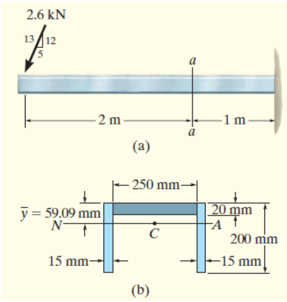
Example 15: A beam has a rectangular cross section and is subjected to the stress distribution shown in Figure. Determine the internal moment M at the section caused by the stress distribution (a) using the flexure formula, (b) by finding the resultant of the stress distribution using basic principles.



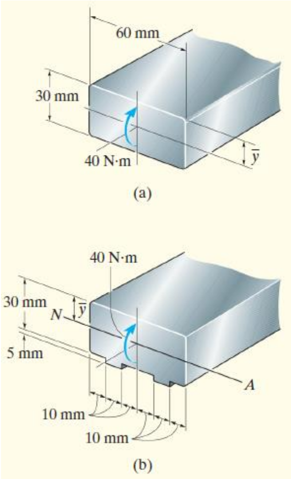
Example 16: The simply supported beam in Figure (a) has the cross-sectional area shown in Figure (b). Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location. Also, what is the stress at point B?



Example 17: The beam shown in Figure (a) has a cross-sectional area in the shape of a channel, Figure (b). Determine the maximum bending stress that occurs in the beam at section a–a.

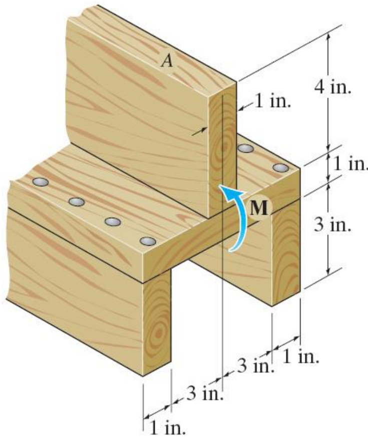


Example 18: The member having a rectangular cross section, Figure (a), is designed to resist a moment of 40 N. m. In order to increase its strength and rigidity, it is proposed that two small ribs be added at its bottom, Figure (b). Determine the maximum normal stress in the member for both cases.

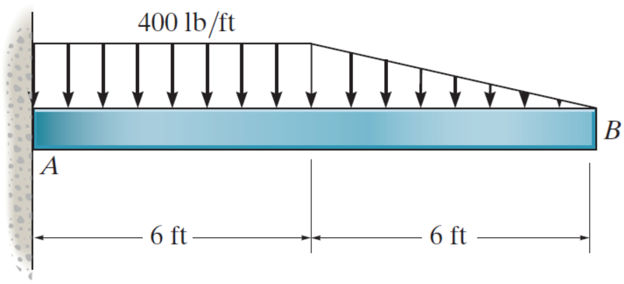


Sheet No. 2

Q 1: If the beam is subjected to an internal moment of M = 2 kip·ft, (a) determine the maximum tensile and compressive stress in the beam. Also, sketch the bending stress distribution on the cross section, (b) determine the resultant force of the bending stress distribution acting on the top vertical board A.

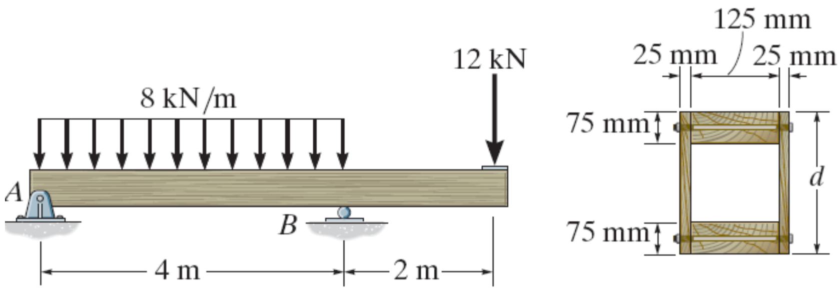


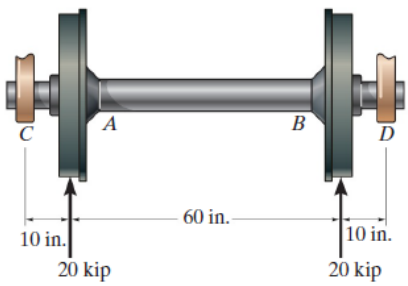
Q 2: If the allowable tensile and compressive stress for the beam are $(\sigma_{allow})_t$ = 2 ksi and $(\sigma_{allow})_c$ = 3ksi, respectively, determine the maximum allowable internal moment M that can be applied on the cross section.



Q 3: If the beam has a square cross section of 6 in. on each side, determine the absolute maximum bending stress in the beam.

Q 4: If d = 450 mm, determine the absolute maximum bending stress in the beam.





Q 5: The axle of the freight car is subjected to a wheel loading of 20 kip. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in.