CHAPTER 7 TRANSVERSE SHEAR

3. SHEAR FLOW IN BUILT–UP MEMBERS

Many times elements are constructed by adding together several simpler elements with the objective of producing a more resistant structural element.

These built up members are made of pieces nailed, bolted or glued together.

Therefore the connectors must be able to carry the shear along the beams length.

In order to design these fasteners or determine their spacing it is necessary to know the shear force that they need to support.

This loading, when measured as force per unit length is called: ***shear flow*** (***q***).

Thus by defining the ***shear flow*** (***q***) as the force per unit length:

$$q = \frac{VQ}{I}$$

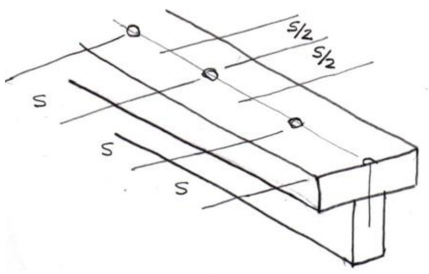
Then, when analyzing a built up member, that is a beam that is made of pieces nailed, bolted or glued together, the connectors must be able to carry the shear along the beams length.

In the case of a glued joint, it is possible to calculate the stress in the glue using:

$$\tau = \frac{VQ}{It}$$

In the case of a nail or bolt, the connectors must be spaced along the length of the beam appropriately so as to carry the shear flow.

Consider two pieces nailed together with nails spaced ***s*** apart.



Each nail must carry the shear flow (***q***) over the length of beam which extends half way to the nail on either side. Therefore, the nail must carry the shear flow over the length ***s***.

The net force, ***q s*** , must be less than the net force ***Fn*** the nail can carry.

Since there could be multiple nails side be side attaching two pieces, a factor ***n*** for the number of connectors is included. Thus,

$$nF_n = qs$$

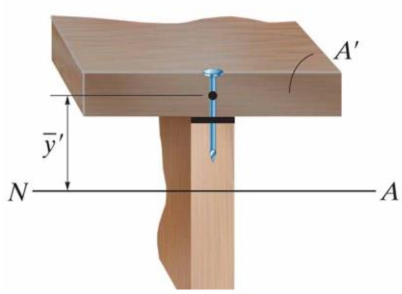
When finding Q, the cut must be through the cross section at the place where the connectors hold the pieces together.

Thus, when determining $\boldsymbol{q = VQ/I}$, the parameters I and Q need to be calculated with respect to the neutral axis of the beam.

To apply properly this equation is very important to identify Q correctly when calculating the shear flow (q) at a particular location.

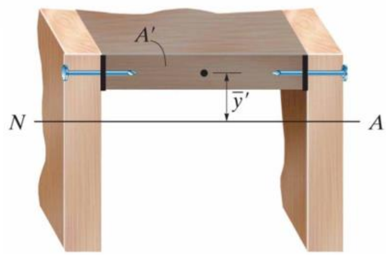
Consider the beam shown, the flange (darker segment) is connected to the web by a nail at a plane identified by a black line..

q is calculated using Q determined from A' and $\bar{y}'$.

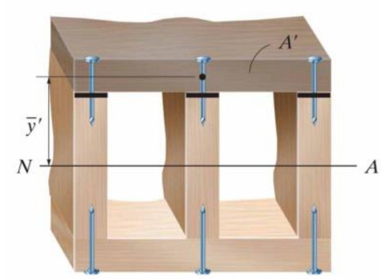


In this particular example, q will be resisted by a single nail (n = 1)

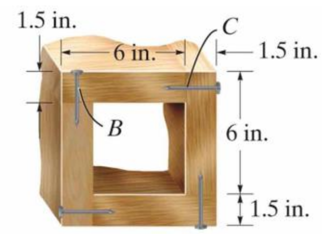
In this second case, the area A' is held by two nails, therefore (n = 2) and q is calculated using Q determined from A' and $\bar{y}'$.



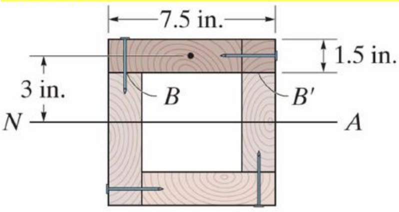
A third case shows the A' held by 3 nails, where the shear flow (n = 3) is once again determined using Q determined from A' and $\bar{y}'$.

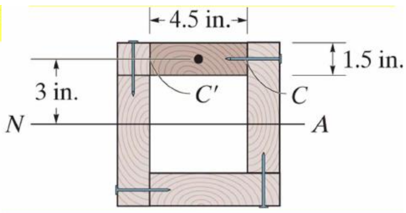


A final case occurs when dealing with a closed cross section, as the one shown. To determine the shear flow for B and C, q_B and q_C, respectively, the areas are sectioned as shown.



q_B: $Q_B = \bar{y}A'$; $\bar{y}$ = 3", A' = 7.5×1.5

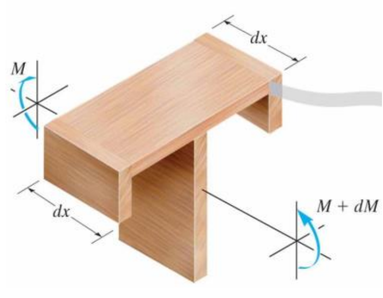




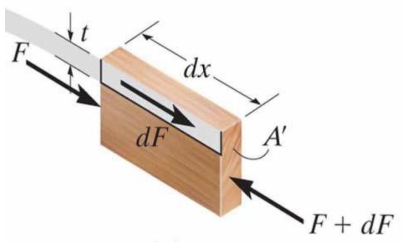
q_c: $Q_c = \bar{y}A'$; $\bar{y}$ = 3", A' = 4.5×1.5

Consider the juncture where the segment is connected to the flange of the beam.

As shown through the differential element, three forces act over this segment. F and $F + dF$ are due to the normal stresses produced by M and $M + dM$, respectively. The third force which is required to satisfy the equilibrium of the system is equal to dF.



Thus, since: $\sigma = \frac{My}{I}$ Then: $dF = \frac{dM}{I}\int y dA'$



As defined previously, the integral represents Q. Now, since the length of the segment is dx, the shear flow (force per unit length) along the beam is $q = dF/dx$. Therefore, since $V = dM/dx$:

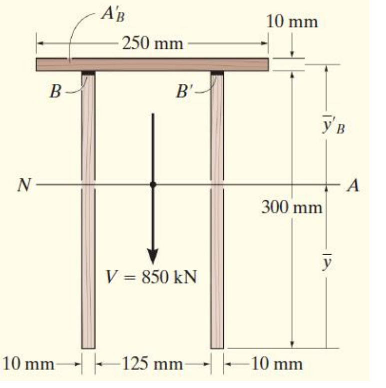
$$q = \frac{VQ}{I}$$

- ***q*** is the shear flow, measured as a force per unit length along the beam.
- V is the internal resultant shear force.
- I is the moment of inertia of the *entire* cross section area calculated about the neutral axis.
- $Q = \tilde{y}'A'$, where A' is the cross section area of the segment that is connected to the beam at the juncture where the shear flow is to be alculated and $\tilde{y}'$ is the distance from the neutral axis to the centroid of A'.

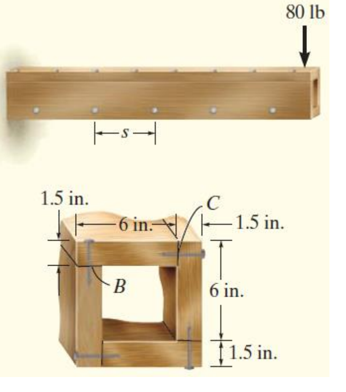
Final Note

Shear flow **(*q*)** is a measure of the force per unit length along the axis of a beam. This value is calculated from the shear formula and is used to determine the shear force developed in fasteners and glue that holds various segments of a composite beam together.

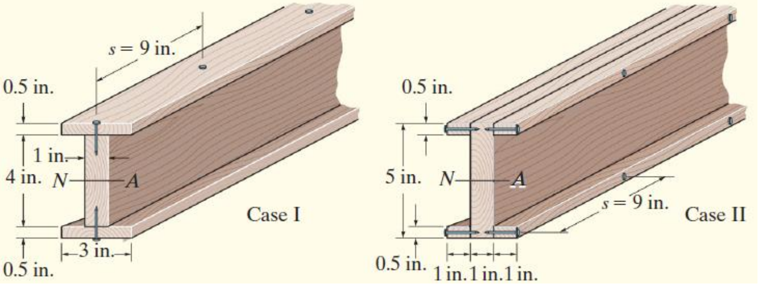
Example 5: The beam is constructed from three boards glued together as shown in Figure. If it is subjected to a shear of V = 850 kN, determine the shear flow at B and B' that must be resisted by the glue.



Example 6: A box beam is constructed from four boards nailed together as shown in Figure. If each nail can support a maximum shear force of 30 lb, determine the maximum spacing s of the nails at B and at C so that the beam can support the force of 80 lb.

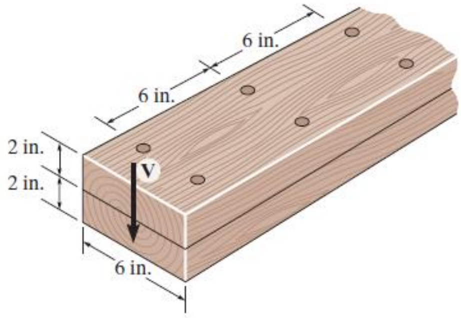


Example 7: Nails, each having a total shear strength of 40 lb, are used in a beam that can be constructed either as in Case I or as in Case II, Figure. If the nails are spaced at 9 in., determine the largest vertical shear that can be supported in each case so that the fasteners will not fail.



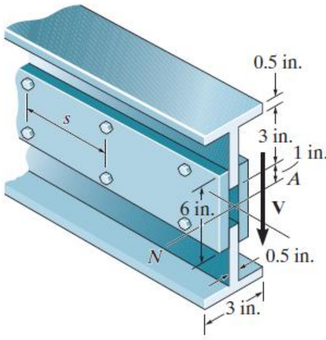
Sheet No. 2

Q 1: The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in.



(1) If each nail can support a 500-lb shear force, determine the maximum shear force V that can be applied to the beam.

(2) If an internal shear force of V = 600 lb is applied to the boards, determine the shear force resisted by each nail.

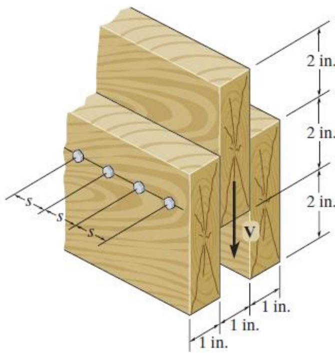


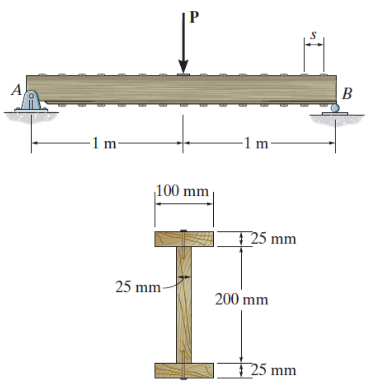
Q 2: The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. Each bolt can resist a shear force of 15 kip.

(1) If a shear of V = 600 kip is applied to the cross section, determine the maximum spacing of the bolts.

(2) If the bolts are spaced at s = 8 in., determine the maximum shear force V that can be applied to the cross section.

Q 3: Three identical boards are bolted together to form the built-up beam. Each bolt has a shear strength of 1.5 kip and the bolts are spaced at a distance of s = 6 in. If the wood has an allowable shear stress of τ_{allow} = 450 psi, determine the maximum allowable internal shear V that can act on the beam.





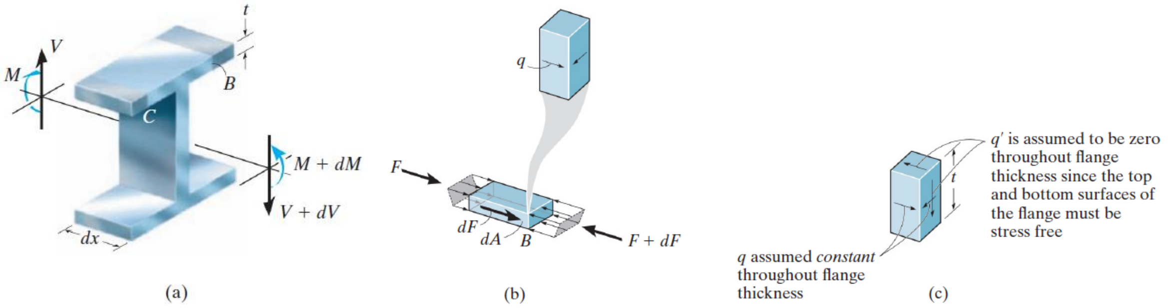
Q 4: The simply supported beam is built-up from three boards by nailing them together as shown. The wood has an allowable shear stress of τ_{allow} = 1.5 MPa and an allowable bending stress of σ_{allow} = 9 MPa. The nails are spaced at s = 75 mm, and each has a shear strength of 1.5 kN. Determine the maximum allowable force **P** that can be applied to the beam.

4. SHEAR FLOW IN THIN-WALLED MEMBERS

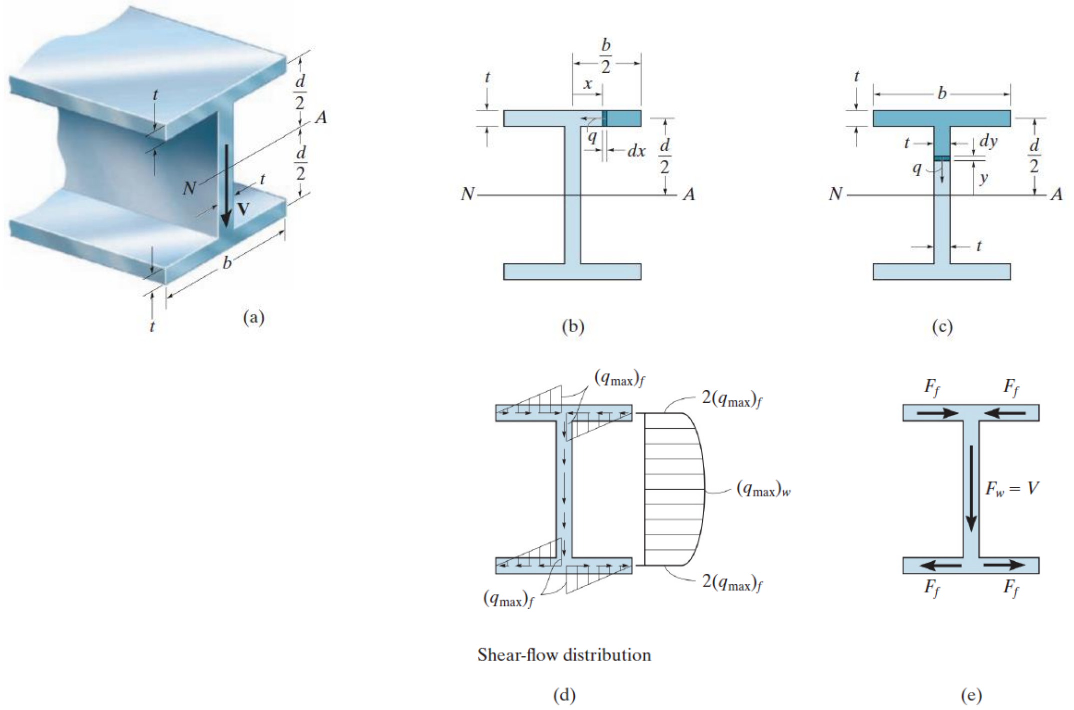
To describe the shear-flow *distribution* throughout a member's cross-sectional area. As with most structural members, we will assume that the member has *thin walls*, that is, the wall thickness is small compared to its height or width.

Before we determine the shear-flow distribution, we will first show how to establish its direction. To begin, consider the beam in Fig. *a*, and the free-body diagram of segment *B* taken from the top flange, Fig. *b*. The force *dF* must act on the longitudinal section in order to balance the normal forces *F* and *F* + *dF* created by the moments *M* and *M* + *dM*, respectively. Because *q* (and t) are complementary, *transverse components* of *q* must act on the cross section as shown on the corner element in Fig. *b*.

Although it is also true that *V* + *dV* will create a *vertical* shear-flow component on this element, Fig. *c*, here we will neglect its effects. This is because the flange is thin, and the top and bottom surfaces of the flange are free of stress. To summarize then, only the shear flow component that acts *parallel* to the sides of the flange will be considered.



• **Shear Flow in Flanges**



The shear-flow distribution along the top flange of the beam in Fig. *a* can be found by considering the shear flow q, acting on the dark blue element dx, located an arbitrary distance x from the centerline of the cross section, Fig. *b*. Here

$Q = \bar{y}'A' = [d/2](b/2 - x)t$, so that

$$q = \frac{VQ}{I} = \frac{V[d/2](b/2 - x)t}{I} = \frac{Vtd}{2I}\left(\frac{b}{2} - x\right)$$

By inspection, this distribution varies in a *linear manner* from $q = 0$ at $x = b/2$ to $(q_{\max})_f = Vt\,db/4I$ at $x = 0$. (The limitation of $x = 0$ is possible here since the member is assumed to have "thin walls" and so the thickness of the web is neglected.) Due to symmetry, a similar analysis yields the same distribution of shear flow for the other three flange segments. These results are as shown in Fig. *d*.

The total *force* developed in each flange segment can be determined by integration. Since the force on the element dx in Fig. *b* is $dF = q\,dx$, then

$$F_f = \int q\,dx = \int_0^{b/2} \frac{Vt\,d}{2I}\left(\frac{b}{2} - x\right)dx = \frac{Vt\,db^2}{16I}$$

We can also determine this result by finding the area under the triangle in Fig. *d*. Hence,

$$F_f = \frac{1}{2}(q_{\max})_f\left(\frac{b}{2}\right) = \frac{Vt\,db^2}{16I}$$

All four of these forces are shown in Fig. *e*, and we can see from their direction that horizontal force equilibrium on the cross section is maintained.

• **Shear Flow in Web.**

A similar analysis can be performed for the web, Fig. *c above*. Here q must act downward, and at element dy we have

$Q = \Sigma\bar{y}'A' = [d/2](bt) + [y + (1/2)(d/2 - y)]t(d/2 - y) = bt\,d/2 + (t/2)(d^2/4 - y^2)$, so that

$$q = \frac{VQ}{I} = \frac{Vt}{I}\left[\frac{db}{2} + \frac{1}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

For the web, the shear flow varies in a parabolic manner, from

$q = 2(q_{\max})_f = Vt\,db/2I$ at $y = d/2$ to $(q_{\max})_w = (Vtd/I)(b/2 + d/8)$ at $y = 0$, Fig. *d*.

Integrating to determine the force in the web, F_w, we have

$$F_w = \int q\, dy = \int_{-d/2}^{d/2} \frac{Vt}{I}\left[\frac{db}{2} + \frac{1}{2}\left(\frac{d^2}{4} - y^2\right)\right] dy$$

$$= \frac{Vt}{I}\left[\frac{db}{2}y + \frac{1}{2}\left(\frac{d^2}{4}y - \frac{1}{3}y^3\right)\right]\Bigg|_{-d/2}^{d/2}$$

$$= \frac{Vtd^2}{4I}\left(2b + \frac{1}{3}d\right)$$

Simplification is possible by noting that the moment of inertia for the cross-sectional area is

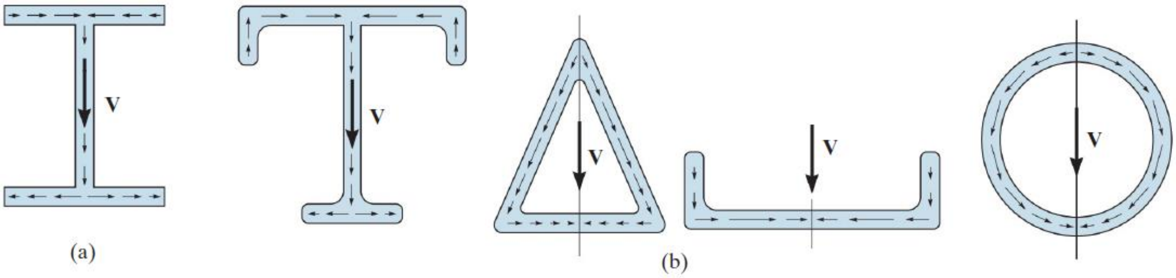
$$I = 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d}{2}\right)^2\right] + \frac{1}{12}td^3$$

Neglecting the first term, since the thickness of each flange is small, then

$$I = \frac{td^2}{4}\left(2b + \frac{1}{3}d\right)$$

Substituting this into the above equation, we see that $Fw = V$, which is to be expected, Fig. *e*.

From the foregoing analysis, three important points should be observed. First, q will vary *linearly* along segments (flanges) that are *perpendicular* to the direction of **V**, and *parabolically* along segments (web) that are *inclined or parallel* to **V**. Second, q will *always act parallel to the walls* of the member, since the section of the segment on which q is calculated is always taken perpendicular to the walls. And third, the *directional sense* of q is such that the shear appears to "*flow*" through the cross section, *inward* at the beam's top flange, "combining" and then "flowing" *downward* through the web, since it must contribute to the downward shear force **V**, Fig. *a*, and then separating and "flowing" *outward* at the bottom flange. If one is able to "visualize" this "flow" it will provide an easy means for establishing not only the direction of q, but *also* the corresponding direction of t. Other examples of how q is directed along the segments of thin-walled members are shown in Fig. *b*. In all cases, symmetry prevails about an axis that is collinear with **V**, and so q "flows" in a direction such that it will provide the vertical force **V** and yet also satisfy horizontal force equilibrium for the cross section.

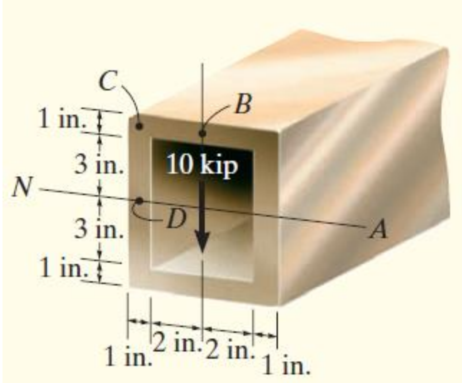


(a)

(b)

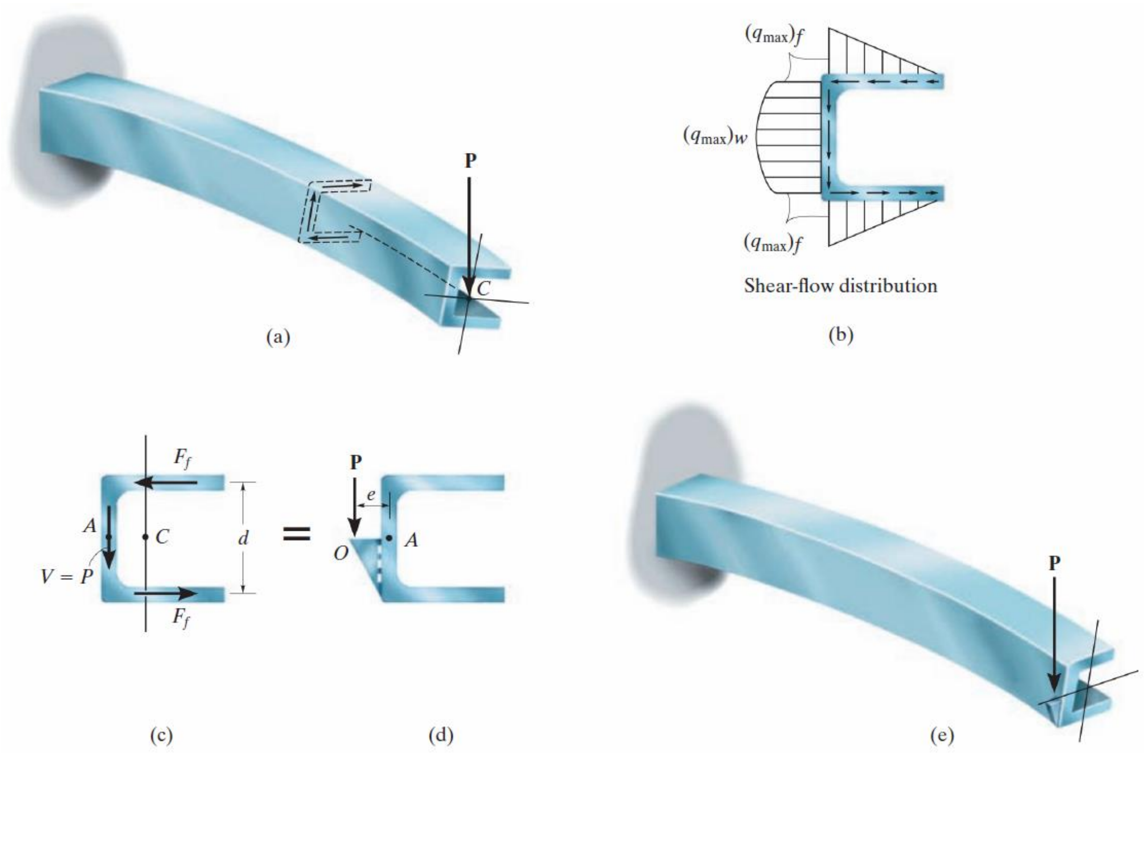
Shear flow q

Example 8: The thin-walled box beam in Fig. a is subjected to a shear of 10 kip. Determine the variation of the shear flow throughout the cross section.



5. SHEAR CENTER FOR OPEN THIN-WALLED MEMBERS

In the previous section, the internal shear **V** was applied along a principal centroidal axis of inertia that *also* represents an *axis of symmetry* for the cross section. In this section we will consider the effect of applying the shear along a principal centroidal axis that is *not* an axis of symmetry. As before, only open thin-walled members will be analyzed, where the dimensions to the centerline of the walls of the members will be used. A typical example of this case is the channel shown in Fig. *a*. Here it is cantilevered from a fixed support and subjected to the force **P**. If this force is applied through the *centroid C* of the cross section, the channel will not only bend downward, but *it will also twist* clockwise as shown.

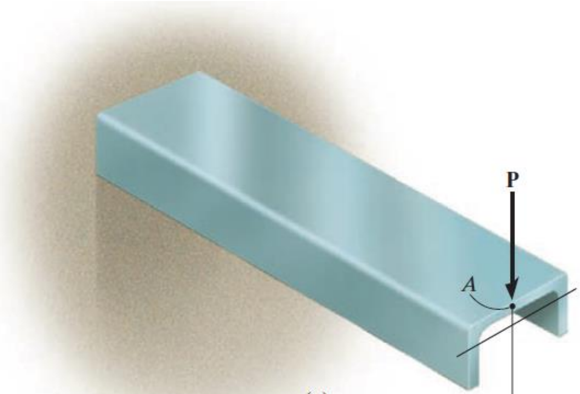


The reason the member twists has to do with the shear-flow distribution along the channel's flanges and web, Fig. *b*. When this distribution is integrated over the flange and web areas, it will give resultant forces of F_f in each flange and a force of $V = P$ in the web, Fig. *c*. If the moments of these three forces are summed about point *A*, the unbalanced couple or torque created by the flange forces is seen to be responsible for twisting the member. The actual twist is clockwise when viewed from the front of the beam, as shown in Fig. *a*, because *reactive* internal "equilibrium" forces F_f cause the twisting. In order to *prevent* this twisting and therefore cancel the unbalanced moment, it is necessary to apply **P** at a point *O* located an eccentric distance *e* from the web, as shown in Fig. *d*. We require $\Sigma MA = F_f\, d = Pe$, or

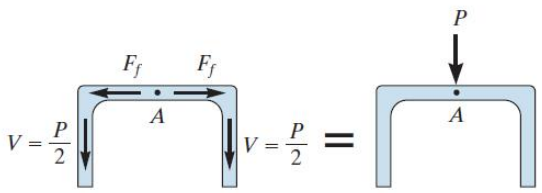
$$e = \frac{F_f d}{P}$$

The point *O* so located is called the ***shear center*** or ***flexural center***. When **P** is applied at this point, the ***beam will bend without twisting***, Fig. *e*. Design handbooks often list the location of the shear center for a variety of thin-walled beam cross sections that are commonly used in practice.

From this analysis, it should be noted that ***the shear center will always lie on an axis of symmetry*** of a member's cross-sectional area. For example, if the channel is rotated 90° and **P** is applied at *A*, Fig. *a below*, no twisting will occur since the shear flow in the web and flanges for this case is *symmetrical*, and therefore the force resultants in these elements will create zero moments about *A*, Fig. *b below*. Obviously, if a member has a cross section with *two* axes of symmetry, as in the case of a wide-flange beam, the shear center will coincide with the intersection of these axes (the centroid).



(a)



(b)

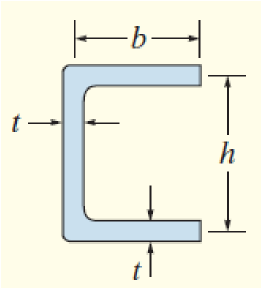
IMPORTANT POINTS

• The *shear center* is the point through which a force can be applied which will cause a beam to bend and yet not twist.

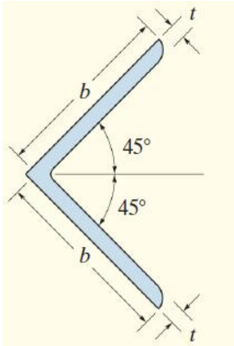
• The shear center will always lie on an axis of symmetry of the cross section.

• The location of the shear center is only a function of the geometry of the cross section, and does not depend upon the applied loading.

Example 9: Determine the location of the shear center for the thin-walled channel having the dimensions shown in Figure.



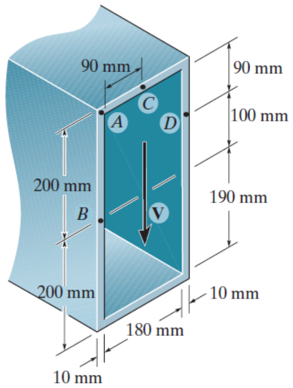
Example 10: Determine the location of the shear center for the angle having equal legs, Figure. Also, find the internal shear-force resultant in each leg.

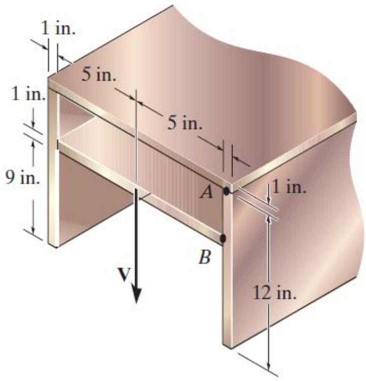


Sheet No. 3

Q 1: (1) A shear force of V = 300 kN is applied to the box girder. Determine the shear flow at points A and B.

(2) A shear force of V = 450 kN is applied to the box girder. Determine the shear flow at points C and D.

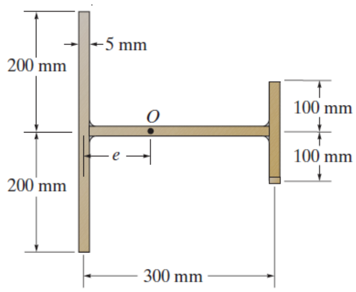




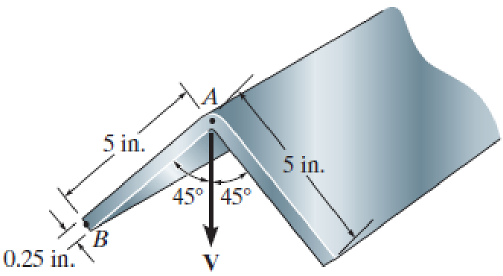
Q 2: The beam is subjected to a shear force of V = 50 kip.

(1) Determine the shear flow at points A and B.

(2) Determine the maximum shear flow in the cross section.

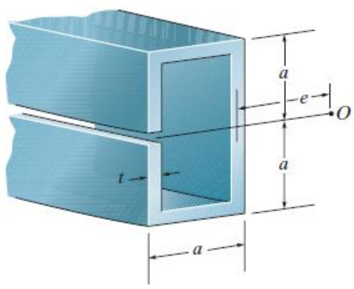


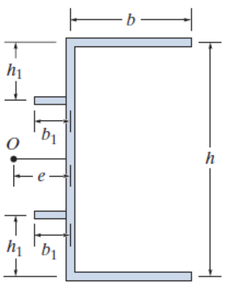
Q 3: The built-up beam is formed by welding together the thin plates of thickness 5 mm. Determine the location of the shear center O.



Q 4: The angle is subjected to a shear of V = 2 kip. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks.

Q 5: Determine the location **e** of the shear center, point O, for the thin-walled member having a slit along its side. Each element has a constant thickness t.





Q 6: Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.