# CHAPTER ONE
# DC MACHINE BASICS

DC machines include dc motors and dc generator in both cases, machine operation is based on two fundamental electromagnetic interactions:

I) A conductor moving in a magnetic field will have an electromotive force induced in it.

ii) A conductor carrying current and lying in a magnetic field have a mechanical force developed on it. This chapter reviews these principles, and explains how dc machines are arranged to make use of them.

## 1.1 Basic Interactions

Consider a straight conductor laying in a magnetic field as in fig. 1.1 the field is uniform along the length of the conductor and perpendicular to it. Let

b =magnetic flux density in tesla, and

L =active length of the conductor in meters.

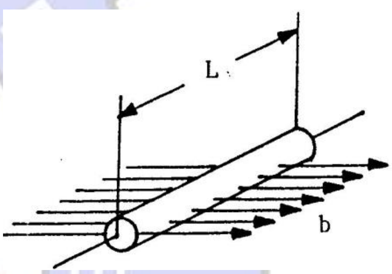


Fig (1.1)

The active length of the conductor is the part which is actually subjected to the field. If, for example, the conductor extends beyond the region of the field, the part that is outside the region is not included in L.

Now, the first electromagnetic interaction occurs if the conductor is moving:

Motion +field ~ induced electromotive force (emf).

The value of the emf is given by:

$$e = ubL\sin(\alpha)\text{volts} \quad (1.1)$$

Where u =speed of conductor perpendicular ·to its length in meters per second.

and.α =small angle (ie less than 180°) from direction of motion (u) to direction of field b. The direction of the induced emf e is given by the right-hand (RH)rule applied as follows: the fingers of the right hand are extended in the direction or motion in such a way that they can be rotated to the direction of the field through the small angle between them; the extended thumb then gives the direction of the emf in the conductor. Fig. 1.2 shows examples of the application of the RH rule; in fig. 1.2a. e=ubL because sin(α) = 1, but in.fig. 1.2b the emf is less because sin(α)< 1; if the conductor moves horizontally in the figure, there would be no emf induced in it.

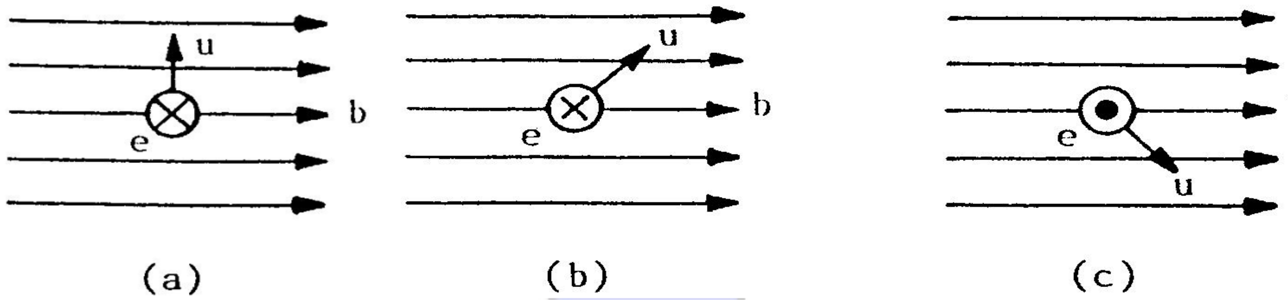



Fig. 1.2 Examples of the application of the right-hand rule to determine thedirection of the emf induced in a conductor moving in a magnetic field

It is important to understand that eqn. 1.1 does not tell us whether or not there is any current in the conductor, nor does it depend on whether the force causing the motion is produced by the conductor itself or by some external means.

The second electromagnetic interaction occurs if the conductor of fig. 1.1carries a current:

current +field ==> mechanical force.

The value of the developed force is given by

$$f = i\,bL \text{ newtons} \qquad (1.2)$$

Where

i =current flowing in the conductor in amperes.

f acts on the entire active length of the conductor in a direction given by the RH rule as follows : rotate the right-hand fingers from the direction of the current to the direction of the field through the small angle between them; the extended thumb then gives the direction of the developed force.

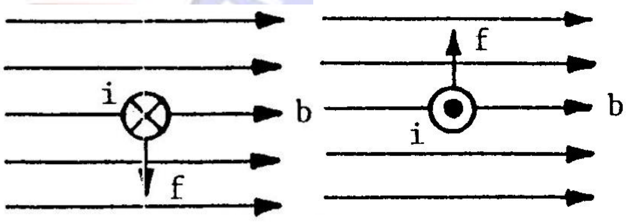



Fig. 1.3 Examples of the application of the RH rule

Fig.1.3shows examples of the application of the RH rule.

It is important to understand that eqn. 1.2 does not tell us whether or not the conductor is moving, nor does it depend on whether the source driving the current is the conductor itself or some external source.

To help you remember the application of the RH rule, note that for both types of interaction, the fingers of the right hand are rotated towards the direction of the magnetic field.

In general, an active conductor in a dc machine will be in motion and carrying current, both at the same time. Therefore, it will have both an induced emf and a developed force. The relative directions of the various quantities involved depend on whether the machine is operating as a generator or as a motor, as we shall see in section 1.7.

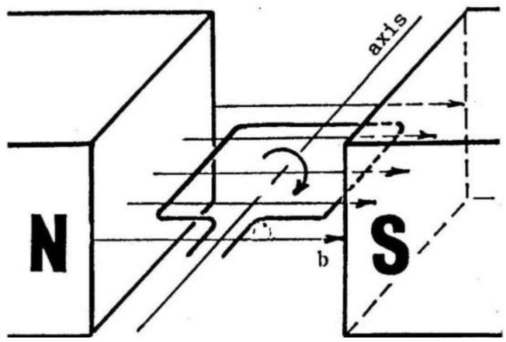


Fig. 1.4 Wire loop in magnetic field.

## 1.2 Wire Loop

In rotary machines, the active conductors are formed into loops. Fig. 1.4 shows a wire loop placed in a uniform magnetic field. The loop has two active sides marked x and y in fig. 1.5; these are the parts of the loop that are perpendicular to the field and in which the two fundamental interactions occur. The end-connections are necessary to complete the circuit, and the leads are necessary to connect the loop to an external circuit.

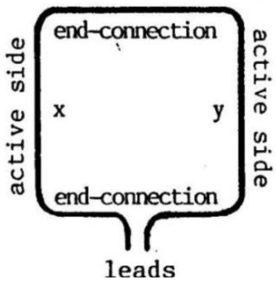


Fig. 1.5 Loop parts.

Let us first see what happens when the loop is rotating about its axis, as shown in cross-section in fig. 1.6a. Here we have motion in a magnetic field, which results in induced emf. Application of the RH rule to sides x and y yields the emf directions shown. The emfs add up around the loop, fig. 1.6b, so that the loop emf e is equal to $(e_x+e_y)$.

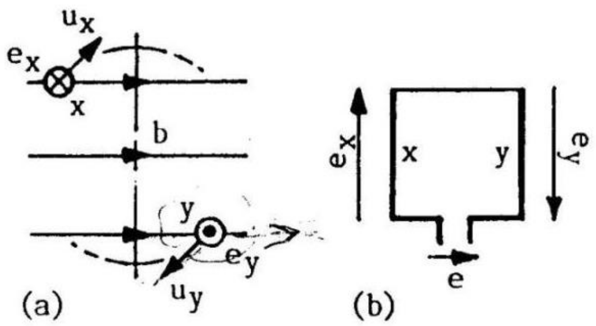


Fig. 1.6 Loop emf due to rotation in uniform field

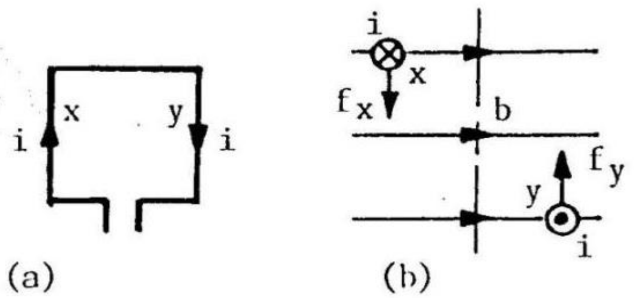


Fig. 1.7 Torque developed by current carrying loop in uniform field.

Next, we see what happens when there is current flowing around the loop, fig. 1.7a.· Here we have conductors carrying current in a magnetic field, which results in forces. Applying the RH rule from side x and y yields forces in the directions shown. The resultant force on the loop is zero because $f_x$ and $f_y$ oppose each other. However, the two forces form a couple, and hence produce a torque about the axis; the developed torque acts on the loop and attempts to rotate it.

## 1.3 Commutator

The electrical connection between the rotating wire loop and the stationary external circuit requires some form of sliding contact. In principle, this can be achieved using slip-rings and brushes as in fig. 1.8. The slip-rings are made of conducting material, and rotate with the loop; each terminal of the loop is soldered to one of the rings. The brushes are stationary; they are connected to the external circuit. The brushes press against the rings to make sliding contact. Clearly, the terminal emf between brushes $e_t$ is equal to the loop emf e.

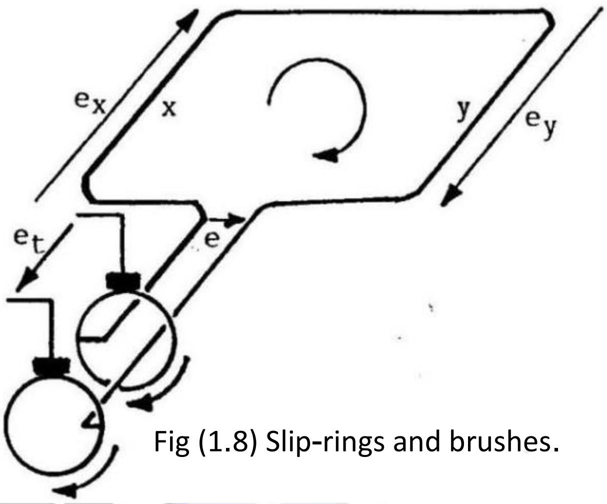


Fig (1.8) Slip-rings and brushes.

But slip-rings are not suitable for dc machines because the loop emf e, and hence the terminal emf $e_t$, are alternating and not steady as required. The alternating nature of e is demonstrated in fig. 1.9. The emfs in the two sides of the loop, at a number of positions, are shown in fig. 1.9a for rotation in a uniform magnetic field. The figure shows that as the loop passes through the vertical position 3, the emfs in the two sides are reversed, resulting in the alternating wave shown in fig. 1.9b.

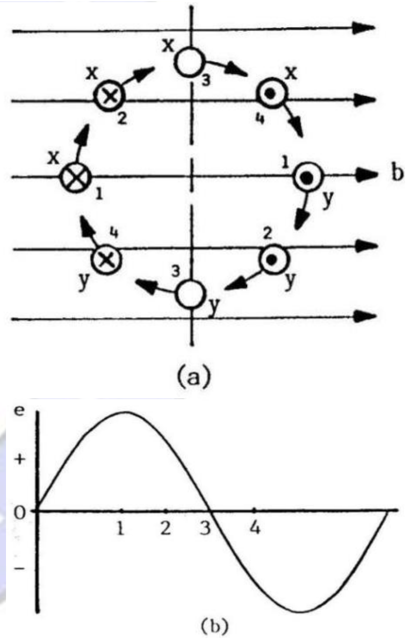


Fig. 1.9 Alternating loop emf due to rotation in uniform field.

The value of the emf at any given position depends on the angle between motion and field, ie $\alpha$ in eqn. 1.1; the emf is zero in the vertical position 3 because, at that position, motion is parallel to the field.

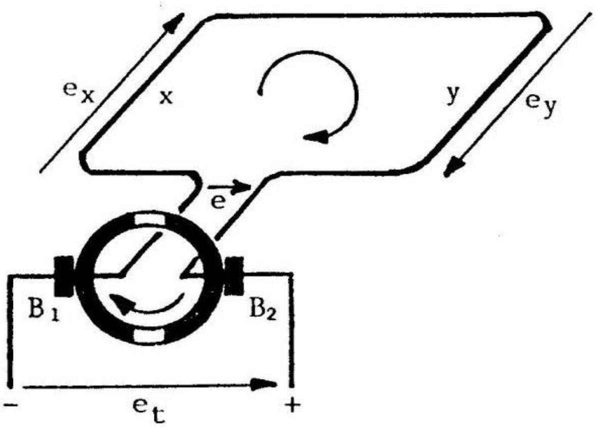



Fig. 1.10 Commutator and

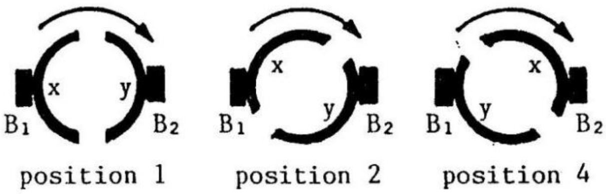



Fig 1.11 Principle of the commutator

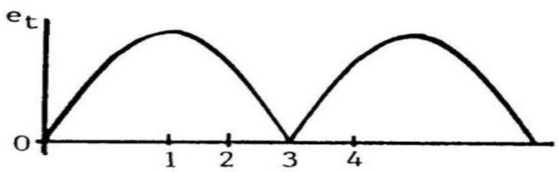



Fig. 1.12 Unidirectional emf obtained between brushes

Thus slip-rings may be used in ac machines, but not in dc machines. DC machines use commutators which not only provide sliding contact, but also rectify the alternating loop emf to a unidirectional emf between brushes. The commutator is basically a conducting ring split into insulated segments; it rotates with the loop, with each terminal of the loop soldered to one of the segments, fig. 1.10. The brushes are stationary, and are connected to the external circuit; they make sliding contact with the commutator segments. The idea of the commutator is that when the loop emf e reverses as the loop passes through the vertical position 3 of fig.1.9, the contact between segments and brushes is also reversed as shown in fig. 1.11. Thus, brush $B_1$ will always be at low potential, while brush $B_2$ will always be at high potential. Although the loop emf e is still alternating as in fig. 1.9, the terminal emf between brushes $e_t$ is now unidirectional as in fig. 1.12; that is, $e_t$ is always in the same direction, although its value is not constant.

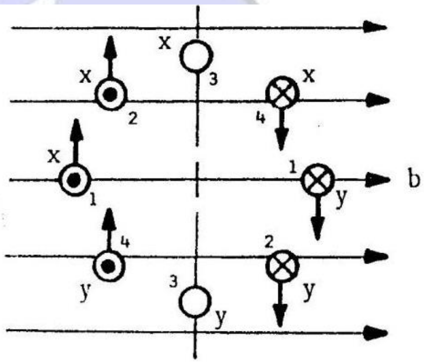



Fig. 1.13 Loop current and developed forces at different positions.

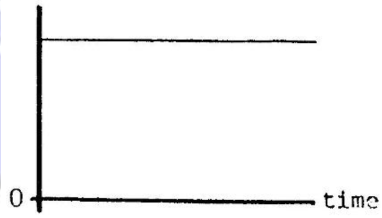



Fig. l.14 Ideal dc conditions

Consider next the effect of the commutator on the developed force and torque. As the machine is a dc machine, the current in the external circuit is always in the same direction. Let us assume that the current from the external circuit goes into brush $B_2$, flows through commutator and loop, and comes out again through brush $B_1$. Thus, in positions 1 and 2 of fig.1.13, the current flows into conductor y and out of conductor x; but when the loop passes through the vertical position 3, the contact between segments and brushes is reversed, so that in position 4 the current goes into conductor x and out of conductor y, that is, the loop current is forced to reverse by the commutator and brushes. Fig.1.13 also shows the resulting forces on conductors x and y at the different positions. It is clear that the torque is always in the same direction, clockwise about the loop axis in this case. If the current in the external circuit is truly dc, i.e. not only unidirectional but also constant, the developed torque will have the same wave shape as $e_t$ shown in fig.1.12. If slip-rings are used, the loop current would not reverse at position 3, and the torque would have the wave shape of fig.1.9; that is, the torque would oscillate in both directions, which is useless in a machine.I, e thus conclude that the emf and current inside the loop and the force on its two sides are alternating, but because of the commutator, the emf and current between brushes and the loop torque are unidirectional.

## 1.4 Armature:

Ideally, the emf and current at the brushes and the torque should be constant with time as in fig.1.14. But we have seen that the emf and torque have the wave shape shown in fig.1.12; although it is unidirectional, it is not constant :it reaches maximum when the loop is in position I of figs.1.9 and 1.13, and goes down to zero when the loop is in position 3. To improve the situation, we can use two loops instead of on1y one, as in fig.1.15. The two loops are displaced from each other by $90^0$in space, but are mechanically coupled together to form a single rigid system so that they rotate together. Each loop will give a curve similar to that of fig.1.12, but the two curves will be out of phase :when one loop has maximum emf, the other loop will have zero emf, as can be seen in fig. 1. 16a. now, if the two loops can be connected in series, the total emf will be the sum of the two emfs; also, the two torques add, and their resultant will act on the rigid mechanical system composed of the two loops. Fig. 1.16c shows the curves for the two loops, and also shows their instantaneous sum.

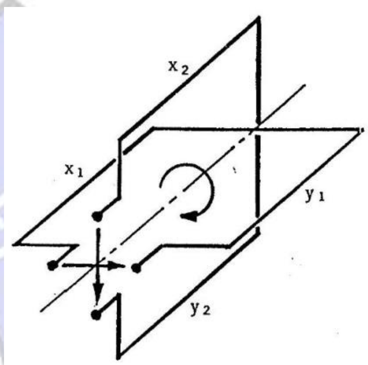


Fig. 1.15 Rigidly coupled loops displaced $90^0$

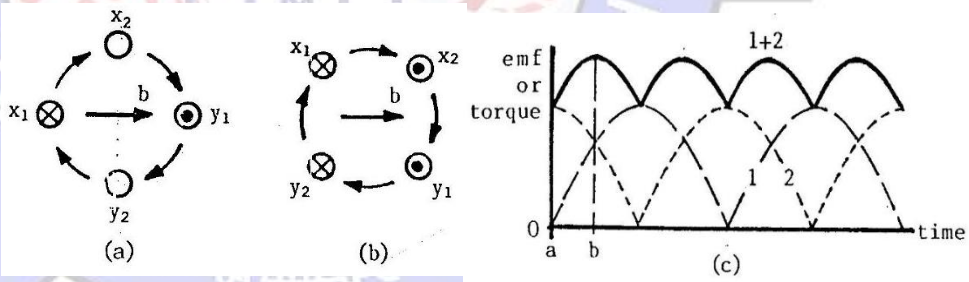


Fig.1.16 Two loops in uniform magnetic field

Clearly, the resultant curve is closer to the ideal dc conditions of fig. 1.14than the individual loop curves. The ripple is the instantaneous difference between the curve and its average value; ideally, the ripple should be zero as in fig. 1.14.

The ripple of the resultant curve in fig. l.16b is seen to be smaller than the ripple of the curve in fig. 1.12. The ripple can be made smaller by using more loops with suitable displacements in space; fig. 1.17 shows four loops with $45^0$displacements. By using a large number of loops, the ripple can be made very small, practically negligible; however, it cannot be eliminated entirely. The rigid mechanical system composed of all the loops together is called the armature of the dc machine; in most practical machines, the copper loops are mounted on an iron cylinder as we shall see in section 1.5

In section 1.3, the principle of the commutator was explained for a primitive machine having one loop only. The commutator had two segments, one connected to each terminal of the loop. For practical machines having multiple loops as described here, the commutator has a corresponding number of segments. The necessary number of segments, and the connection of the loops to the segments to ensure that they are in series, will be explained in chapter 3.

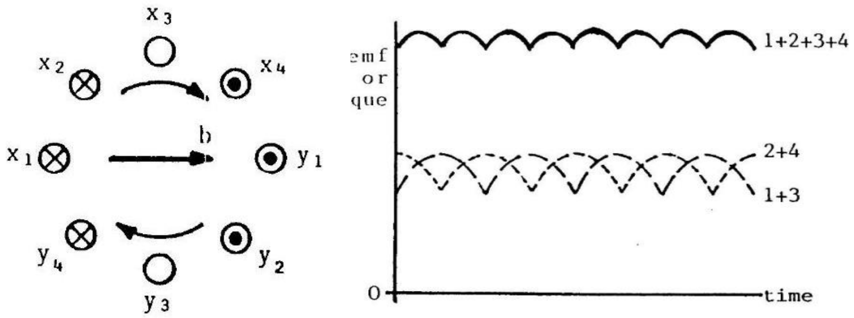


Fig. 1.17 Four loops in uniform magnetic field

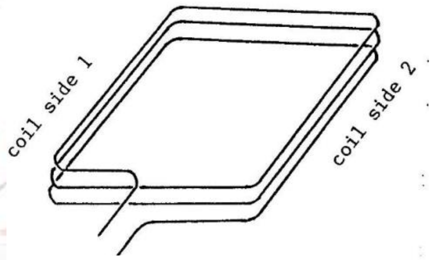


Fig. 1.18 Three-turn coil.

Increasing the number of loops displaced in space not only reduces the ripple, but also increases the emf and torque, as seen in figs. 1.16c and1.17. The emf and torque can also be increased by using multi-turn coils instead of single-turn wire loops.Fig. 1.18 shows a coil having three turns; each turn is in fact a loop, with all loops connected permanently in series so that their emfs add up, and the same current flows through them. Moreover, the loops making up the coil are all located at practically the same position in space, and not displaced from each other as in fig.1.15; in fact, each of the two loops of fig. 1.15 can be replaced by a multi-turn coil. Thus, all the active conductors in each side of the coil see the same flux density, and hence all the turns of the coil have the same emf and the same torque; the loop torques, of course, aid each other, see the same flux density, and hence all the turns of the coil have the same emf and the same torque; the loop torques, of course aid, each other.

For a coil of N turns. There are N active conductors in each coil side as can be seen in fig.1.18. The end-connections at the front and back are necessary to complete the circuit; no useful electromagnetic interactions occur in them. The two Leads of the coil are soldered to commutator segments as was done for the leads of single-turn loops, fig. 1.10.

## 1.5 Magnetic Field

In the preceding sections we studied the active conductors where the electromagnetic interactions, leading to the production of emf and torque, occur.

Both interactions require the presence of a magnetic field, represented by the magnetic flux density b in eqns. 1 and 2. So far, we have simply assumed that there is a uniform magnetic field in the space around the active conductors or the armature. In this section, we study how the field is produced, and we shall see that it is not truly uniform.

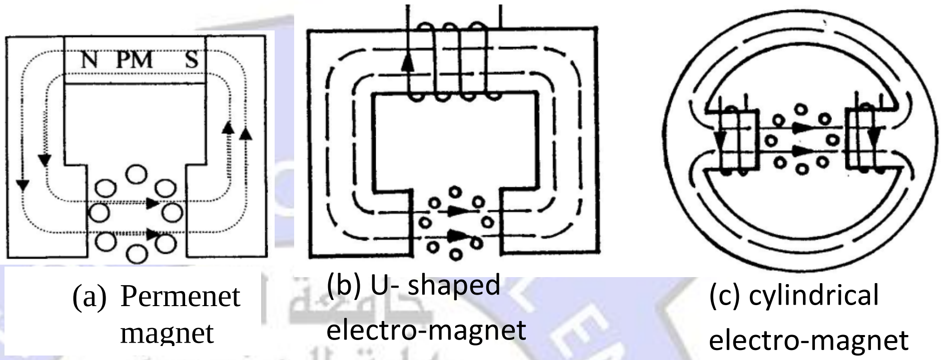



Fig. 1.19 producing the magnetic field.

The field may be obtained by means of a permanent magnet (PM) as in fig.1.19a, or an electromagnet as in fig. 1.19b. In both cases, the iron is shaped to direct the magnetic flux to the region where it is needed, that is, the region where the active conductors of the armature are placed. Fig. 1.19c shows an alternative way of constructing the electromagnet. From the view point of the armature conductors, all three arrangements in fig. 1.19 are the same :they provide a north pole on the left, and a south pole on the right.

Now, the strength of the magnetic field is inversely proportional to the reluctance in its path. In the arrangements of fig.1.19, most of the flux path is in iron, which has a high permeability, and hence low reluctance. But the flux has to pass through the region containing the active conductors; this region is air which presents a high reluctance to the flux. To obtain a strong field, the path in the air must be made as short as possible. By mounting the armature coils on an iron core, as in fig. 1.20b, the air path becomes much shorter than in fig. 1.20a. It can be made even shorter by cutting channels, called slots in the surface of the cylindrical core, fig. 1.20c, and placing the active sides of the armature coils in the slots.

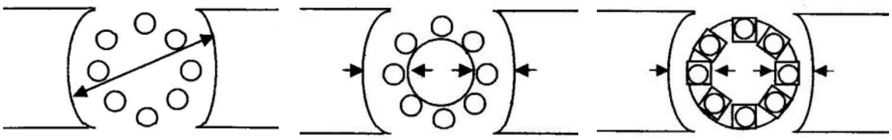


Fig. 1.20 Shortening the air path

we can seen that the armature is composed of an iron cylinder, with slots in which the coil sides are placed; the end-connections of the coils remain outside the iron cylinder, in front of it and behind it. The whole mechanically-rigid structure is mounted on a shaft through its axis; the torques developed by the coils act on the structure as a whole. Terminals of the coils are soldered to the segments of the commutator, which is also mounted on the same shaft. The clearance between the armature surface and the pole faces is called the air gap. The air gap is necessary to allow the armature to rotate but it is usually made very small so as to obtain the largest possible magnetic flux.

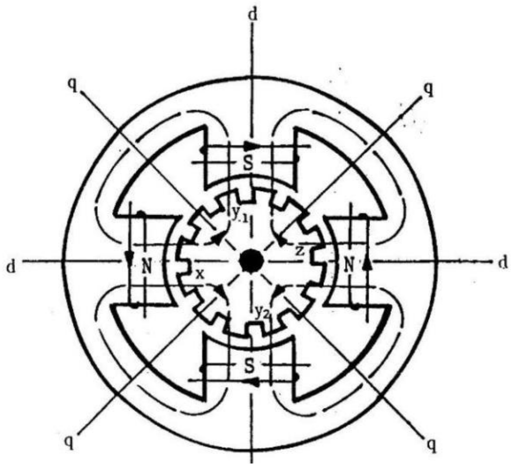


Fig. 1.21Cross-section of a four-pole machine

In figs. 1.19 and 1.20, there are only two poles. One north and the other South; or we can say that the machine has *one* pair of poles. Machines' can have more pole pairs; let

p =number of pole pairs, Then 2p =number of poles,

So that the number of poles is always even :for every north pole, there must be a south pole. Fig. 1.21 shows a machine having four poles, ie. 2p =4, and p =2. Opposite poles follow each other around the armature :every north pole is followed by a south pole, and every south pole is followed by a north pole :note the flux paths in fig. 1.21. The center line of the pole is called the pole axis, or direct axis, or d-axis for shortness; all d-axes are marked with the letter d in the figure. The axis passing through the midpoint between adjacent poles is called the quadrature axis, or q-axis; all q-axes are marked with the letter Q in the figure. Now, let

D= diameter of the armature in meters;

Then the perimeter of the armature is πD meters. This distance faces 2p poles, so to each pole there corresponds a distance of πD/2p meters which is called the pole pitch:

Pole pitch = $\frac{\pi D}{2p}$ meters (1.3)

Clearly, the distance between consecutive d-axes, measured on the armature surface, is a pole pitch; similarly, for the distance between consecutive q-axes. The distance between a d-axis and the adjacent q-axis is half a pole pitch. In fig.1.21, the pole pitch is πD/4 meters. Sometimes it is useful to measure the pole pitch as a number of slots; in fig. 1.21 there are 12 slots and 4pole. so that the pole pitch is 12/4=3 slots. The pole pitch can also be measured as an angle in degrees or radians :the full circle of the armature is 360 degree (or 2π radians). so that the pole pitch is 360/2p degrees (or2π/2p radians). For the 4-pole machine for fig.1.21, the pole pitch is 90degrees (or $\pi/2$ radians). For a 2-pole machine as in figs. l.19 and 1.20, the pole pitch is 180 degrees (or π radians).

Fig. 1.22 shows the emf directions in a 4-pole machine :in each pole pitch corresponding to one pole, i.e. enclosed between two consecutive q-axes, the emfs are in the same direction; the emfs in the next pole pitch are in the opposite direction, and so on. Because of the iron of the armature, the flux tends to be perpendicular to the armature surface, so that the angle between motion and field, α in eqn. 1.1, is $90^0$ and sin(α) = 1; thus eqn. 1.1becomes e =ubL for the armature conductors. It is very important to observe that if a coil has one side at the position marked x in figures 1.21 and 1.22, then the second side of that coil must be at position $y_1$ or $y_2$ so that the emfs add up around the coil as in fig.1.6b; if the two sides of the coil are placed at x and z, then the emfs oppose each other around the coil, and the coil emf will be zero.

Thus, the distance between the two sides of each coil, which is called the coil span; must be a pole pitch; when one side passes under a north pole, the other side passes under a south pole. Since the pole pitch depends on the number of poles in the machine, eqn. 1.3 then the coil span also depends on the number of poles.

Fig. 1.23 shows the current pattern in the armature conductors and the resulting forces. All the forces are seen to aid each other in developing a torque, CCW in this case. Because the flux is perpendicular to the armature surface, application of the right-hand rule yields forces that are tangential to the surface. Again, it is noted that x and z cannot be the two sides of the same coil; current goes into one side, and comes out through the other, as in x and $y_1$, or x and $y_2$.

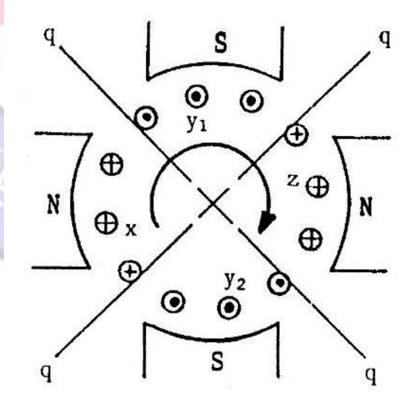


Fig.1.22 EMF's induced in the armature conductors of a 4-pole machine

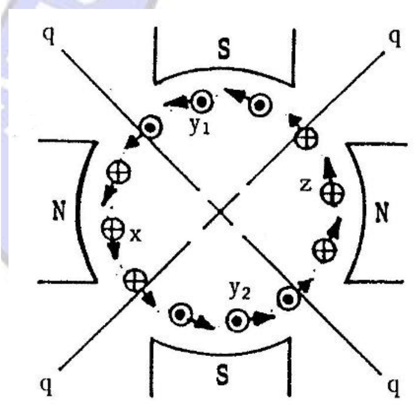


Fig. 1.23 Forces developed in the armature conductors of a 4-pole machine

Conversely, if x and z are assumed to be the two sides of the same coil, then one of the crosses must be changed to a dot, and the forces on the two sides will produce opposing torques; which of course is wrong. Thus, the currents in fig. 1.23 reverse from one pole pitch to the next.

The general path of flux in the machine is shown in fig. 1.21. The actual distribution of field is quite complicated but we shall be mainly interested in the flux density distribution at the surface of the armature because that is where the active conductors are, i.e. where the two main electromagnetic interactions occur; remember that the symbol b in eqns 1.1 and 1.2 represents the magnetic flux density at the location of the conductor. Fig. 1.24 shows a rough sketch of the flux lines in the air gap, ie. at the armature surface.

Note that the machine is shown in fig. 1.24 in developed form, obtained by making an imaginary radial cut in the cross-section shown in fig. 1.21, and straightening the poles and armature; the horizontal length in fig. 1.24 is actually the diameter of the armature.

The curve of the air gap flux density is shown in fig. 1.24. It alternates from one pole pitch to the next because the polarity of the pole's alternates :south followed by north followed by south, etc. Within each pole pitch, the flux density b is high and constant under the pole face because the air gap is the short and has constant length; the absence of pole iron near the q-axis causes the air gap flux density to decrease till it reaches zero at the q-axis; it then reverses as we go into the next pole pitch.

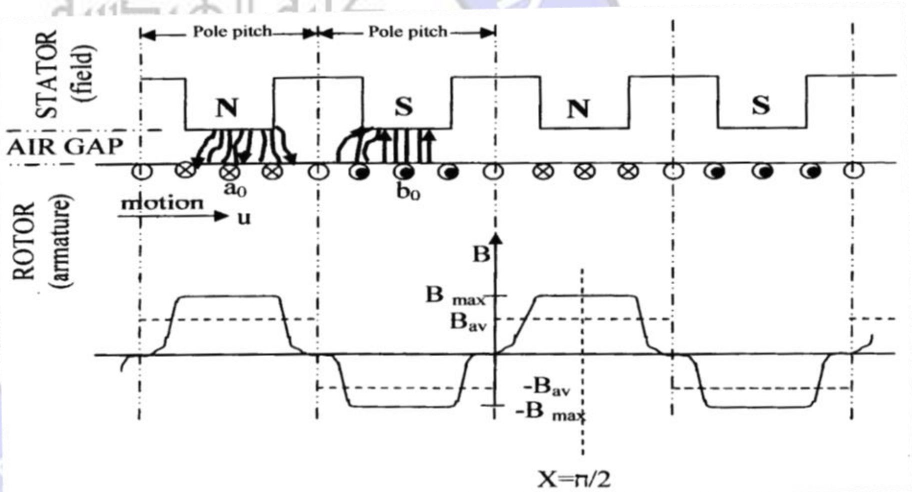



Fig. 1.24 Developed diagram of a 4-pole machine corresponding air gap flux density distribution.

It is clear from fig. 1.24 that the air gap flux density b is periodic :the curve repeats itself every two pole pitches. Thus, electrically, two pole pitches represent 360 degrees (or 2π radians), and we can say that a pole pitch is 180 electrical degrees (or π electrical radians). Care is needed not to confuse the electrical angle described here, and the mechanical angle described all previews page; it should be clear that:

Electrical angle =p x mechanical angle (1. 4) for example, the full circle around the armature is always 360 mechanical degrees, but it is P x 360 electrical degrees :a 4-pole machine has two electrical cycles around the armature, ie 720 electrical degrees. Moreover.

Pole pitch $=\frac{360}{2p}$mechanical degrees or $=\frac{2\pi}{2p}$mechanical radians (1. 5a)

But, Pole pitch =180 electrica1 degrees or π electrical radians (1.5b)

Note that the pole pitch in electrical degrees (or electrical radians) is constant, and does not depend on the number of poles. Electrically, everything repeats after two poles, so that one pole pitch always represents half a cycle, as expressed by eqn. 1.5b.

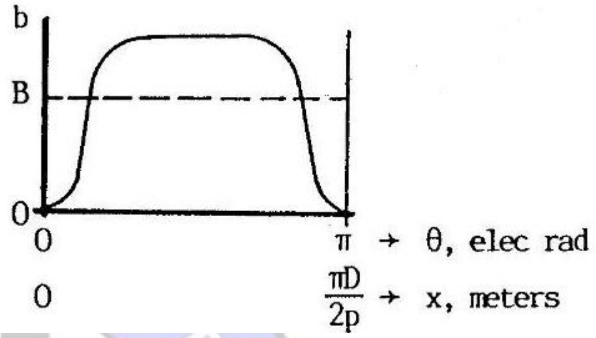


Fig. 1.25 Flux density distribution over one pole pitch.

A typical curve for the air gap flux density is shown in fig. 1.25; the details of the curve may differ a little from machine to machine.

Position along the periphery of the armature be measured as an electrical angle, θ, or as distance in meters, x. Let

B =average flux density in the air gap in tesla.

B is a constant, shown dotted in fig. 1.25. The actual flux density b, on the other hand, is a function of position since it may change from point to point around the periphery of the armature. i.e. b=b(θ), or b=b(x). The average B can be obtained by integrating b over one pole pitch, and dividing by a pole pitch:

$$B = \frac{1}{2\pi}\int_0^{\pi} b d\theta = \frac{1}{\frac{\pi D}{2p}}\int_0^{\frac{\pi D}{2p}} b dx \qquad (1.6)$$

If b is known as a function of position, the above integration can he perform to determine B.

From fig. 1.26, the cylindrical surface area of the armature is πDL. Let:

$A_p$=area on the armature surface that corresponds to one pole pitch, $m^2$;

Then

$A_p = \frac{\pi DL}{2p}$ (1.7)

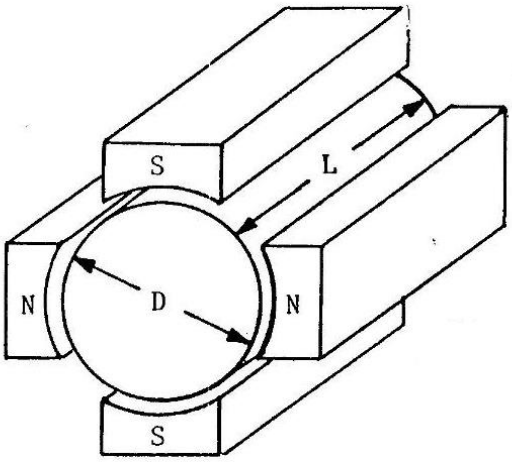


Fig. 1.26 Armature dimensions

Next, let Φ =flux per pole in webers; that is, Φ is the total magnetic flux going out through the face of a north pole and entering the armature, or going from the armature into the face of a south pole. Φ has the same value for all poles. It can be obtained by integrating b over $A_p$:

$$\Phi = \iint_{A_p} bdA = \int_0^L \int_0^{\frac{\pi D}{2p}} bdxdL \quad (1.8a)$$

But b is constant along the length of the armature in the direction of L; therefore

$$\Phi = L\int_0^{\frac{\pi D}{2p}} bdx = L\frac{\pi D}{2p}B = A_p x B \quad (1.8b)$$

Thus, if the average flux density B is known, the flux per pole Φ can be found by multiplying B by the area per pole $A_p$; conversely, if Φ is known, B maybe found from

$$B = \frac{\Phi}{A_p} \quad (1.9)$$

This, it will be noted, fits the basic definition of flux density as the flux per unit area.

## 1.6 Armature Coil Equations

In this section we are going to derive the emf and torque equations for armature coils. The derivations start from eqns. 1.1 and 1.2 which give the instantaneous emf e and instantaneous developed force $f_d$.

For a given armature conductor, e and $f_d$ vary with time because the conductor sees different values of flux density b as it rotates, fig. 1.24. In fact, e and $f_d$ have the same wave-shape as b in fig. 1.24b since the other factors in eqns. 1.1 and 1.2 are constant; this wave-shape has two kinds of time variation:

(i) It alternates from one pole pitch to the next, and
(ii) Within each pole pitch, it varies between maximum under the pole face and zero at the q-axis.

But we have seen how the commutator rectifiers the alternation, fig. 1.12, and how the use of multiple coils cancels variations about the average, figs. 1.16and 1.17. We are interested in the resulting average commutated values, which are constant, and not in the instantaneous values within conductors, which are time-varying. To distinguish between the two kinds of variables, we shall use capital letters to denote the average commutated values; thus

E =average commutated emf, volts;

$F_d$ = average commutated developed force, newtons;

I= average commutated current, amperes;

$T_d$= average commutated developed torque, newton meters.

Recall that the average flux density B and the flux per pole Φ are also constant quantities.

**1. 6.1 Coil EMF**

From eqn. 1.1, the instantaneous emf in an armature conductor is

$$e_{con} = ubL \qquad (1.10)$$

where sin(α) = 1 because the flux is perpendicular to the armature iron. A loop has two active conductors placed a pole pitch apart, so that whatever the flux density seen by one conductor, the flux density seen by the other is equal and opposite, fig. 1.24; thus the two conductor emfs add around the loop, fig. 1.6b, giving

$$e_{loop}=2xe_{cond} = 2ubL \qquad (1.11)$$

A coil having N turns is composed of N loops in series, fig. 1.18, so that

$$e_{coil} = Nxe_{loop} = 2NubL \qquad (1.12)$$

To obtain the average commutated coil emf, we replace the instantaneous flux density b by the average flux density B, so that

$$E_{coil} = 2NuBL \qquad (1.13a)$$

This is the coil emf equation that we require. It can be written in other forms. For example, it is usual to express the speed of rotating bodies in revolutions per second rather than in meters per second. Let

n =rotational speed of the armature, revolutions/second (rps).

Consider a point on the armature surface; the time it takes to complete one revolution is simply (l/n) seconds per revolution. The distance it covers in one revolution is the perimeter of the armature, ie πD. But speed is equal to distance divided by time; therefore

$$u = \frac{\pi D}{\frac{1}{n}} = \pi D n \qquad (1.14)$$

Substituting for u in eqn. 1 .13a, we get

$E_{coil} = 2\pi DLNnB$ (l.13b)

Next, let

ω = angular speed of armature, radians/second.

A point on the armature surface covers 2π radians when it completes one revolution in (l/n) seconds; thus

$$\omega = \frac{2\pi}{\frac{1}{n}} = 2\pi n \qquad (1.15)$$

Substituting for n in terms of ω, eqn. 1.13b becomes

$E_{coil} = DLN\omega B$ (1.13c)

finally, we can substitute for the average flux density B in terms of the flux

per pole Φ using eqn. 1. 9 in this case, eqn. 1.13b becomes

$$E_{coil} = 2\pi DLNn\frac{\Phi}{A_p} = 2\pi DLNn\frac{\Phi}{\frac{\pi DL}{2p}} = 4pNn\Phi = \frac{2p}{\pi}N\omega\Phi \qquad (1.\ 13d)$$

Eqns. 1.13a. 1.13b. 1.13c. and 1.13d are all expressions for the same quantity, the average commutated coil emf ($E_{coil}$). In each form, there is a factor representing speed, and a factor representing the magnetic field, multiplied by some constant; this reflects the fact that, fundamentally, the emf is induced by the interaction between motion and field.

It is instructive to show how the expression for $E_{coil}$ can be derived in an entirely different way. Faraday's law states that time-varying magnetic flux induces emf in a coil enclosing it.

$$e_{coil} = N\frac{d\Phi}{dt} \qquad (1.16)$$

where Φ is the flux linking the coil, that is, the total flux passing through the space enclosed by the N turns of the coil. Note that this is not the same as Φ which represents the flux per pole in the machine. Eqn. 1.16 gives the instantaneous emf; the average emf over an interval Δt is obtained by simplifying eqn. 1.16 to

$$E_{coil} = N\frac{d\Phi}{\Delta t} = N\frac{\Phi_2 - \Phi_1}{t_2 - t_1} \qquad (1.17)$$

Where $\Phi_1$ is the flux linking the coil at the instant $t_1$ and $\Phi_2$is the flux linking it at $t_2$. Let us take $t_1$ as the instant at which the coil sides are at consecutive q-axes enclosing a north pole as in fig. 1.27a; the flux linking the coil is the flux per pole Φ directed downward, so that $\Phi_1$=-Φ.

Next, we take $t_2$ as the instant at which the coil has moved so that it encloses the following south pole, fig. 1.27b; the flux linking the coil is still Φ, but it is directed upward so that $\Phi_2$ = Φ Eqn. 1.17 now becomes:

$$E_{coil} = N\frac{\Phi - (-\Phi)}{t_2 - t_1} = N\frac{2\Phi}{\Delta t} \qquad (1.18)$$

Now, Δt =$t_2$-$t_1$ is the time it has taken the coil to move one pole pitch. As we have seen, the time to complete one revolution is (l/n) seconds. But one revolution covers 2p poles, so that the time to move one pole pitch is:

$$\Delta t = \frac{1/n}{2p} = \frac{1}{2pn} \qquad (1.19)$$

Substituting this value for Δt back into eqn. 1. 18, we get

$$E_{coil} = N\frac{2\Phi}{(1/2pn)} = 4pNn\Phi$$

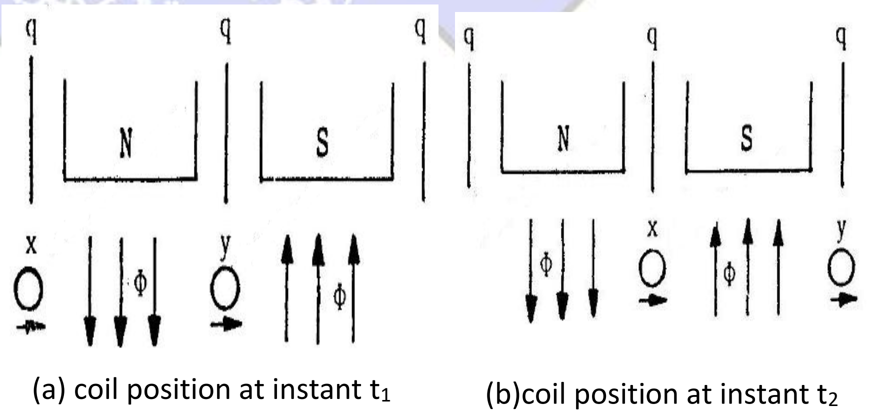
Which is the same as eqn. 1.13d.

### 1.6.2 Coil torque

From eqn. l.2, the instantaneous force developed in an armature conductor is

$$f_{cond} = i_{cond}\, bL \qquad (1.\ 20)$$

where $i_{cond}$ is the current flowing in the conductor. Because the




(a) coil position at instant $t_1$ (b)coil position at instant $t_2$

Fig. 1.27 Motion of an armature coil through one pole pitch.

flux is perpendicular to the armature iron, this force is tangential to the armature surface as shown in fig. 1.23. Therefore, this force produces a torque about the axis of the armature

$\tau_{cond} = f_{cond}$ x D/2 $= \frac{1}{2} i_{cond} bLD$ (1. 21)

where D/2 is the torque arm. The two active conductors in a loop carry equal and opposite currents, fig. 1.7, and see equal and opposite values of flux density, fig. 1.24, so that them torques are equal and in the same direction about the armature axis, fig. 1.23; thus

$\tau_{loop} = 2$ x $\tau_{cond} = i_{cond}$ bLD $= i_{loop}$ bLD

Where $i_{loop} = i_{cond}$. A coil of N turns is composed of N loops in series, so that $i_{coil} = i_{loop}$ and the total torque developed by the coil is:

$\tau_{coil} = N$ x $\tau_{loop} = NDLi_{coil}$ b (1. 23)

τ is the instantaneous torque. The average commutated torque is obtained by replacing the instantaneous i and b by I and B:

$T_{coil} = NDLI_{coil}B$ (1.24a)

This is the coil torque equation that we require. It can be written ina different form by substituting for B in terms of Φ using eqn. 1.9:

$$T_{coil} = NDLI_{coil}\frac{\Phi}{A_p} = NDLI_{coil}\frac{\Phi}{\frac{\pi DL}{2p}} = \frac{2p}{\pi}NI_{coil}\Phi \quad (1.24b)$$

Eqns. 1.24a and 1.24b express the same quantity, the average commutated coil torque $T_{coil}$In both forms, current appears as a factor multiplied by a factor representing the magnetic field; this is to be expected because, fundamentally, the torque is produced by the interaction between current and field.

### 1.6.3 Coil Resistance

The resistance of a conductor of length ℓ meters and cross-sectional area s $m^2$is given by

$$R = \frac{\rho L}{s}\ \Omega \quad (1.\ 25)$$

Where ρ is the resistivity of the material from which the conductor is made.

Machine coils are usually made from copper, for which

$\rho = 1.72 * 10^{-8}$ Ωm=1.72 μΩcm at $20^0$ C

The resistivity, and hence resistance, increase with temperature according to the ratio

$$\frac{\rho_2}{\rho_1} = \frac{R_2}{R_1} = \frac{T_2 - T_0}{T_1 - T_0} \qquad (1.26)$$

Where $\rho_1$ and $R_1$ are at a temperature $T_1$ , and $\rho_2$ and $R_2$ are at a temperature $T_2$. $T_o$ is a constant of the material; for copper it is

$T_0 = -234.5^0C$

Thus, if the resistivity is known at some temperature $T_1$, it can be found at a different temperature $T_2$ using eqn. 1.26; similarly, for the resistance $\ell$ in eqn. 1.25 represents the total length of the conductor. For a wire loop as shown in fig. 1.5,

$$\ell_{loop} = 2L + \ell_{end} \qquad (1.27)$$

Where $\ell_{end}$ represents the length of the end-connections at the back and front. Note that the entire length of the loop contributes to its resistance, whereas only the active parts contribute to the emf and torque. Thus

$$R_{loop} = \frac{\rho \ell_{loop}}{s} \qquad (1.28)$$

As a coil is made of N loops in series, the coil resistance is given by

$$R_{coil} = N x R_{loop} = N \frac{\rho \ell_{mean}}{s} \qquad (1.29)$$

Where $\ell_{mean}$ is the mean length of the turns of the coil, since, for a coil of many turns, not all the turns will have exactly the same length.

## 1.7 Electromechanical Power Conversion

The voltage between the two terminals of an armature loop or coil is equal to the emf induced in the coil so long as there is no current flowing in it. When current does flow in the coil, it causes a voltage drop due to the resistance of the coil, and the terminal voltage will be somewhat different from the emf.

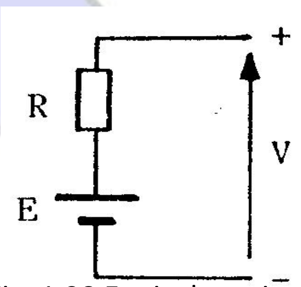


Fig. 1.28 Equivalent circuit of an armature coil

Electrically, then, the coil is represented by a simple equivalent circuit composed of the coil emf in series with the coil resistance, as shown in fig. 1.28. For simplicity, in this section we shall drop the subscript 'coil' on the understanding that all quantities of interest (E, R, V, I, and T) correspond to an armature coil. It is recalled that the emf E is induced in the active sides of the coil due to their rotation in a magnetic field; the resistance R, and hence the voltage drops IR, exist along the entire length of the wire from which the coil is made, including both the active parts as well as the end-connections and leads.

For any circuit element having two terminals, the power is the product of the voltage between the terminals and the current through the element. The power in the emf-element of fig. 1.28 is thus given by

$P_c = E\ I$ (1. 30)

Where E and I are the coil emf and current. If we now substitute for E from eqn. 1.13d, and collect terms, we find

$$P_c = \frac{2p}{\pi} N\omega\Phi I = \omega\left\{\frac{2p}{\pi} NI\Phi\right\}$$

The quantity in brackets is the coil torque as given in eqn. 1.24b; therefore

$P_c = \omega T$ (1.31)

But the product of angular speed and torque is the mechanical power of a rotating system Thus, eqns. 1.30 and 1.31 tell us that the electrical power in the emf-element of the equivalent circuit of the coil is the same as the mechanical power of the rotating coil:

$P_c = EI = \omega T$ (l .32)

These equation summaries the electromechanical power conversion that occurs in the coil by means of the two fundamental electromagnetic interactions of emf and torque production. $P_c$ is called the conversion power; it is equal to EI on the electrical side, and ωT on the mechanical side. It is the power that is changed from mechanical form to electrical form in a generator, and from electrical form to mechanical form in a motor. Let us look at the conversion processes of the two cases in more detail.

### 1.7.1 Generator Action

A generating coil acts as a source in the electrical circuit :it supplies electrical power to the circuit, fig. 1.29a. The emf-element supplies $P_c$ =EI to the circuit; part of this power is lost as heat in the resistance of the coil itself, $I^2R$, and the rest goes to the electrical load, VI. Conservation of energy and power requires that the power fed into the circuit must come from somewhere; in the generating armature coil, it comes from the mechanical side as $P_c=\omega T$. Therefore, there must be a mechanical source, or prime mover, providing the mechanical power to rotate the coil.

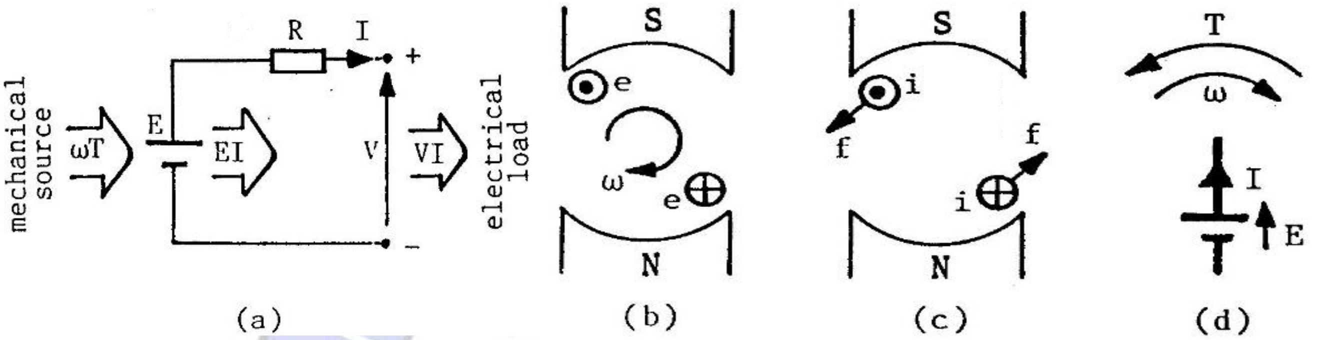



Fig. 1.29 Rotating armature coil in generating mode

Assuming the coil is rotating in the CW direction, application of the RH rule at the instant shown in fig. 1.29b -gives the instantaneous emf directions in the two sides of the coil. Since the coil is generating, it acts as the source in the electrical circuit, and hence drives the current in the same direction as the emf as shown in the equivalent circuit of fig. 1.29a. Thus, the current in the coil sides shown in fig. 1.29c is in the same direction as the emf's shown in fig. 1.29b. But current in a magnetic field produces force, and application of the RH rule gives the force directions shown in fig. 1.29c.

Clearly, the torque is in the CCW direction, that is, the developed torque opposes the rotation. Therefore, there must be an external torque that forces the coil to rotate in the CW direction; the mechanical source provides this torque which acts against the opposition of the torque developed electro-magnetically in the coil.

Fig. 1.29d gives the relative directions for the generating mode. On the electrical side, the emf drives the current into the load so that

E and I are in the same direction;

on the mechanical side, the developed torque opposes rotation so that

T and ω are in opposite directions.