



Finally, let us write down the circuit equation for the generating coil. Applying Kirchhoff's voltage law to the equivalent circuit of fig. 1.29a. We have

$$V = E - IR \quad (1.33)$$

That is, the terminal voltage V is less than the induced emf by the amount of voltage drop in the coil resistance IR . Multiplying throughout current by the I , eqn. 1.33 gives

$$VI = EI - I^2R \quad (1.34)$$

The output power going to the load VI is less than the conversion power $P_c = EI$ by the amount of power lost as heat in the coil itself I^2R . We shall see in later chapters that the process of electromechanical conversion is accompanied by other types of loss, such as friction between moving parts, and hysteresis and eddy currents in iron cores.

1.7.2 Motor Action

A motoring coil acts as a load in the electrical circuit :it receives electrical power from the circuit, fig.1.30a. The electrical source supplies the input power VI to the circuit; part of this

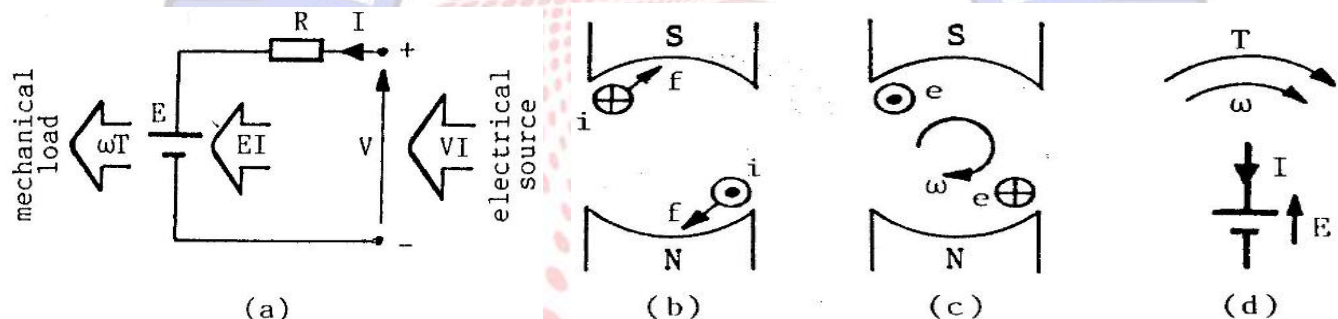


Fig. 1.30 Rotating armature coil in motoring mode

power is lost as heat in the resistance of the coil, $I^2 R$, and the rest goes to the emf-element of the coil as the electric power EI . Conservation of energy and power requires that the power fed into the emf-element of the circuit must go somewhere; in the motoring coil, it goes to the mechanical side as $P_c = \omega T$, where it rotates the mechanical load.

Let us assume that at a certain instant the coil current is in the direction shown in fig. 1.30b; application of the RH rule then gives the force directions in the two sides of the coil, and these produce a CW torque. Since the coil is motoring, it causes the rotation of the shaft and load; that is, rotation is in the same direction as the torque developed electromagnetically, which is CW as shown in fig. 1.30c. But motion in a magnetic field induces emf, and application of the RH rule gives the instantaneous emf directions shown in fig. 1.30c. Clearly, the induced emf opposes current.



Therefore, there must be an external voltage applied to the electric circuit that forces the current to flow against the emf; the electrical source provides this voltage which acts against the emf induced electromagnetically in the coil.

Fig. 1.30d gives the relative directions for the motoring mode. On the electric side, the external source forces the current against the induced emf so that

E and I are in opposite directions;

on the mechanical side, the developed torque causes the rotation, so that

T and ω are in the same direction.

The circuit equation for the motoring coil is obtained by applying Kirchhoff's voltage law to the equivalent circuit of fig. 1.30a:

$$V = E + IR \quad (1.35)$$

The terminal applied voltage V must be greater than the induced emf E by the amount of voltage drop in the coil resistance IR . Multiplying throughout by the current I , eqn. 1.35 gives

$$VI = EI + I^2R \quad (1.36)$$

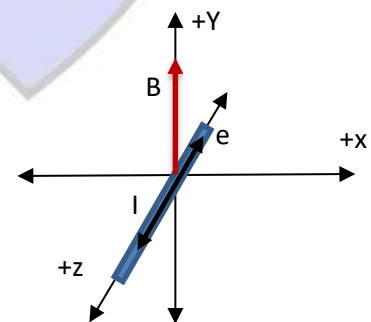
The input power from the electric source VI is greater than the conversion power $P_c = EI$ by the amount of power lost as heat in the coil, I^2R .

Example1: A straight conductors moving at constant speeds through uniform magnetic fields. Active length $60(z)$ mM, Flux density $150(+y)$ m T, Resistance 10 m Ω , Current $0.5(+z)$ A, Induced emf, $40(-z)$ mV The indicated directions follow a right-hand Cartesian coordinate system. Find

- Velocity and direction.
- What action? and why?
- Terminal voltage.
- Developed force and direction.
- Conversion power.
- Copper loss.
- What happen when the direction of flux is reversed?

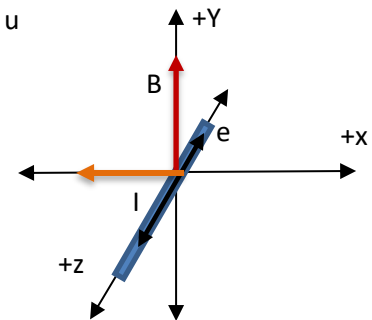
Solution:

$$a) \quad u = \frac{e}{BL} = \frac{40 \cdot 10^{-3}}{150 \cdot 10^{-3} \cdot 60 \cdot 10^{-3}} = 4.44 \text{ M/s } (-x)$$





- 1- By applying the RH rule, the conductor must be moving u perpendicular to the direction of the flux B , so the moving of velocity must in the direction of x or z axis.
- 2- The conductor is in the long z axis so the moving must in x axis only.
- 3- From 1 and 2 can to get the direction of velocity u moving the fingers from I to B then **get u in the direction of $(-x)$** also can conclude the direction of emf e is in the $(-z)$.
- b) From a can conclude **the action is motor** because of the direction of current is opposite to the direction of emf.
- c) For motor $v = e + IR = 40 * 10^{-3} + 0.5 * 10 * 10^{-3} = 45 * 10^{-3} V$
- d) $F = i BL = 0.5 * 150 * 10^{-3} * 60 * 10^{-3} = 0.0045 N$ ($-x$) in motor action the force is in the direction of velocity ($-x$).
- e) $P_c = EI = 40 * 10^{-3} * 0.5 = 20 * 10^{-3} W$
- f) Copper loss $= I^2 R = 0.5^2 * 10 * 10^{-3} = 2.5 * 10^{-3} W$
- g) When the direction of the flux is reversed the direction of rotation speed is reversed also ($+x$) using RH rule.



Example2: the coil span should be approximately equal to a pole pitch. Discuss the effect of making the coil span very different from the pole pitch on (a) the coil emf and (b) the coil torque. Use a diagram like that shown in fig. 1.24, and assume first a short-pitched coil spanning half a pole pitch, then a long-pitched coil spanning one and a half pole pitches.

Solution:

a short-pitched coil spanning half a pole pitch:

as see in figure for a short-pitched coil spanning half a pole pitch the emf in the coils x_1-y_1 that is laying under the same pole will introduce the same voltage but opposite polarity x_1 and y_1 so the resultant emf introduced is zero $E_{x_1} - E_{y_1}$, the also the resultant force is zero because no current will passing through the coil, in the other coils x_2-y_2 the emf will introduced in x_2 but in y_2 there is no voltage introduced that is mean the resultant emf is $\frac{1}{2}$ of coil emf also the torque introduced is in the part x_2 only and zero in part y_2 but as a resultant the torque is $\frac{1}{2} T_{coil}$. a long-pitched coil spanning one and a half pole pitches:

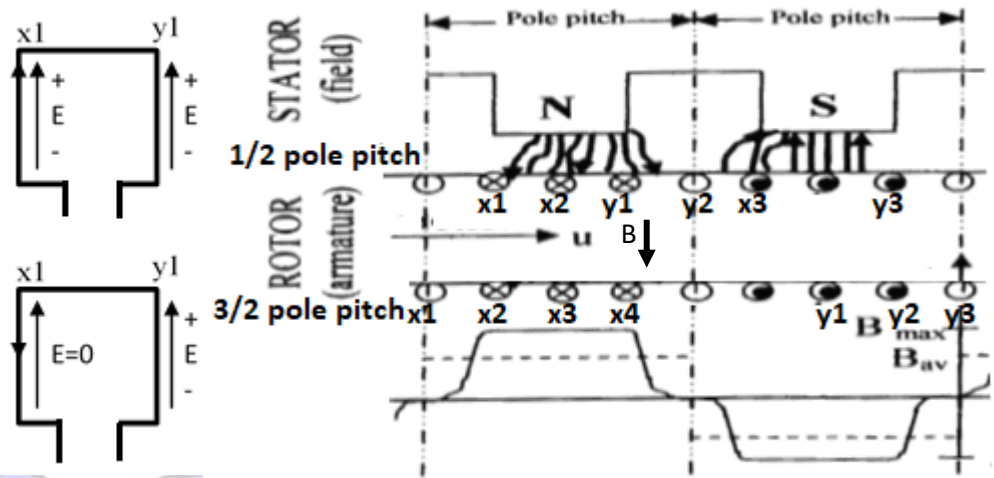
we can see from figure the coil side x_1 is lying between poles so no magnetic field will be cutting the conductors but the other side y_1 is laying under the pole and will introduced emf so the resultant is $\frac{1}{2}$ emf of the coil and $\frac{1}{2} T_{coil}$. The coil x_2-y_2 will introduced emf coil and T_{coil} because of each coil side is under different pole. The coil x_3-y_3 is the same of coil x_1-y_1



but in other polarity.
 The coil x_4-y_4 is under the pole polarity so the induced emf is zero also the torque.

Exercise

1.1: The table below relates to straight conductors moving at constant speeds



through uniform magnetic fields. The indicated directions follow a right-hand Cartesian coordinate system (a screw turning from x to y advances with z). Complete the table giving values, as well as directions where applicable.

	I	ii	iii	Iv	V	vi
Flux density, m T	80(+z)	80(+z)	300(-y)	150(+y)	?	120(+y)
Active length, mM	300(y)	300(y)	150(x)	60(z)	250(x)	200(z)
Resistance, mΩ	5	5	8	10	0.8	2.5
Velocity, m/s	1.6(+x)	1.6(-x)	?	?	?	2(-x)
Current, A	2(-y)	2(-y)	?	0.5(+z)	12(-x)	4(-z)
Induced emf, mV	?	?	90(-x)	40(-z)	-x	?
terminal voltage, mV	?	?	105	?	60	?
Developed force, mN	?	?	?	?	750(+z)	?
Conversion power, mW	?	?	450	?	840	?
Copper loss, mW	?	?	?	?	?	?
Input power, mW	?	?	?	22.5	?	?
Output power, mW	?	?	?	?	?	?
Action	?	?	mot	Mot	Gen	?



1.2 The armature of an 8-pole dc machine has 72 slots, and its diameter is 48 cm. Find the pole pitch in (a) meters, (b) mechanical degrees, (c) mechanical radians, (d) electrical degrees, (e) electrical radians, and (f) number of slots. Also find (g) the electrical angle corresponding to a mechanical

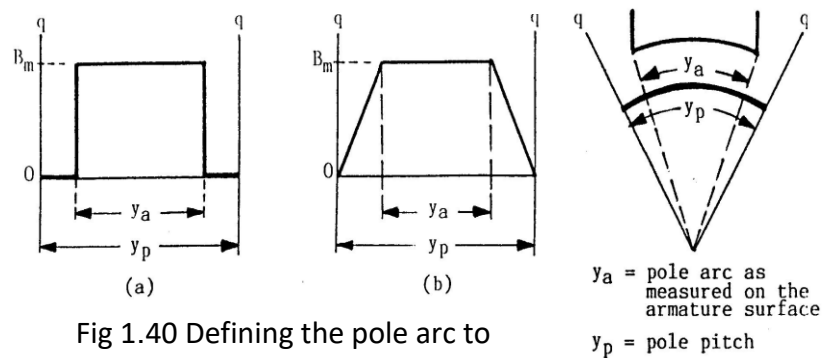


Fig 1.40 Defining the pole arc to pole pitch ratio, $\alpha = y_a/y_p$.

angle of 35° , (h) the mechanical angle corresponding to an electrical angle of 35° , and (i) the distances along the armature surface corresponding to the angles in g and h.

1.3 The slot pitch is the distance from the centre of one slot to the centre of the adjacent slot, measured along the surface of the armature. For the armature of question 1.2, find the slot pitch in (a) meters, (b) mechanical degrees, and (c) electrical degrees.

1.4 (a) Assuming that the air gap flux density of fig. 1.25 can be approximated as in fig. 1.40a, shows that the flux per pole and the average air gap flux density are given by:

$$\Phi = \alpha \pi B_m D \ell / 2p \quad \& \quad B_{av} = \alpha B_m, \quad (\alpha = y_a/y_p)$$

(b) Assuming that the air gap flux density of fig. 1.25 can be approximated as in fig. 1.40b, shown that the flux per pole and the average air gap flux density are given by.

$$\Phi = (\alpha + 1) \pi B_m D \ell / 4p \quad \& \quad B_{av} = (\alpha + 1) B_m / 2, \quad (\alpha = y_a/y_p)$$

1.5 Assuming that the air gap flux density of fig. 1.25 can be approximated by a sine wave, show that the flux per pole and the average air gap flux density are given by:

$$\Phi = B_m D \ell / p \quad \& \quad B_{av} = (2/\pi) B_m$$

where B_m is the maximum flux density.

1.6 The armature of a 6-pole dc machine is 80 cm long and has a diameter of 50 cm. The maximum air gap flux density is 1.5 T. The pole arc covers 70% of the pole pitch (see fig. 1.40). The armature speed is 500 rpm (revolutions per minute). Assume the air gap flux density is distributed as in fig. 1.40b.

- a. Find the flux per pole and the average air gap flux density.
- b. Find the emf, developed torque, and conversion power for a wire loop on the armature carrying a current of 9 A.



1.7 The armature of a 4-pole dc machine rotates at 840 rpm. The armature length and diameter are 40 cm and 30 cm respectively. The flux per pole is 65 mWb. Each armature coil has 5 turns, and develops an average torque of 3.0 Nm.

a. Find the coil emf, current, and conversion power.

b. Make a rough estimate of the copper ($I^2 R$) loss in the coil assuming it is made of 1.0 mm² copper wire, and the losses during operation raise the temperature to 80 °C.

1.8 The armature of a small 2-pole dc machine rotates at 1200 rpm. The armature length and diameter are 22 mm and 26 mm respectively. Each armature coil is made of 30 turns, with a resistance of 4 mΩ per turn. The coil terminal voltage and induced emf are 2.0 V and 1.8 V respectively. Find the average air gap flux density and the flux per pole. Also find the coil torque and conversion power.

1.9 A constant dc voltage of 10 V is applied, through brushes and commutator, to a 7-turn coil placed on a 4-pole armature of length and diameter 24 cm and 36 cm respectively. The coil resistance is 50 mΩ, and the average flux density in the air gap is 658 mT.

a. Find the average developed torque when the armature rotates at 240 rpm.

b. Find the average developed torque when the speed is (i) decreased by 10%, and (ii) increased by 10%.

c. Find the copper ($I^2 R$) loss in the coil for each of the three cases in parts a and b.

d. What is the mode of operation of the coil in each of the cases in parts a and b?

e. Derive an equation that gives the coil torque as a function of speed; sketch the relationship.

1.10 If the diagrams in figs. 1.6 and 1.7 correspond to the same armature loop, is that loop operating as a motor or as a generator? If the direction of the field is reversed in both figures, would the mode of operation change? How would you change fig. 1.6 to obtain the opposite mode of operation? How would you change fig. 1.7 to obtain the opposite mode of operation?

1.11 Repeat question 1.10 for figs. 1.9a and 1.13. Repeat it again for figs. 1.22 and 1.23.

1.12 For the developed diagram of the 4-pole machine shown in fig. 1.24a:

a. The coils on the poles are called field coils. They should be connected in series; draw the necessary interconnections between them.



b. Indicate on the diagram suitable directions for motion, currents, emfs, and forces in the armature conductors for motor operation.

c. Repeat part b for generator operation.

1.13 The armature of a 6-pole dc motor has 24 slots and rotates in the clockwise direction. Draw a cross-section of the machine showing (a) north and south poles, (b) all direct and quadrature axes, (c) a pole pitch, (d) flux paths, (e) field coils, with their interconnections and current directions, (f) armature conductors with current and emf directions, (g) directions of torque and rotation.

1.14 Repeat question 1.13 for a 4-pole generator whose armature has 16 slots and rotates in the counterclockwise direction.

1.15 The armature of a 10-pole dc machine has a diameter of 48 cm and rotates at 570 rpm. Find (a) the speed in rps (revolutions per second) (b) the speed in meters per second, (c) the angular speed, (d) the time it takes to complete one revolution, (e) the time it takes to complete one electrical cycle, (f) the time it takes to move one pole pitch, (g) the distance a point on the armature surface covers in one second.

1.16 In fig. 1.37c, let α denote the angle between the inclined surface and the horizontal plane, and β the angle between the applied force f_a and the inclined surface. Use the symbols defined on the figure, together with α and β , in the following requirements:

a. Write out the force equation along the axis parallel to the surface.

b. Write out the force equation along the axis perpendicular to the surface.

c. Assuming that the body moves up the inclined surface at a constant speed u , what is the energy lost by the agent applying the force f_a in a time interval Δt ? Where does this energy go? What is the power exerted by the moving agent?

d. where does the lost energy in part c go if the speed changes as the body moves up the incline?

1.8 APPENDIX :Review of mechanics

This appendix reviews some basic principles from mechanics which are important to the study of electrical machines. First, we consider rectilinear motion (ie motion in a straight line), then modify our equations for rotary motion.