CHAPTER 5 TORSION

6. SOLID NONCIRCULAR SHAFTS

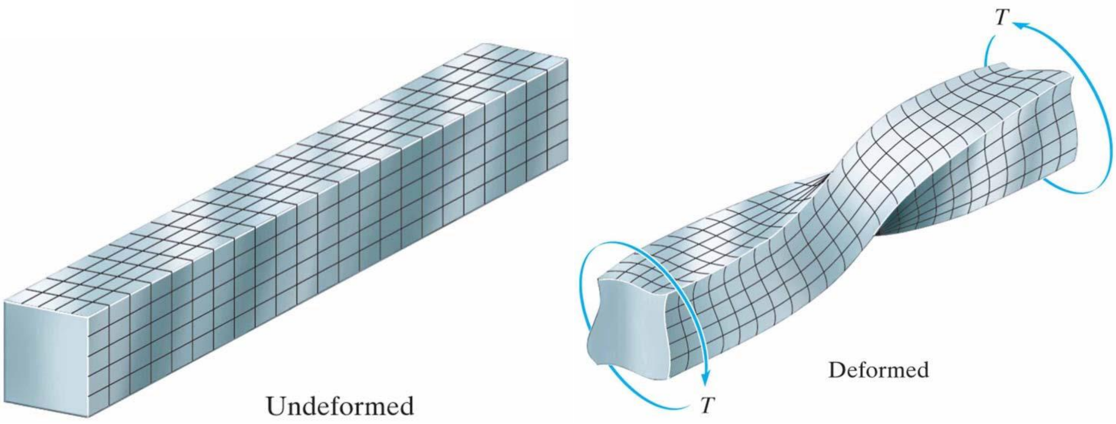
As it was established previously, when torque is applied on a shaft with circular cross–section area:

• The shear stress varies linearly from zero at it center to a maximum at its outer surface.

• The shear strain is the same at all points on the same radius.

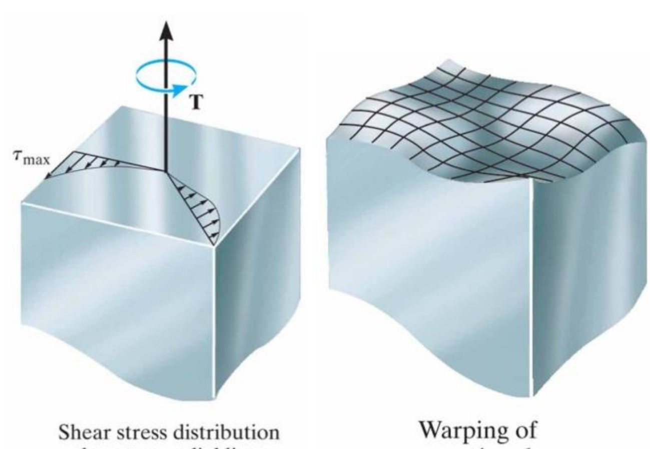
• The cross section area do not deform but instead remains plain after the shaft has twisted.

For shafts with non–circular cross section area, due to the lack of axisymmetry the cross section will bulge or warp when subjected to torque.



For solid noncircular cross sections, the theory is beyond the scope of this course.

Table below presents the equations for 3 common solid cross sections.

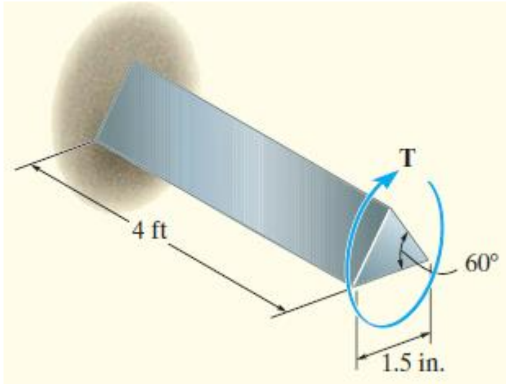


Shear stress distribution along two radial lines

Warping of cross-sectional area

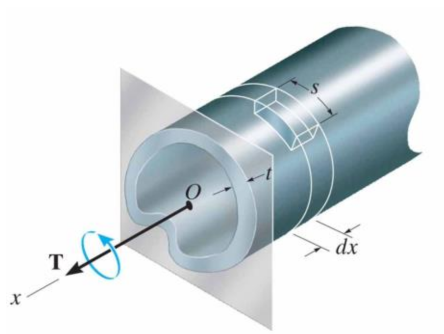
Shape of cross section	τ_{max}	ϕ
Square (side a)	$\frac{4.81\,T}{a^3}$	$\frac{7.10\,TL}{a^4G}$
Equilateral triangle (side a)	$\frac{20\,T}{a^3}$	$\frac{46\,TL}{a^4G}$
Ellipse (semi-axes a, b)	$\frac{2\,T}{\pi ab^2}$	$\frac{(a^2+b^2)TL}{\pi a^3b^3G}$

Example 11: The 6061-T6 aluminum shaft shown in Figure has a cross-sectional area in the shape of an equilateral triangle. Determine the largest torque T that can be applied to the end of the shaft if the allowable shear stress is τ_{allow} = 8 ksi and the angle of twist at its end is restricted to fallow = 0.02 rad. How much torque can be applied to a shaft of circular cross section made from the same amount of material?



7. THIN-WALLED TUBES HAVING CLOSED CROSS SECTIONS

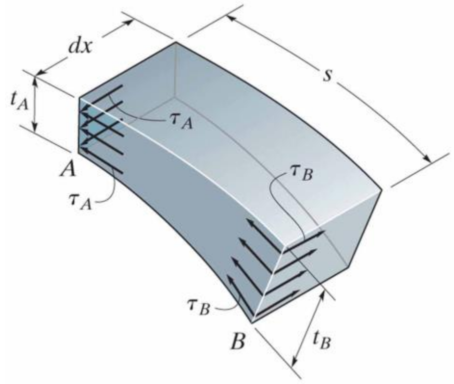
Objective: To study the effects of applying a torque to a thin–walled tube with a *closed* cross–section.



Consider a shaft with thin walls, an arbitrary cross–section area and a variable thickness *t*.

Under these considerations it is possible to assume that the shear stress, is *uniformly distributed* across the thickness of the tube at any given point.

Thus, it is assumed a constant shear stress through the thickness of the shaft being tangent to the wall centerline.



***Shear Flow* (*q*).** Considering a differential element with width *dx*, thickness t_A at end and t_B at the other.

Due to internal torque, shear stresses are created on the front face of the element (τ_A and τ_B).

Since shear stress always occurs in a set of four stresses, τ_A and τ_B must also act on the longitudinal sides of the element.

By applying $\Sigma F_x = 0$:

$$\tau_A t_A dx = \tau_B t_B dx$$ **Or** $$\tau_A t_A = \tau_B t_B$$

This relation states that ***the product of the average shear stress times the thickness of the tube is the same at any point on the tubes cross–section area.***

This product is called ***shear flow* (*q*)**. And it is generally expressed as:

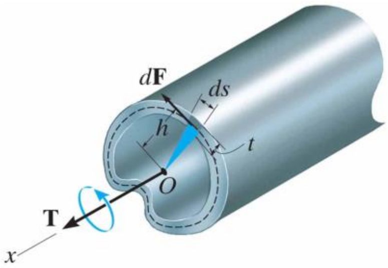
$$q = \tau_{avg} t$$

Since q is constant, then the *largest* average shear stress occurs where the thickness of the tube is the *smallest*.

The shear flow around the circumference of the wall must be statically equivalent to the applied torque.

Average Shear Stress. To relate the average stress to the torque T, consider the shear stress produced by a force acting on a differential element of the tube.

$$dF = \tau_{avg} dA = \tau_{avg} t dS$$



This force is tangential to the centerline of the tube's wall.

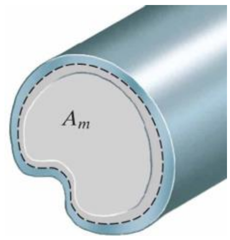
If the arm of the moment is h, then

$$dT = hdF = h(\tau_{avg} t dS)$$

Where: $dF = q\ ds$ and $q = \tau_{avg}\, t$

By integrating over the entire cross section area:

$$T = \int h\, \tau_{avg} t dS$$



Since $q = \tau_{avg}\, t$ = const.:

$$T = \tau_{avg} t \int h\ dS$$

By noticing that $dA_m = ½\ hds$

$$T = \tau_{avg} t \int dA_m = 2\tau_{avg} t A_m$$

Where A_m is the area enclosed by the wall centerline.

$$\tau_{avg} = \frac{T}{2tA_m}$$

Here

τ_{avg} = the average shear stress acting over a particular thickness of the tube

T = the resultant internal torque at the cross section

t = the thickness of the tube where τ_{avg} is to be determined

A_m = the mean area enclosed within the boundary of the centerline of the tube's thickness.

Finally, since $q = \tau_{avg}\, t$, then the shear flow throughout the cross section becomes:

$$q = \frac{T}{2A_m}$$

The angle of twist can be found as:

$$\phi = \frac{TL}{4A^2{}_m G}\int \frac{dS}{t}$$

$$\int \frac{ds}{t}$$

is evaluated knowing how ***t*** varies with the circumference variable ***s***

If ***t*** is constant,

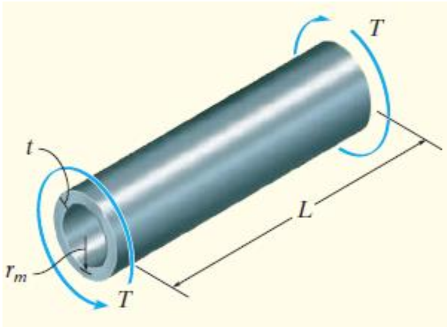
$$\int \frac{ds}{t} = s/t \quad and \quad \phi = \frac{TL}{4A_m^2 G}\frac{s}{t}$$

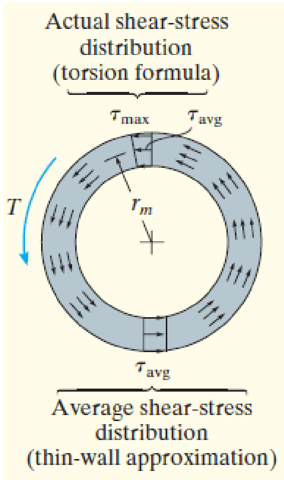
where ***s*** is the length of circumference centerline

If t is constant in sections of the wall,

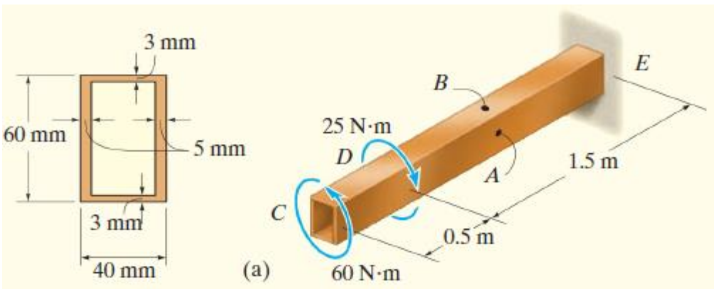
$$\int \frac{ds}{t} = \sum (s/t)_i$$

Example 12: Calculate the average shear stress in a thin-walled tube having a circular cross section of mean radius r_m and thickness t, which is subjected to a torque T, Figure. Also, what is the relative angle of twist if the tube has a length L?





Example 13: The tube is made of C86100 bronze and has a rectangular cross section as shown in Figure. If it is subjected to the two torques, determine the average shear stress in the tube at points A and B. Also, what is the angle of twist of end C? The tube is fixed at E. Take G = 38 GPa.



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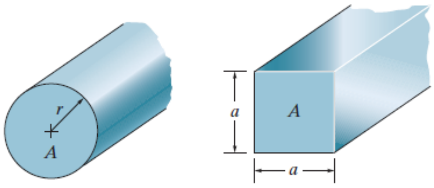
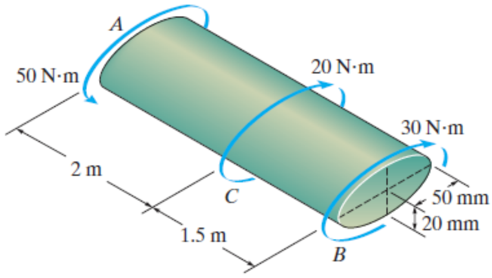
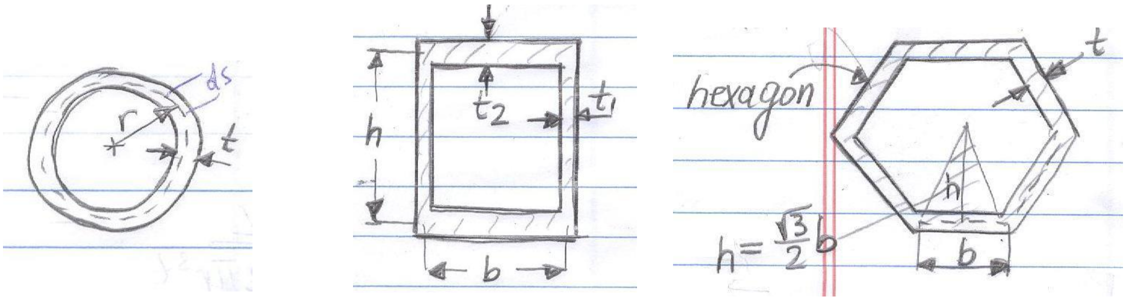


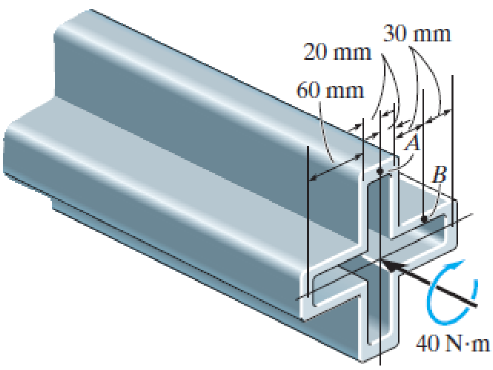
Q 1: Compare the values of the maximum elastic shear stress and the angle of twist developed in 304 stainless steel shafts having circular and square cross sections. Each shaft has the same cross-sectional area of length of 36 in., and is subjected to a torque of 4000 lb . in.

Q 2: The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions AC and BC, and the angle of twist of end B relative to end A.



Q 3: Determine f and t. T, G and L are given.





Q 4: The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of T = 40 N. m, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.