CHAPTER 3
ARMATURE WINDING

In chapter 1 we saw that the armature has a number of coils displaced from each other in space. The coils are connected to each other at the commutator segments to form the armature winding. This chapter explains the distribution of coils in slots, the connection of coil terminals to commutator segments, and the resultant equivalent circuit of the armature.

3.1 Winding details

The coil span is the distance, along the armature surface, between the two sides of the coil, fig. 3.1; it is equal to a pole pitch so that when one coil side is under a north pole, the other coil side is under a south pole, and hence their emfs add, section 1.5. The coil span can be measured in units of length, in mechanical degrees, or in electrical degrees; it can also be measured in slot pitches, where a slot pitch is the distance from the center of one slot to the center of the next slot. Thus in fig. 3.1, the coil span is four slot pitches, or four slots for short; clearly, the coil spans an integer number of slots.

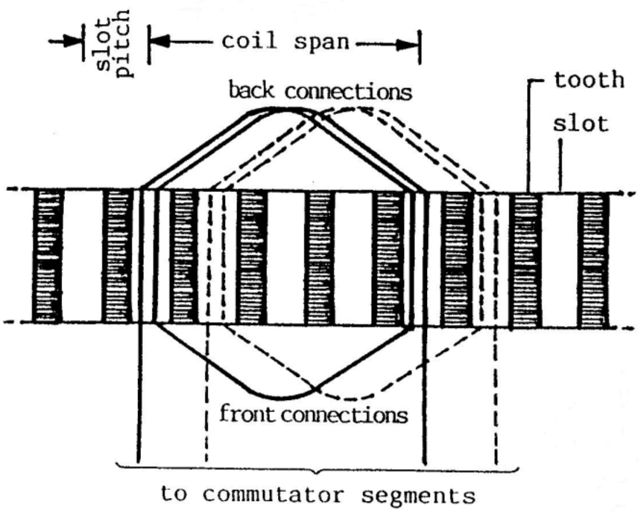


Fig. 3.1 Developed side view of armature showing locations of two coils.

Let

N =number of turns per coil;

C =total-number of armature coils;

Z= total number of armature conductors.

Each turn has 2 active conductors, so that each coil has2N active conductors; then

$Z = 2NC$ (3.1)

It should also be clear that the total number of coil sides is $2C = Z/N$, and that the total number of armature loops or turns is NC.

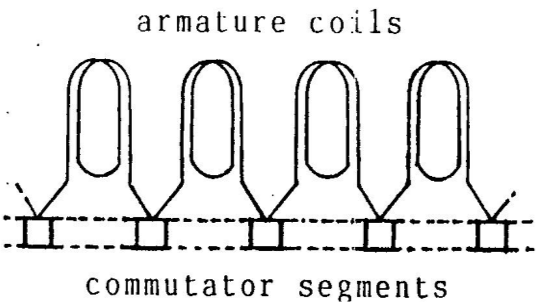


Fig. 3.2 Interconnection of armature coils on commutator segments.

The two terminals of each coil are soldered to the risers of two different commutator segments. Moreover, the riser of each segment has two terminals, coming from different coils, soldered to it. Since there are C coils, there are 2C coil terminals, and hence 2C /2 =C commutator segments; that is

Number of commutator segments =number of armature coils =C.

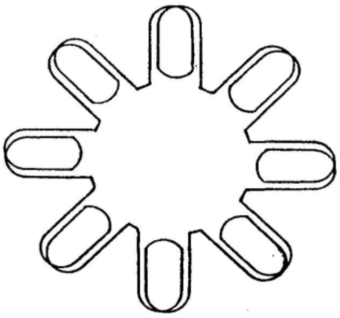


Fig. 3.3 Armature coils are connected to form a closed winding.

All the coils on the armature are connected together, on the commutator, to form a single closed winding :starting with any coil, its end is connected to the beginning of a second coil; the end of the second coil is connected to the beginning of a third coil, and so on, as in fig. 3.2. this is repeated until the last coil is reached :its end is connected to the beginning of the first coil, and the winding is closed, fig. 3.3. The diagrams in figs. 3.2 and 3.3 do not show which coils are connected to which segments; the sequence of interconnected coils and segments will be explained in section 3.2. What is important to understand here is that the armature coils form a closed winding, and that the interconnections between coils are made on the commutator segments. Note in particular that the closed armature winding, as shown in fig. 3.3, has no terminals, and we cannot say which coils are in series or parallel; in fact, it is the brushes, contacting the commutator segments, which provide the terminals of the winding, and divide the coils into series and parallel groups.

Armature coils are arranged in two layers :for each coil, one side is placed in the top half of its slot (top layer), and the other side is placed in the bottom half of its slot (bottom, layer); the bottom half of the first slot and the top half of the second slot are filled by other coil sides, fig. 3.4.

Two-layer windings allow coil end connections, in the front and the back, to be arranged in a regular manner; otherwise, the end-connections of adjacent coils will be in each other's way. All the coils in a two-layer winding can be made identical, which simplifies manufacture, and results in a symmetric and compact armature winding. Small machines usually have one coil side per slot per layer, and hence two coil sides per slot, fig. 3.4. Large machines may have more coil sides per slot; for example, in fig. 3.5 there are three coil sides per slot per layer, and hence six coil sides per slot. Because there are two layers, the number of coil sides per slot is always an even number :two in fig. 3.4, and six in fig. 3.5.

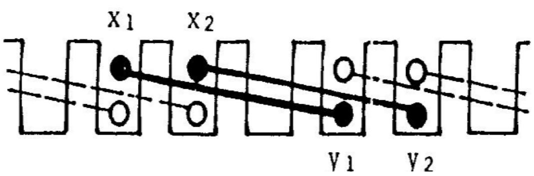


Fig. 3.4 Two-layer winding.

In fig. 3.5, x_1 and y_1 are the two sides of one coil; clearly, the coil has 4 turns, and each coil side has 4 conductors. The two terminals of the coil are soldered to commutator segments. Similarly, x_2 and y_2 are the two sides of another coil which has 4 turns, and whose terminals are also soldered to commutator segments.

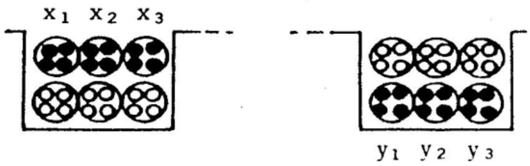


Fig. 3.5 Two-layer winding with 3 coil sides per slot per layer.

Let S = number of armature slots;

M= number of coil sides per slot per layer.

In fig. 3.5, m=3; also, N=4, so that the number of conductors per slot per layer is 3x4 = 12, and the number of conductors per slot is 2x12 =24. In general, there are mN conductors/slot/layer, and hence 2mN conductors/slot.Clearly, then.

$Z = 2mNS$ (3.2)

Substituting for Z from eqn. 3.1, we find

$C = mS;$ (3.3)

thus, the number of coils C is an integer multiple of the number of slots S. The two are equal when m =1, that is, for windings having one coil side per slot per layer, or two coil sides per slot, as in fig. 3.4.

In section 1.5, the pole pitch was defined as $\pi D/2p$, the distance on the armature surface corresponding to one pole; it was also shown how the pole pitch may be measured as a mechanical or electrical angle. It is often useful to measure the pole pitch by the number of slot pitches it covers :the armature has a total of S slots, so that to each pole there correspond S/2pslot pitches, or simply slots; thus

pole pitch $= s/2p$slot pitches, or slots. (3.4)

For example, if the armature of an 8-pole machine has 120 slots, the pole pitch is 120/8 =15 slot pitches, or slots. Recalling that the coil span should be equal to a pole pitch, it is also 15slots in this example. Now suppose that an 8-pole machine has 122 slots in the armature; the pole pitch is given by eqn. 3.4 as 122/8 =15.25 slot pitches, or slots. But the coil span, when measured in slots, has to be an integer because it represents the number of slots covered in moving from the first coil side to the second coil side, both of which are placed in slots; in this example, then, the coil span cannot be exactly equal to the pole pitch. In such cases, the coils are wound with a span just below, or just above, the pole pitch; that is, 15 slots or 16 slots in our example. Coils whose span is approximately, but not exactly, equal to the pole pitch are described as being chorded; the following terms are also in common use:

coil span =pole pitch → full-pitched coils

coil span < pole pitch → short-pitched coils

coil span > pole pitch → long-pitched coils

chorded coils

Thus, for a pole pitch of 15 slots, as in the first example above, the coils are full-pitched with a span of 15 slots. In the second example, with a pole pitch of 15.25 slots. the coils cannot be full-pitched; they may be short-pitched with 15 slots, or long-pitched with 16 slots. In all cases, the coil span should not differ too much from the pole pitch to ensure that when one coil side is under a north pole, the other coil side is under a south pole.

3.2 Some Numbers

C= total number of coils on armature;

N = number of turns in each coil;

S = total number of a slots in armature;

Z= total number of armature conductors; = 2NC.

$\frac{S}{2P}$ = total number of slots / poles, = pole pitch measured in slots.

NC =total number of loops;

2C= Z / N = total number of coil sides;

Z / S = 2NC / S = number of conductors / slots;

2C / S = number of coil sides / slot;

NC / S = number of conductors / slot/ layers;

C/ S = number of coil sides / slot / layer.

3.3 Winding schemes

In the last section, it was emphasized that the armature coils form a closed winding, fig. 3.3, and that the interconnections between coils are made on the commutator segments, fig. 3.2. This section explains the sequence of interconnected coils and segments, that is, which coils are soldered to which segments.

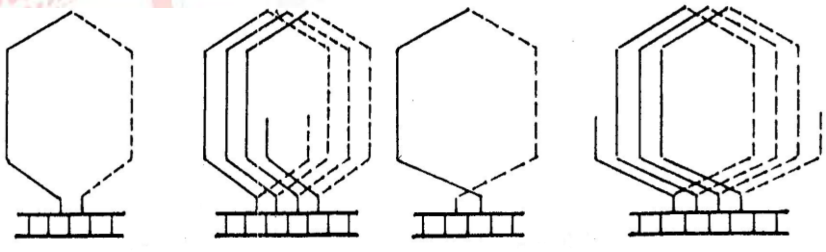
Recalling that the two terminals of each coil are soldered to two different commutator segments, let us define the commutator pitch as the distance, on the commutator, between the coil terminals. This distance need not be measured in units of length, but simply in number of segments advanced as we go from the segment of the first terminal to the segment of the second terminal; we shall use the symbol y_c for the commutator pitch, and it should be clear that y_c is always an integer number of segments.

There are two ways for interconnecting coils and segments, lap and wave, which will be explained in the following sections. Lap windings are easier to understand, and hence will be studied first; but it is wave windings that are more widely used. To keep the diagrams relatively clear, each coil is shown to have a single turn; moreover, the coil side lying in the top layer is shown in full-line, while the coil side lying in the bottom layer is shown dashed.

In certain diagrams, there are arrows on coil sides; these may represent the direction of induced emf, or the direction of current.

3.3.1 Lap winding

In the lap winding scheme, each coil is connected to the coil that lies immediately next to it on the armature. Such adjacent coils are in approximately the same position relative to the poles; that is, they see almost the same flux density. The terminals of each coil are soldered to adjacent commutator segments, so that the commutator pitch y_c is one. Fig. 3.6 shows that there are two ways of forming lap windings, progressive and retrogressive; the choice between these has no important effects. The commutator pitch is taken to be positive for progressive winding, and negative for retrogressive winding; thus, for lap windings in general



(a) progressive, $y_c = +1$; (b) retrogressive, $y_c = -1$.

Fig. 3.6 Lap winding

$$y_c = \pm 1 \text{(lap)}. \quad (3.5)$$

In both cases, the connection of the end of one coil to the beginning of the next coil is continued until the last coil, the Cth coil, is reached, whose end then closes on the beginning of the first coil.

As an example, let us consider a 4-pole armature with 22 coils (2p=4, C =22). For simplicity, we assume the armature has 22 slots, so that there are only two coil sidesin each slot, one in the top layer, and one in the bottom layer (m =1, and C=S in eqn. 3.3). The pole pitch is 22/4=5.5 slots, so that the coils cannot be full-pitched; they can be short-pitched with a coil span of 5 slots, or long-pitched with a coil span of 6 slots. In our example, we choose short-pitched coils, which have the advantage of shorter end-connections. The complete winding is shown in fig. 3.7 as a developed diagram. Let us first study the coils and commutator segments, both of which are on the rotor; that :is, they move together. The coils and segments are numbered sequentially such that the first terminal of coil 1 is soldered to segment 1; of course, any coil could have been chosen as number 1. The winding is progressive, $y_c+ = 1$, so that the second terminal of coil 1 is soldered to segment 2. Also soldered to segment 2 is the first terminal of coil 2, which lies immediately after coil 1 on the armature; the second terminal of coil 2is soldered to segment 3, on which coil 3 starts. You can thus follow the coils, one after the other, passing through their interconnections on the commutator segments. When you reach coil 22, you will see that it starts on segment 22, and ends on segment 1; that is, it closes on coil 1.

Consider now the slots. Although they are not drawn explicitly in the figure, their positions are indicated. Each slot contains two coil sides, one in the top layer, drawn in full-line, and one in the bottom-layer, drawn dashed.

For example, slot 1 has the first side of coil 1 in the upper layer, and the second side of coil 18 in the bottom layer. The coils are short-pitched with a coil span of 5 slots; thus coil 1, which has its first side in slot 1, has its second side in slot (6=1+5). Coil 20 has its first side in slot 20 and its second side in slot (3= 20+5-22). Each coil has its first side in the top layer (drawn in full-line), and its second side in the bottom layer (drawn dashed). Note that the two coil sides in each slot are at the same position, although in the diagram they appear to follow each other around the armature.

Consider, finally, the poles; these actually lie above the active conductors, but have been drawn as shown in fig. 3.7 for clarity. The brushes and quadrature axes positions are also indicated. The distance between consecutive q-axes is a pole pitch, and it is slightly greater than the coil span; compare, for example, the distance from q_4 to q_l with the distance from slot 11 to slot 16, which represents one coil span, short-pitched.

The poles, q-axes, and brushes are not part of the armature winding; they are stationary, while the armature is in motion. Assuming the armature is moving to the right, in a short while slot 1 will be in the position of slot 2, which will have moved to the position of slot 3, and so on; meanwhile, poles, q-axes, and brushes will remain in their positions. Now, we know from chapter 1that poles alternate in polarity :if P1 is north, then P2 is south, P3 north, and P4 south, and vice versa. The arrows on the active conductors may be taken to indicate the directions of current flow :all the arrows under each pole are in the same direction, and are opposite to the arrows under the next pole; we have already studied this in chapter 1 -see, for example, figs. 1.22 and 1.23.

Coils 6, 11,17, and 22 have no arrows on their active conductors :each of the coils is short-circuited by a brush at the instant shown in fig. 3.1; such coils are said to be “undergoing commutation”, ie their currents and emfs are being reversed. To understand this, consider, for example, coil 10; at the instant shown in fig. 3.7, coil 10 is active. But in a short while, coil 10will have moved to the position of coil 11, so that brush B_3 short-circuits it. A little while later, coil 10 will come out from the short, and it will take the position of coil12, so that its arrows will be reversed with respect to the arrows of coil 10 in fig. 3.7. This reversal occurs in each coil every time it passes through a brush short-circuit. But the brushes are placed in such positions that the coils they short-circuit are the ones which have their active sides at or near the q-axes; that is, the reversal occurs in each coil when it passes through a q-axis (ie when its two sides are at q-axes). The q-axis is therefore sometimes called the brush axis : although the brushes do not themselves lie there, the coils which are in direct contact with the brushes have their active sides; there. It is noted that the instantaneous emf induced in a coil passing through the q-axis is zero because the flux density b is zero there, fig. 1.24b.

It should now be clear that in fig. 3.7 the armature coils, slots :and commutator segments are in motion, while the poles, brushes, q-axes, as well as the arrows, are stationary. The arrows do not change their directions, although the conductors to which they are attached are changing all the time.

The closed winding formed by the coils is shown in fig. 3.8; the diagram shows the sequence with which the coils are connected to each other. The brushes divide the coils into four groups; for example, coils 1, 2, 3, 4, and 5 are in series between brushes B1 and B2; similarly, coils 10, 9, 8, and 7 are in series between brushes B3 and B2. Each group of series coils provides a current path between two brushes, and the inter-brush connections connect the four paths in parallel with each other, as shown in fig. 3.9.

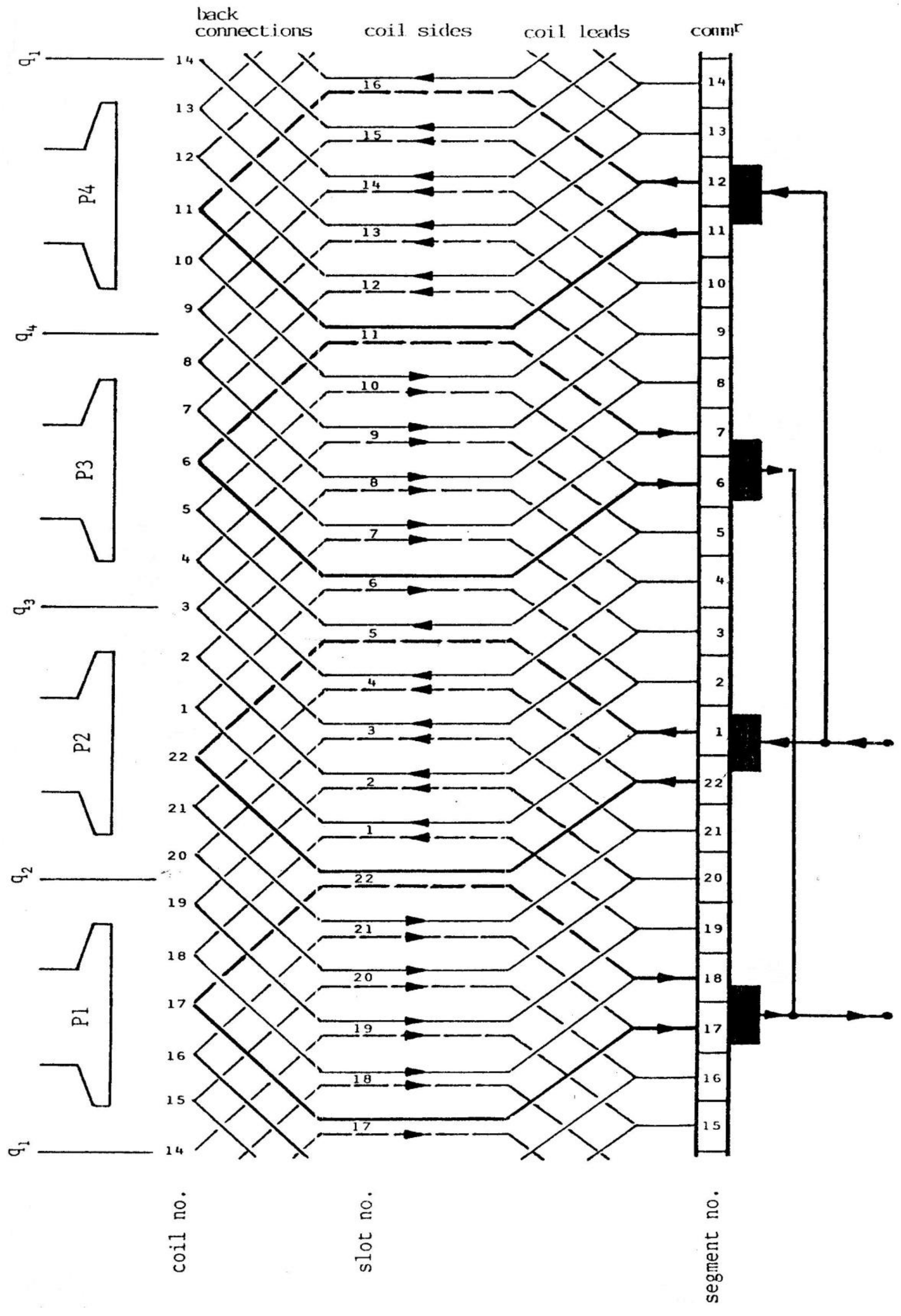


Fig.3.7 Simple lap winding :developed diagram for 4 poles and 22 armature coils

To understand these connections, let us follow some of the paths :the current entering into the machine divides between brushes B1 and B3; the current into B1 divides between two paths, coils 1-2-3 -4-5 to the right, and coils 21-20-19-18 to the left; the right path terminates on B2, and the left path on B4; but B2 and B4 are connected externally to each other; therefore, these two paths are in parallel with each other between brush Bl and brushes B2/B4.

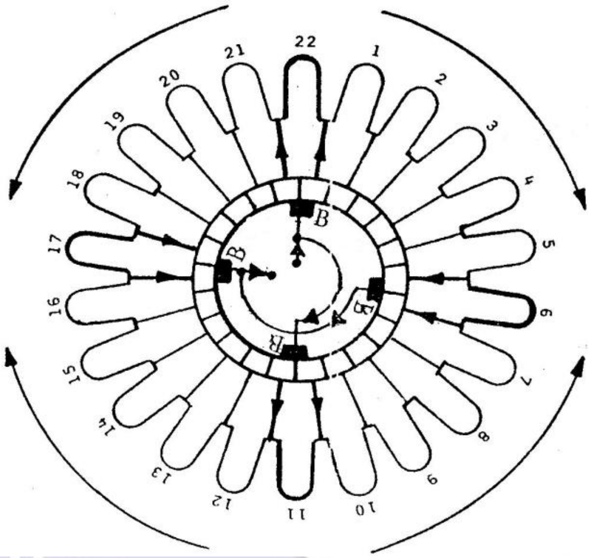


Fig. 3.8 Simple lap winding: sequence diagram corresponding to fig.3.7

Similarly, the path 10-9-8-7 and the path 12-13-14-15-16 are in parallel with each other between brush B3 and brushes B2/B4. Since, moreover, B1 and B3 are connected to each other externally, all paths are in parallel as shown in fig. 3.9. The machine in our example has 4 poles, 4 brushes, and 4 parallel paths; in general, for lap windings, the number of paths is equal to the number of brushes which, in turn, is equal to the number of poles. Denoting the number of parallel paths by 2a (ie a is the number of pairs of parallel paths), we have

$2a = 2p$ (lap). (3.6)

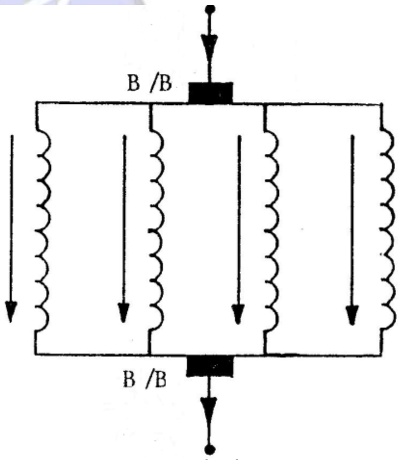


Fig. 3.9 Simple lap winding :parallel paths between brushes.

Of course, because the armature is rotating and the brushes are stationary, the coils forming the paths are changing all the time. Let us assume c1ockwise rotation, and consider coil 10. At the instant shown in figs. 3.7 and3.8, coil 10 is part of the path 10-9-8-7. But in a short while, coil 10 will enter into short-circuit by brush B3, and hence will leave this path. When it comes out from the short-circuit, coil 10 will join the next path, which will be 10-11-12-13-14; in this new path, the current in coil 10 will be reversed :as already explained with reference to fig. 3.7, the arrows in fig. 3.8 are stationary in space.

The above discussion helps explain the function of the commutator and brushes. Consider a coil that is part of a path, with a given direction of current, and a given direction of emf. When the coil reaches the q-axis, it will also be short-circuited by a brush. When the short-circuit ends, the coil will join the next path, with the direction of current reversed; the direction of the emf is also reversed because the coil sides are now under poles of reversed polarity. Thus, the emf and current of the coil are alternating, but the emf and current at the brushes are unidirectional.

3.3.2 Wave Winding

In the wave winding scheme, each coil is connected to a coil that lies approximately two pole pitches away from it on the armature, fig. 3.10. Since the air gap flux density curve is a wave that repeats itself every two pole pitches, fig. 1.24, such coils see almost the same flux density; remember that the same thing was achieved in the lap winding by connecting adjacent coils, section 3.2.1. To connect the coils in wave, the two terminals of each coil are soldered to commutator segments that are separated by approximately two pole pitches on the commutator, fig. 3.10; that is, the commutator pitch y_c is approximately (C/2p)X2 =C/p.

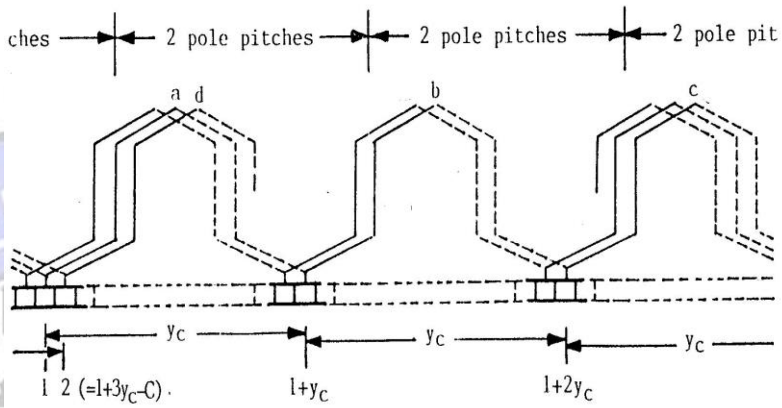


Fig. 3.10 Progressive wave winding for a 6-pole

To find out the exact value of y_c let us follow the coil connections in fjg. 3.10. Coil a start from a segment which can be taken as number 1; the coil then ends on segment number (1 +y_c) according to the definition of the commutator pitch y_c on page 3.4. At segment (1 +y_c) starts coil b, which is approximately 2 pole pitches after coil on the armature; that is, the slot of the first side of coil b is approximately two pole pitches away from the slot of the first side of coil a, and similarly for their second sides. Coil b ends on segment number (l+y_c)+ yc= (l +2y_c).

At segment (1 +2y_c) starts coil c, which is approximately 2 pole pitches after coil b; coil c ends on segment number (l+2y_c) + y_c = (l + 3y_c) at which starts coil d which is approximately 2 pole pitches after coil c. But this means that we have moved 6 pole pitches :2 from a to b, 2 from b to c, and 2 from c to d.

Since the machine in fig. 3.10 has 6 poles, we are back approximately where we started; that is, coil d is next to coil a, and we have gone around the entire armature once. Similarly, we have gone around the entire commutator once, and segment number (l +3y_c) is actually segment number 2, right after segment 1from which we started. Since the number (l +3y_c) is greater than the total number of segments C, we must have

$$1 + 3y_c - c = 2 \rightarrow y_c = \frac{c+1}{3} = \frac{c+1}{p} \qquad (3.7a)$$

This is the exact value of the commutator pitch, and it is slightly different from the approximate value *C/p*. If we attempt to make the commutator pitch exactly *C/p*, then coil c will end on segment 1 instead of 2, and the three coils a, b, and c will form a closed circuit by themselves, which is not what we want :we must go through all the armature coils before closing the circuit, fig. 3.3. The +1 in eqn. 3.7a ensures that, after moving through p coils, we reach the segment after the one we started from. The winding shown in fig. 3.10is progressive; for a -retrogressive winding, we reach the segment before the one we started from. That is, coil c ends on the segment to the left of segment number 1, ie segment number C; thus

$$1 + 3y_c = c \rightarrow y_c = \frac{c-1}{3} = \frac{c-1}{p} \qquad (3.7b)$$

where the -1 ensures that after moving through p coils we reach the segment before the one we started from. The general expression for the commutator pitch is thus

$$y_c = \frac{c \pm 1}{p} \quad \text{wave} \qquad (3.8)$$

With the positive sign for progressive windings, and the negative sign for retrogressive windings. In both cases, the connection of the end of one coil to tile beginning of a coil approximately 2 pole pitches after it is repeated until the last coil is reached, whose end then closes on the beginning of the first coil to form a closed winding including all the armature coils. Every time we go through p connected coils, we complete one turn around the armature; in lap winding, on the other hand, we have to go through all the armature coils to complete one turn around the armature, figs. 3.6 and 3.7.

As an example, let us try to reconnect the armature of the previous section in wave. The armature slots and coils, the commutator segments, and the poles are all unchanged; the only difference is that the terminals of the coils will now be soldered to different segments. The example machine had 4 poles and 22 armature coils; to connect the coils in wave, the commutator pitch must be

$$y_c = \frac{22 \pm 1}{2} = 11.5 \text{ or } 10.5$$

which is impossible :the commutator pitch has to be an integer because it represents a number of segments, and it is meaningless to have a fraction of a segment. Therefore, this machine, with 2p=4 and C=22. cannot be connected in wave. In general, to be able to connect a winding in wave, the number of poles 2p and the number of coils C must be such that eqn. 3.8 gives an integer answer for y_c; otherwise, the winding cannot be connected in wave. There is no such difficulty with the lap scheme :any number of coils can be connected in lap, since the consecutive coils simply follow each other around the armature.

Let us then change our example a little :instead of 22 coils, we take the number of armature coils to be 21; the number of poles is still 4. The commutator pitch is now

$y_c = \frac{21\pm1}{2} = 11 \text{ or } 10$

With a commutator pitch of 11, the winding is progressive, and with a commutator pitch of 10 it is retrogressive; we choose a progressive winding, ie y_c= 11segments. The resulting developed diagram is shown in fig. 3.11. Coil 1 starts from segment 1 and ends on segment 12 (=1 + 11); coil 12 then starts from segment 12 and ends on segment 2 (=12 +11 -21). Thus, by tracing two coils, we have gone once around the armature, and advanced one commutator segment, from segment 1 to segment 2. Coil 2 followed by coil 13 then take us to segment 3, and so on. The last coil which brings us back to segment 1 is coil 11; the sequence of coil connections is shown in fig. 3.12.

Consider next the slots. Assuming one coil side per slot per layer (m =1), the number of slots is equal to the number of coils, S =21 according to eqn. 3.3. The pole pitch is 21/4 =5.25 slots, and the coil span has to be either 5 or 6; the figure is drawn for a short-pitched coil span of 5 slots.It is interesting to note that the winding connections would be unchanged if the armature had only 7 slots, with 6 coil sides per slot (m =3); the coil sides in slots 1, 2, and 3 in the figure would all be placed in a single slot, and the coil sides in slots 4, 5, and 6 would be placed in the next slot, and so on.

As discussed with the lap winding, the coils and commutator segments are in motion, while the poles, q-axes, brushes, as well as the arrows, are stationary. The brushes are placed such that they contact the coils passing through the q-axes. As a coil passes through the q-axis, its current and emf are reversed, which is the principle of commutation.

However, the short-circuiting of the coils by the brushes in the case of wave is different from that in lap. We shall see that a wave winding requires only two brushes; let us therefore assume that brushes B3 and B4 in fig. 3.11are removed. Consider coil 21 :it starts from segment 21, and ends on segment 11, which is free (since B3 is removed). At segment 11 is the start of coil 11, which ends on segment 1. Therefore, brush Bl, which is in contact with segments 21 and 1, short-circuits coils 21 and 11 in series. Similarly, brush B2 short-circuits coils 5 and 16in series. It is noted that all these short-circuited coils, 2l, 11, 5, and 16, are passing through q-axes. Tracing the remaining unshort-circuited coils, they will be found to be in the sequence shown in fig. 3.11, forming two parallel paths between brushes B1 and B2 :the coils in each path are in series; that is the same current flows through them, and their emfs add up, being in the same direction along the path. As each coil passes through the q-axis it leaves one path to join the other, with its current and emf reversed. In general, then, wave windings require only two brushes, and have only two parallel paths, fig. 3.13; thus

$$2a = 2 \qquad \text{(wave)}, \qquad (3.9)$$

Irrespective of the number of poles. This is one of the major differences between wave and lap windings -see eqn. 3.6.

Let us now see what happens when brushes B3 and B4 are put back in place. Brush B3 is connected externally to B1; it is also connected to B1 internally through coils 21 and 10 in parallel. Both coils are in the q-axis with negligible emf, and coil 21 is already short-circuited by B1. Therefore, the only effect of B3 is to remove coil 10 from the active path and add it to the short-circuited coils. Similarly, brushes B2 and B4 are connected externally, as well as internally through the coils 5 and 15 lying in the q-axis. Coil 5 is already short-circuited by brush B2, so that the effect of B4 is to remove coil 15 from the active path and add it to the short-circuited coils.

Therefore, the presence or absence of the additional brushes has little effect on the performance of the armature winding :only q-axis coils, with zero emf, are short-circuited or included with the active paths. The reason for this is that similar brushes (say Bl and B3) are placed two pole pitches apart, and the commutator pitch of the coils is also approximately two pole pitches, so that the brushes are connected internally through coils lying in the q-axis with zero or negligible emf; such coils can be short-circuited, or added to the active path without changing its total series emf. Thus, a wave winding can have 2p brushes, but only two are necessary; the additional brushes are sometimes kept to obtain better current distribution over the commutator. If only two brushes are to be kept, we can choose B1 and B2 as above, or B2 and B3, or B3 and B4, or B4 and BI; that is, one brush from each group.