Irrespective of the number of poles. This is one of the major differences between wave and lap windings -see eqn. 3.6.

Let us now see what happens when brushes B3 and B4 are put back in place. Brush B3 is connected externally to B1; it is also connected to B1 internally through coils 21 and 10 in parallel. Both coils are in the q-axis with negligible emf, and coil 21 is already short-circuited by B1. Therefore, the only effect of B3 is to remove coil 10 from the active path and add it to the short-circuited coils. Similarly, brushes B2 and B4 are connected externally, as well as internally through the coils 5 and 15 lying in the q-axis. Coil 5 is already short-circuited by brush B2, so that the effect of B4 is to remove coil 15 from the active path and add it to the short-circuited coils.

Therefore, the presence or absence of the additional brushes has little effect on the performance of the armature winding :only q-axis coils, with zero emf, are short-circuited or included with the active paths. The reason for this is that similar brushes (say Bl and B3) are placed two pole pitches apart, and the commutator pitch of the coils is also approximately two pole pitches, so that the brushes are connected internally through coils lying in the q-axis with zero or negligible emf; such coils can be short-circuited, or added to the active path without changing its total series emf. Thus, a wave winding can have 2p brushes, but only two are necessary; the additional brushes are sometimes kept to obtain better current distribution over the commutator. If only two brushes are to be kept, we can choose B1 and B2 as above, or B2 and B3, or B3 and B4, or B4 and BI; that is, one brush from each group.

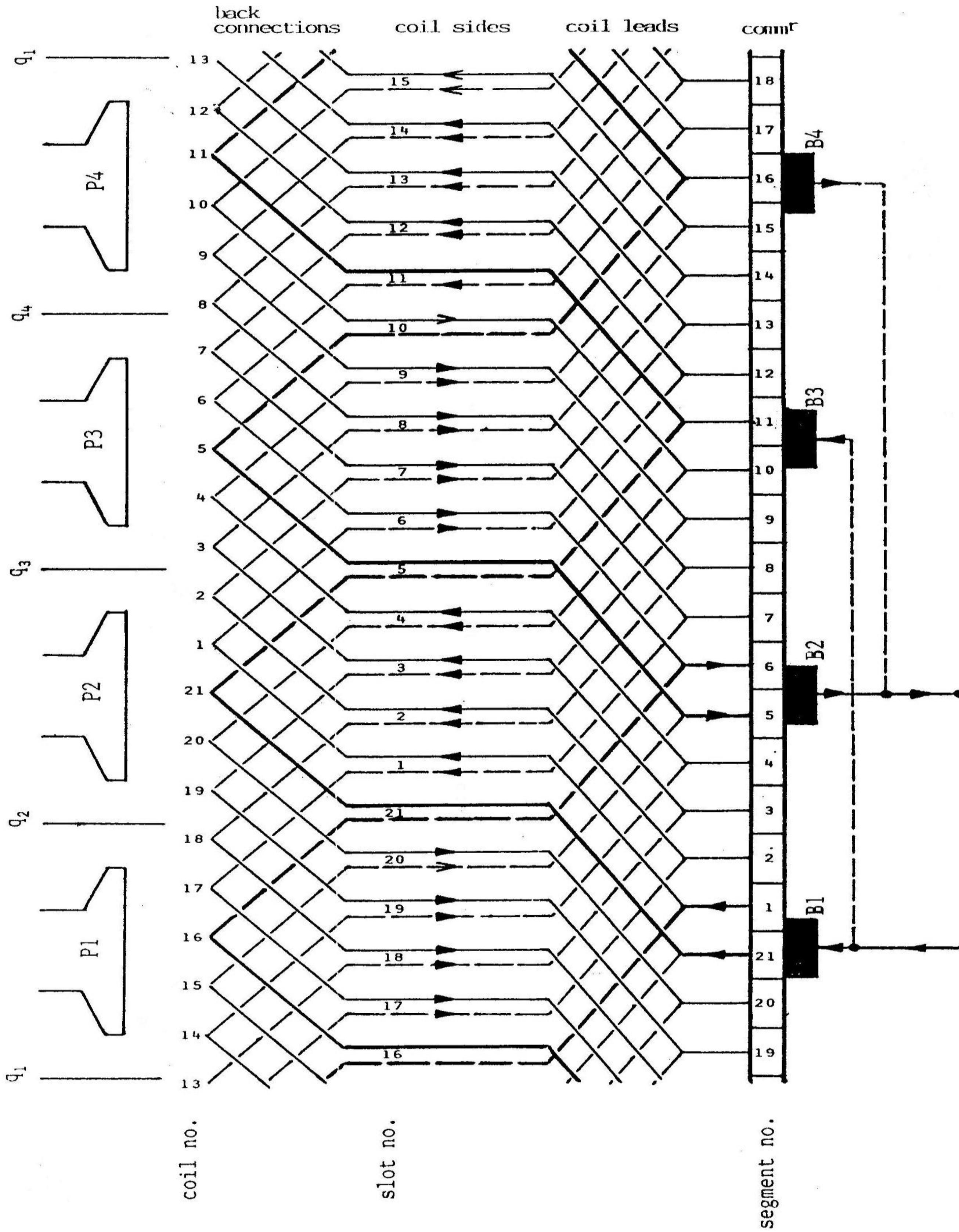


Fig.3.11 Simple wave winding developed diagram for 4 poles and 21 armature coils

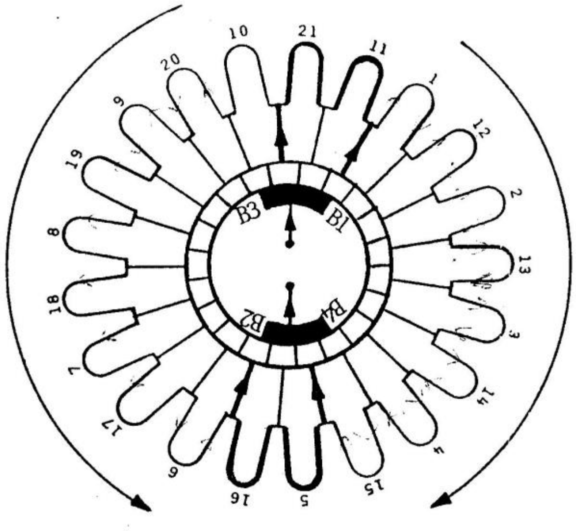


Fig. 3.12 Simple wave winding :sequence diagram corresponding to fig. 3.11

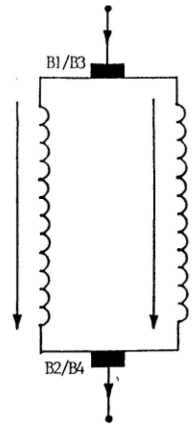


Fig. 3.13 Simple wave winding :parallel paths between brushes

3.3 Armature Calculations

The last section explained how the brushes divide the armature winding into a number of paths, and connect the paths in parallel, figs. 3.9 and 3.13. Each path is composed of a number of coils in series; dividing the total number of armature coils C by the total number of parallel paths 2a, we get

C/2a =average number of coils per path. (3.10)

In chapter 1, section 1.7, it was explained that each coil is represented electrically by its average emf in series with its resistance, fig. 1.28.

Therefore, the electrical circuit of the armature is as shown in fig.3.14a. Summing the coil emfs and resistances in series for each path, the circuit maybe redrawn as in fig. 3.14b. Finally, Thevenin's theorem may be applied to give the simple equivalent circuit of the armature winding shown in fig. 3.14c, where

E_A = equivalent average emf of the armature winding in volts,

And

R_A = equivalent resistance of the armature winding in ohms.

The emf induced in the path is the sum of the series coil emfs in the path; similarly, the path resistance is the sum of the series coil resistances in the path, as there is an average of C/2a coils per path, we have

$$E_{path} = \frac{C}{2a} x E_{coil} \quad \text{and} \quad R_{path} = \frac{C}{2a} x R_{coil}$$

The path emf and resistance are the same for all paths, so that Thevenin's theorem gives

$E_A = E_{path}$ and $\frac{1}{R_A} = \sum \frac{1}{R_{path}} = 2ax \frac{1}{R_{path}}$ → $R_A = R_{path} / 2a$

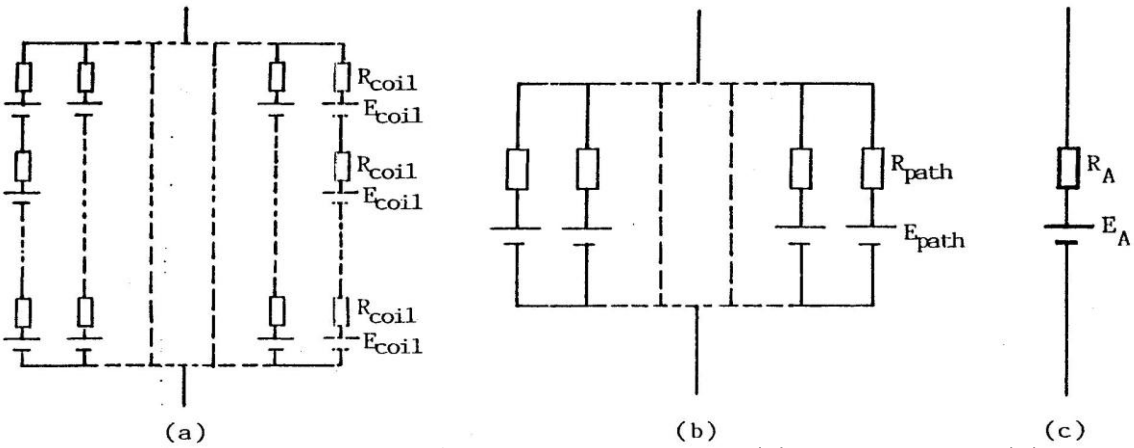


Fig. 3.14 The electrical circuit of the armature winding :(a) detailed circuit; (b) parallel path circuits; (c) Thevenin equivalent.

Using the above resistance equations, together with eqns. 1.29 and 3.1, the armature resistance may be expressed in a number of ways:

$$R_A = \frac{1}{2a} x R_{path} = \frac{1}{2a} \cdot \frac{C}{2a} \cdot R_{coil} = \frac{C}{4a^2} \cdot R_{coil}$$

$$R_A = \frac{C}{4a^2} \cdot NR_{loop} = \frac{NC}{4a^2} \cdot R_{loop} = \frac{Z}{8a^2} \cdot R_{loop} \qquad (3.11)$$

Using, next, the emf equations, together with eqns. 1.13d, 1.15, and 3.1, the armature emf may be expressed in a number of ways:

$$E_A = E_{path} = \frac{C}{2a} x E_{coil}$$

$$= \frac{C}{2a} 4pNn\Phi = \frac{2p\,.2NC}{2a} n\Phi = \frac{pZ}{a} n\Phi = k_e n\Phi$$

$$E_A = \frac{pZ}{a} \cdot \frac{\omega}{2\pi} \Phi = \frac{pZ}{2\pi a} \omega\Phi = k\omega\Phi \qquad (3.12)$$

where k_e=pZ/a and k=pZ/2πa =ke/2π. As already discussed in section 1.6.1, the emf expressions always have a factor representing speed and a factor representing the magnetic field, multiplied by a constant :the emf is induced by the interaction between motion and field.

The voltage between the terminals of the machine and the induced emf in the armature are equal when there is no current flowing in the armature winding; this happens, for example, when the machine operates as an unloaded generator.

However, when current does flow in the armature winding, it will produce a voltage drop in the winding resistance, resulting in a difference between the armature emf and terminal voltage. Current flow through the brushes produces an additional drop composed of two parts. The first is the drop-in brush and commutator segment resistances; this is usually neglected. The second is a potential difference appearing between the brush and commutator surfaces, which are in sliding contact; this is called the 'brush contact drop', and is approximately constant at around (1-2) volts. Now, let

V_A= voltage between machine terminals (armature terminal voltage) in volts,

I_A= total current in the armature winding (armature current) in amperes,

And V_b =brush contact drop in volts.

With the machine operating as a generator, application of Kirchhoff's voltage law to the equivalent circuit as shown in fig. 3.15a gives

$$V_A = E_A - (I_A R_A + V_b) \qquad (3.13)$$

with the machine operating as a motor, fig. 3. 15b, we get

$$V_A = E_A + (I_A R_A + V_b) \qquad (3.14)$$

These are the voltage equations of the dc machine. Under normal operation, the values of induced emf E_A and terminal voltage V_A are close to each other; the difference between them is the voltage drop ($I_A R_A$+ V_b), which is usually small compared to E_A and V_A The usual circuit symbol for the armature is shown in fig.3.16.

The total armature current at the brushes I_A is the sum of the path currents; as the 2a parallel paths are identical, they carry identical currents, so that $I_A = 2a \times I_{path}$. Moreover, the path current is simply the coil current

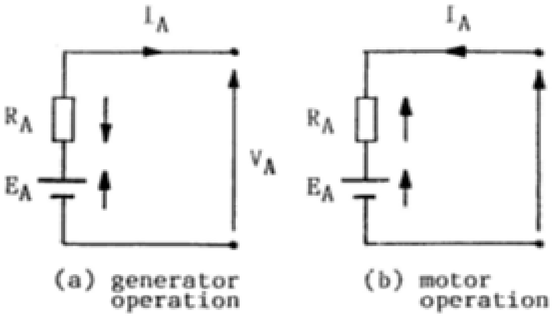


Fig. 3.15 Armature equivalent circuit

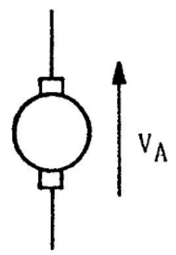


Fig. 3.16 Armature circuit symbol

because the path is just a number of coils in series, fig. 3.14.

Thus

$$I_{coil} = I_{path} = I_A / 2a \quad (3.15)$$

Now, let

T_d = total torque developed electromagnetically by the armature winding (armature developed torque) in newton-meters.

T_d is the sum of the torques developed by all armature coils because the armature winding is arranged so that all coil torques are in the same direction, fig. 1.23, aiding each other. Noting that there is a total of C coils in the armature, and using eqn. 1.24b for the average coil torque, as well as eqns. 3.1 and 3.15, we obtain the following expressions

$$T_d = CXT_{coil} = C\frac{2p}{\pi} \cdot NI_{coil}\Phi = \frac{2pNC}{\pi} \cdot \frac{I_A}{2a} \cdot \Phi$$

$$\therefore \; T_d = \frac{pNC}{\pi a} \cdot I_A\Phi = \frac{Zp}{2\pi a} \cdot I_A\Phi = kI_A\Phi \quad (3.16)$$

where k =(Zp/2πa) as before. As discussed in section 1.6.2, the expression for the developed torque has the current as one factor, and the magnetic field as another, multiplied by a constant the torque is developed by the inter action between current and field.

The power in the emf element of the equivalent circuit is E_AI_A. Using eqns. 3.12 and 3.16, we have

$$E_AI_A = (k\,\omega\,\Phi) \cdot I_A = \omega \cdot (k\,I_A\Phi) = \omega \cdot T_d \quad (3.17)$$

This power, of course, is the conversion power P_c for the whole armature winding; it is given by E_AI_A on the electrical side, and by ωT_d on the mechanical side.

$$P_c = E_A \cdot I_A = \omega \cdot T_d \quad (3.18)$$

The total conversion power in the armature winding is the sum of the conversion powers in the coils; this can be verified using eqns. 3.12 and 3.15:

$$E_AI_A = \left(\frac{C}{2a} \cdot E_{coil}\right)(2a \cdot I_{coil}) = CE_{coil}I_{coil} \quad (3.19)$$

Conversion power and the process of electromechanical power conversion are discussed in section 1.7. The same discussion applies to the armature winding as a whole with, of course, coil quantities (emf, current, resistance, and torque) replaced by the corresponding armature quantities. Multiplying eqn. 3.13 throughout by I_A, we get, for generator operation,

$$V_A I_A = E_A I_A - (I^2_A R_A + I_A V_b) \qquad (3.20)$$

The input power to the electrical circuit is $P_c = E_A I_A$; part of it is lost as heat in the armature winding, $I^2_A R_A$, and in the brush contact, $I_A V_b$, and the rest, $V_A I_A$, is the useful electrical output power obtained from the generator; see also figs. 3.15a and 1.29a. Multiplying now eqn. 3.14 by I_A we get for motor operation

$$V_A I_A = E_A I_A + (I^2_A R_A + I_A V_b) \qquad (3.21)$$

The input power to the electrical circuit is now $V_A I_A$; part of it is lost as heat in the armature winding, $I^2_A R_A$, and in the brush contact, $I_A V_b$, and the rest is the conversion power, $P_c = E_A I_A$, which will appear as, ωT_d, on the mechanical side; see also figs. 3.15b and 1.30a.

3.4 Comparison of Winding Schemes

The equations derived in the previous section apply to both lap and wave windings. The difference between the two types of winding appears through the value of the factor a in the equations :for lap windings, the number of parallel paths 2a is equal to the number of poles 2p, eqn. 3.6; for wave windings, the number of paths 2a is always 2, eqn. 3.9.

First, we note that for a machine having only two poles, there is no difference between lap and wave :in both cases the number of parallel paths is two. In fact, it is meaningless to talk of lap and wave windings when the number of poles is two; there is just one type of winding. Section 3.2.2 explained that in wave winding, each coil is connected to a coil that lies two pole pitches after it on the armature; but if the machine :has only two poles, then moving two pole pitches takes us back to the coil we started from, so that it is connected to the coil next to it, which is precisely the definition of lap winding in section 3.2.1. Also, the commutator pitch for wave is given in eqn. 3.8; for a 2-pole machine, $2p = 2$ and $p = 1$, so that $Y_c = C \pm 1$. Starting from any commutator segment, and moving C segments, we come back to the segment we started from; the ±1 then takes us to the segment just after it, or just before it, which, again, is the commutator pitch for lap.

Clearly, then, it is meaningless to compare lap and wave windings if the number of poles is only two, since in this case they are the same. Therefore, in the following comparisons between lap and wave windings, it is assumed that the number of poles is greater than two, i.e $2p > 2$, and $p > 1$.

Let us suppose that a manufacturer of electrical machines wants to design dc machine having a certain rated voltage and a certain rated current; that is, the values of the terminal voltage V_A and the armature current I_A are given.

The machine can be designed with a lap winding or with a wave winding, and we wish to see the effect of this choice on the design of the individual armature coils. Before starting the analysis, it is recalled from the discussion following eqns. 3.13 and 3.14 that the difference between the terminal voltage V_A and the armature emf E_A is a small voltage drop; neglecting this drop as a first approximation, we can say that the value of E_A is approximately known since the value of V_A is given. Our problem may now be stated as follows:

given the values of the armature emf and current, E_A and I_A, find the values of the coil emf and current, E_{coil} and I_{coil} for the case of lap and the case of wave, and compare the results. From eqn. 3.12, we have:

$$E_{coil} = 2a \cdot E_A / C$$

this gives

$E_{coil,\ lap} = 2p \cdot E_A/C$ and $E_{coil,\ wave} = 2E_A/C$;

thus

$$E_{coil,\ lap} = P \times E_{coil,\ wave}. \quad (3.22)$$

Using, next, eqn. 3.15, we have

$I_{coil,\ lap} = I_A/2p$ and $I_{coil,\ wave} = I_A/2$;

thus

$$I_{coil,lap} = \frac{1}{p} x I_{coil,wave} \quad (3.23)$$

Eqns. 3.22 and 3.23 tell us that the lap coils must be designed for higher emf and lower current compared with the wave coils; conversely, the wave coils must be designed for lower emf and higher, current compared to the lap coils. Assuming the two windings have the same number of coils C, the lap winding will have fewer coils in series per path because it has more paths than the wave winding; therefore, each lap coil must have a greater emf to add up to the required armature emf E_A. Moreover, the total armature current I_A is divided over a larger number of paths in the case of lap, so that each path will have a smaller current than the wave paths, which are only two.

The above comparison indicates that in the case of lap winding, each coil will have more turns (higher emf)of thinner wire (lower current); conversely, in the case of wave winding, each coil will have fewer turns (lower emf)of thicker wire (higher current), and this is the main advantage of wave windings overlap windings :with fewer conductors of larger cross-section, a relatively smaller part of the slot cross-section is wasted on insulation; that is, a larger proportion of the slot cross-sectional area is filled with useful copper. The decreased amount of insulation also means that wave-wound coil span be cooled more easily than lap-wound coils. In practice, wave windings are used in all machines rated at high voltage; they are also used in almost all machines of power rating up to 50 KW. Lap windings are used mostly in large machines rated at low voltage and high current.

Next, let us perform an inverse comparison suppose we have an armature whose construction has been completed except for the soldering of coil terminals to commutator segments, and this can be done either in lap or in wave. In this comparison, the coil emf and current are the same for both cases, and we wish to see the effect of the winding scheme on the overall armature emf and current.

Using eqns. 3.12 and 3.15, we can write

$$E_{A,lap} = \frac{C}{2p} E_{coil} \,, \quad E_{A,wave} = \frac{C}{2} E_{coil} \;\rightarrow E_{A,lap} = \frac{1}{p} E_{A,wave}$$

and

$$I_{A,\,lap} = 2pI_{coil}, \qquad I_{A,\,wave} = 2I_{coil}, \rightarrow \quad I_{A,\,lap} = p\, I_{A,\,wave}$$

These relationships indicate that machines with lap windings tend to have low voltage and high current at the terminals. The overall conversion power is the same for both lap and wove wound machines in this comparison.

A lap winding requires 2p brushes, whereas a wave winding requires only two brushes, but can have up to 2p brushes. This is a minor advantage of wave windings :brushes must be accessible for maintenance, and it is easier to arrange for access to 2 brushes on one side of the commutator than for 2pbrushes placed all around the commutator. It is noted in passing that in large machines of high current, the commutator segments can be quite long, and each brush is replaced by a group of brushes, with independent springs, for better contact along the length of the segments.

The magnetic circuit of the dc machine should be symmetric, with the length of the air gap being exactly the same all around the armature. In practice, however, there might be slight variations in air gap length due to imprecision in manufacture or wear of bearings. As a result, the flux density, and hence the induced emf, under one of the poles may be different. This is no problem for wave windings because the coils of each path are distributed all around the armature, so that both paths are equally affected by the fault in the magnetic circuit. In lap windings, however, the fault affects only some of the paths because each path is located under a specific pair of poles; for example, the coils of the upper right path of fig. 3.8 are located under the poles P2 and P3 in fig. 3.7. As a result, the path emf will not be exactly the same for all paths. Unequal emfs in parallel produce circulating current, that is, current that flows from one path to the other without appearing at the machine terminals. Circulating currents heat the coils unnecessarily, and the heating can be quite heavy because the current is limited only by path resistances which are quite small. To reduce this effect, large lap-wound armatures are sometimes equipped with equalizers, which are connections between points in the winding which are two pole pitches apart, that is, points that should beat the same potential; the equalizing connection forces the points to be at the same potential, and hence forces the path emfs to be approximately equal.

Equalizers add to the cost of the machine, and it is an advantage of wave windings not to require them.

The simplicity of the lap winding means that it can be made to fit any number of coils and poles :each coil is connected to the one following it on the armature until the last coil is reached, which then closes on the first coil, section 3.2.1. A wave winding, on the other hand, can be fitted only if the number of coils C and the number of pole pairs p yield an integer value for the commutator pitch in eqn. 3.8. This is no problem at the design stage because suitable values of C and p can always be found. However, it sometimes happens that a manufacturer needs to modify an armature that has already be unconstructed, and the armature is not suitable for the required wave winding.

For example, suppose that the 22-coil winding of fig. 3.7 is to be reconnected in wave; as we saw in section 3.2.2, this cannot be done because the machine has four poles. However, it is possible to form a proper wave winding using only 21 of the coils, with the remaining coil left unconnected. Such an unconnected coil is called a dummy coil; its terminals are not soldered to any commutator segments. Although dummy coils are useless electrically, they are not removed from the armature to maintain mechanical balance.

Example: The following data is given for the armature of a dc machine :conductors/slot/layer =6; commutator segments =146; Coil sides/slot =4; pole arc/pole pitch =0.65; diameter =28 cm; Length =55 cm ; coil span =12 slots ; flux per pole =70 mWb.

A) Find the total number of armature conductors.

B) Find the commutator pitch when the machine is connected in simple wave, and state whether progressive or retrogressive.
C) Find the pole pitch in meters, mechanical degrees, and electrical degrees

Find the average air gap flux density

Solution

Commutator segment=c=146 coils

Coil sides/slot =4=2C/s , so s=2C/4; s=2*146/4=73 slots

conductors/slot/layer =6=NC/S , N=S*6/C =73*6*146=3 turns

A) Z=2NC=2*3*146=876 conductors.
B) Coil span=12= S/2p , 2p=S/12, 2p=73/12 = 6.08 =6 poles

$y_c = \frac{c \pm 1}{p}$ wave, $y_c = \frac{146 \pm 1}{3} = 49$ progressive.

C) Pole pitch=лD/2p =3.14*28/6 =14.65cm

In mechanical degrees= 360/2p =360/6=60 degree

In electrical degrees = 180 degree

In slots =S/2p = 73/6= 12.16 slot.

D) $A_p = \frac{\pi DL}{2p} = \frac{3.14*28*55*10^{-4}}{6} = 805.93 * 10^{-4} m^2$

$B = \frac{\Phi}{A_p} = \frac{70}{805.93*10^{-4}} = 868$ mWb

Example: The table below lists some data on the armature windings of 8 different dc machines. Use the given information to complete the table.

MACHIN NO	3
Number of poles	
Number of slots	12
Number of coils	
Number of conductors	816
Turns /coil	17
Coil sides /slot	
Conductors /slot	
Conductors /slot/layer	
Coil span (in slots)	3
Pole pitch (in slots)	

2p=S/ coil span

2p=12/3=4 poles

C=Z/2N=816/2*17=24 coils

Coil sides /slot=2C/S=2*24/12=4

Conductors/ slot =Z/S =816/12 = 68 cond.

Conductors /slot /layer =NC/S = 17*24/12 =34

Pole pitch (in slots) =S/2p=12/4=3 slots

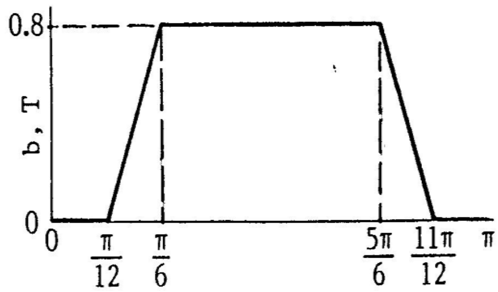
Example: An 8-pole dc generator has 156 slots and 312 commutator segments. The armature coils are connected in simple lap, with each coil made up of 4 turns. The armature rotates at 670 rpm; its length and diameter are 40 cm and 30 cm respectively.

A) How many brushes does the machine have?
B) What is the time for one revolution of the armature?
C) What is the number of conductors per slot per layer?
D) What is the pole pitch? given your answer in meters, electrical degrees, mechanical degrees, and slots.
E) What is the coil span? is it short___, full ___, or long pitched?
F) What will the commutator pitch be if the machine is reconnected in simple wave?

Solution:

A) Number of brushes=2p=8
B) $\frac{t}{1} = \frac{60}{670}$, t=0.0895 sec
C) conductors per slot per layer=NC/S=4*312/156=8
D) pole pitch=л*D/2p=3.14*30/8=11.775 cm
mech. Deg.=360/2p=360/8=45^o
elect.deg.=180^o
in slot =S/2p=156/8=19.5 slots
E) 19 short, long 20 slots
F) $y_c = \frac{c\pm1}{p} = \frac{312\pm1}{4}$=78.25 or 77.75 so cannot wave.

Example: the flux density distribution over one pole pitch of the machine of Question 8 is as shown in the adjacent figure. (a) estimate the pole arc to pole pitch ratio and the width of pole face. (b)plot the flux density distribution over two consecutive poles; indicate all axes. (c)Also find the average air gap flux density and the flux per pole.



Solution:

(a) pole arc to pole pitch ratio $\frac{y_a}{y_p} = \frac{\frac{5\pi}{6} - \frac{\pi}{6}}{\pi} = 0.667$

width of pole face $y_a = \frac{y_a}{y_p}\, y_p = 0.667 * \frac{\pi D}{2p} = 0.667 * \frac{3.14*30}{8} = 7.85\, cm$

(b) plot

(c) $B_{av} = \frac{1}{2\pi}\int_0^{\pi} b d\theta = \frac{1}{\frac{\pi D}{2p}}\int_0^{\frac{\pi D}{2p}} b dx = \frac{1}{y_{p=\pi}} 0.8\left(\frac{4\pi}{6} + \frac{\pi}{12}\right) = 0.6\, T$

$$\Phi = L\int_0^{\frac{\pi D}{2p}} b dx = L\frac{\pi D}{2p} B = A_p x B = \frac{3.14 * 30 * 40 * 10^{-4}}{8} 0.6 = 28.26\, mWb$$