Properties of materials

Stress/strain relationships: the constitutive equation:

If we take a rod of some material and subject it to a load along its axis we expect that it will change in length.

We might draw a load/displacement curve based on experimental data, as shown in figure 1.1.

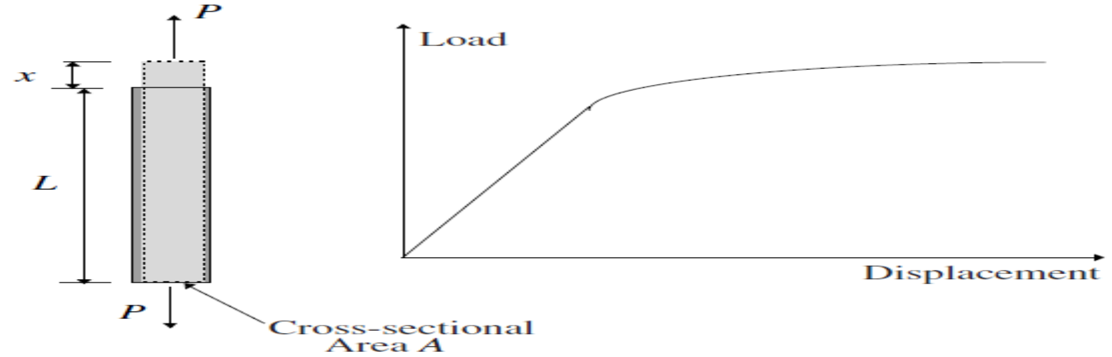

Figure 1.1. *Load/displacement curve: uniaxial tension.*

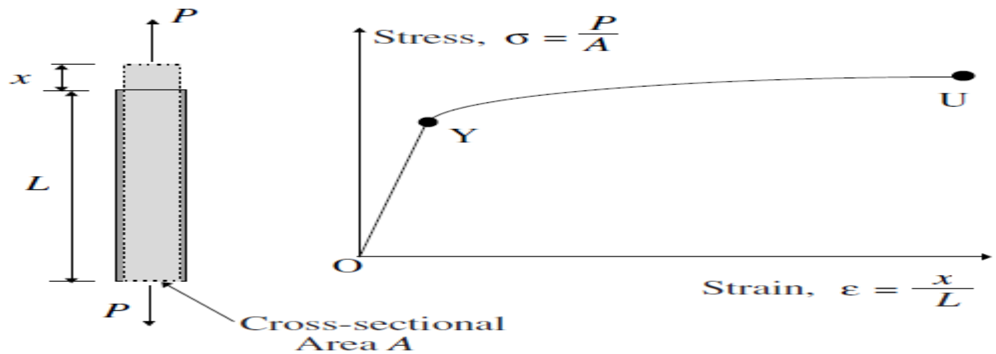

Figure 1.2. *Stress/strain curve: uniaxial tension.*

We might then describe the **displacement in terms of extension per unit length**, which we will call **strain (ε)**, and the load in **terms of load per unit area**, which we will call **stress (σ)**.

We can then redraw the load/displacement curve as a stress/strain curve, and this should be independent of the dimensions of the bar.

The shape of the stress/strain curve illustrated in figure 1.2 is typical of many engineering materials, and particularly of metals and/ alloys. In the context of biomechanics it is also characteristic of bone .

In this region the stress is proportional to the strain. The constant of proportionality, *E*, is called *Young's modulus*,

$$\sigma = E\varepsilon$$

The linearity of the equivalent portion of the load/displacement curve is known as Hooke's law.

According **to Hook`s law** : For many materials a bar loaded to any point on the portion OY of the stress/strain curve and then unloaded will return to its original unstressed length. It will follow the same line during unloading as it did during loading. This property of the material is ***known as elasticity***.

In this context it is not necessary for the curve to be linear: the important characteristic is the similarity of the loading and unloading processes. A material that exhibits this property and has a straight portion OY is referred to as *linear elastic* in this region. All other combinations of linear/nonlinear and elastic/inelastic are possible. The linear relationship between stress and strain holds only up to the point Y.

After this point the relationship is nonlinear, and often the slope of the curve drops off very quickly after this point. This means that the material starts to feel ‘soft’, and extends a great deal for little extra load. *Typically the point Y represents a critical stress in the material.* After this point the unloading curve will no longer be the same as the loading curve, and upon unloading from a point beyond Y the material will be seen to exhibit a permanent distortion. For this reason Y is often referred to as the *yield point* (and the stress there as the yield stress), although in principle there is no fundamental reason why the limit of proportionality should coincide with the limit of elasticity. The portion of the curve beyond the yield point is referred to as the *plastic region*.

The bar finally fractures at the point U. The stress there is referred to as the (uniaxial) *ultimate tensile stress* (UTS).

Materials like rubber, when stretched to high strains, tend to follow very different loading and unloading curves. A typical example of a uniaxial test of a rubber specimen is illustrated in figure 1.3. This phenomenon is known as *hysteresis*, and the area between the loading and unloading curves is a measure of the energy lost during the process. Over a period of time the rubber tends to creep back to its original length, but the capacity of the system as a shock absorber is apparent.

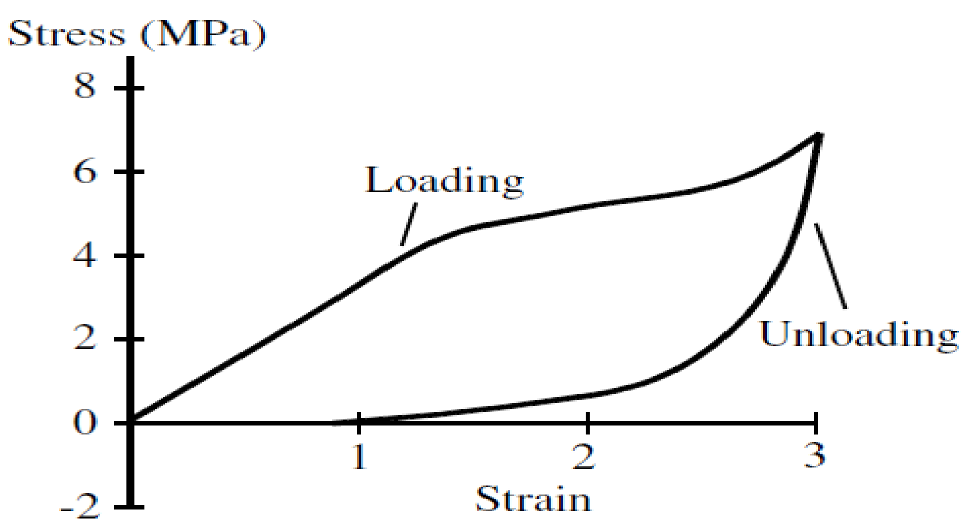

Figure 1.3. *Typical experimental uniaxial stress/strain curve for rubber.*

In fact there are many questions that remain unanswered by a test of this type. These fall primarily into three categories: one associated with the nature and orientation of loads; one associated with time; and one associated with our definitions of stress and strain.

Bone

Bone is a composite material, containing both **organic and inorganic components**. The organic components,about one-third of the bone mass, include the cells, osteoblasts, osteocytes and osteoid. The inorganic components are hydroxyapatites (mineral salts), primarily calcium phosphates.

•The osteoid contains collagen, a fibrous protein found in all connective tissues. It is a low elastic modulus material **($E \approx 1.2$ GPa)** that serves as a matrix and carrier for the harder and stiffer mineral material.The collagen provides much of the tensile strength (but not stiffness) of the bone. **Deproteinized bone** is hard, brittle and weak in tension, like a piece of chalk.

•The mineral salts give the bone its hardness and its compressive stiffness and strength. The stiffness of the salt crystals is about 165 GPa, approaching that of steel. Demineralized bone is soft, rubbery and ductile .

The skeleton is composed of *cortical* (*compact*) and *cancellous* (*spongy*) bone, the distinction being made based on the porosity or density of the bone material. The division is arbitrary, but is often taken to be around 30% porosity (see figure 1.4).

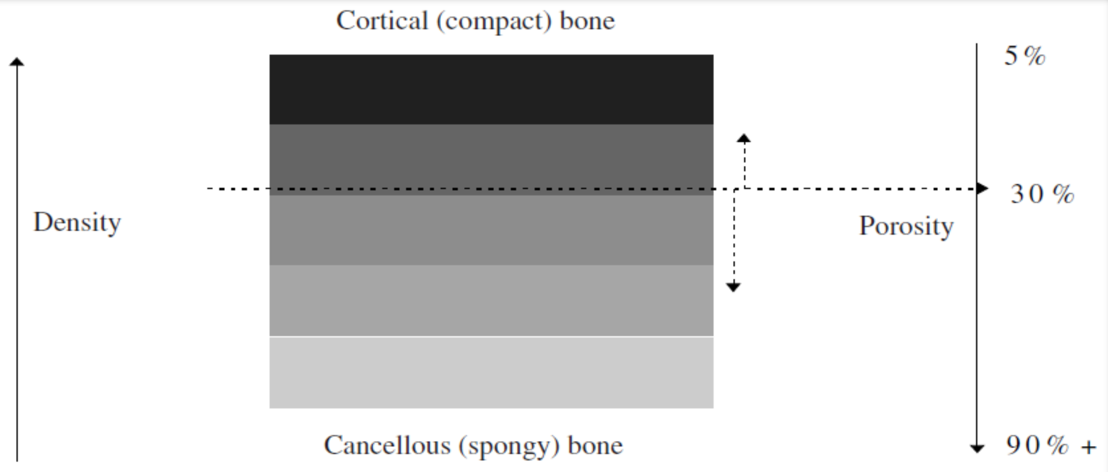

Figure 1.4. *Density and porosity of bone.*

Cortical bone is found where the stresses are high and ***cancellous bone*** where the stresses are lower (because the loads are more distributed), but high distributed stiffness is required. The aircraft designer uses honeycomb cores in situations that are similar to those where cancellous bone is found.

Dry bone is typically slightly stiffer (higher Young's modulus) but more brittle (lower strain to failure) than **wet bone**. A typical uniaxial tensile test result for a wet human femur is illustrated in figure 1.5.

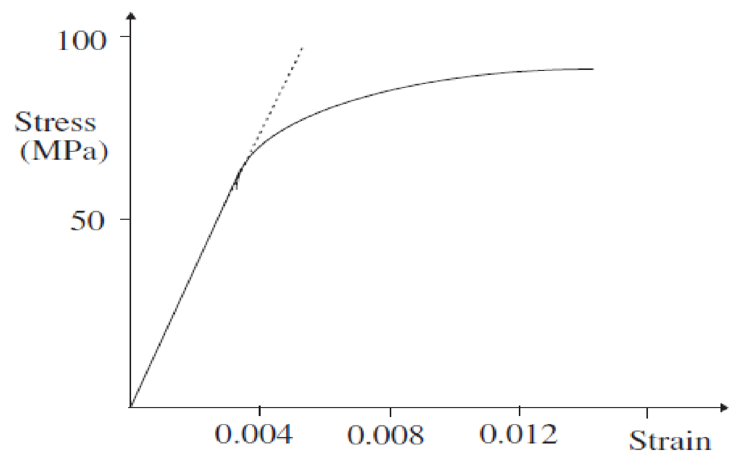

Figure 1.5. *Uniaxial stress/strain curve for cortical bone.*

Table 1.1. *Mechanical properties of bone (values quoted by Fung (1993)).*

	Tension			Compression			Shear		Poisson's ratio
Bone	σ (MPa)	ε (%)	E (GPa)	σ (MPa)	ε (%)	E (GPa)	σ (MPa)	ε (%)	ν
Femur	124	1.41	17.6	170	1.85		54	3.2	0.4

Tissue

Tissue is the fabric of the human body. There are four basic types of tissue, and each has many subtypes and variations. The four types are:

- epithelial (*covering*) tissue;
- connective (*support*) tissue;
- muscle (*movement*) tissue;
- nervous (*control*) tissue.

In this chapter we will be concerned primarily with connective tissues such as tendons and ligaments. Tendons are usually arranged as ropes or sheets of dense connective tissue, and serve to connect muscles to bones or to other muscles. Ligaments serve a similar purpose, but attach bone to bone at joints. In the context of this chapter we are using the term tissue to describe soft tissue in particular.

Viscoelasticity

The tissue model considered in the previous section is based on the assumption that the stress/strain curve is independent of the rate of loading. Although this is true over a wide range of loading for some tissue types, including the skeletal muscles of the heart, it is not true for others. When the stresses and strains are dependent upon time, and upon rate of loading, the material is described as *viscoelastic*.

The models that we shall consider are all based on the assumption that a rod of viscoelastic material behaves as a set of linear springs and viscous dampers in some combination.

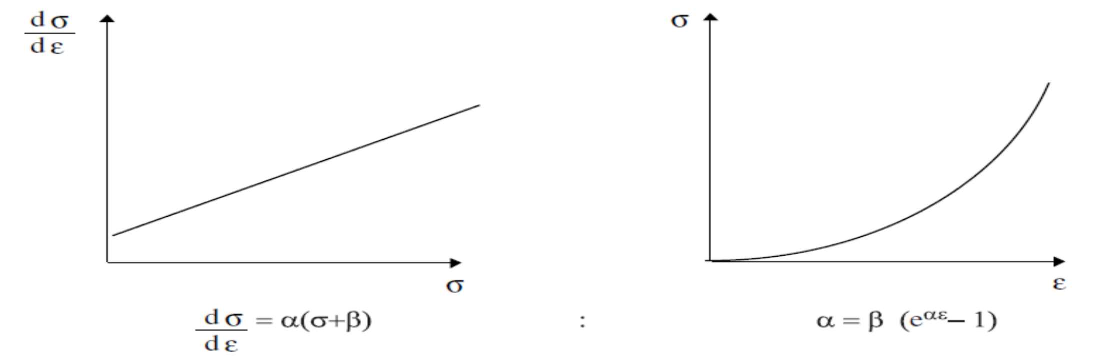

Figure 1.6. *Typical stress/strain curves for some tissues.*

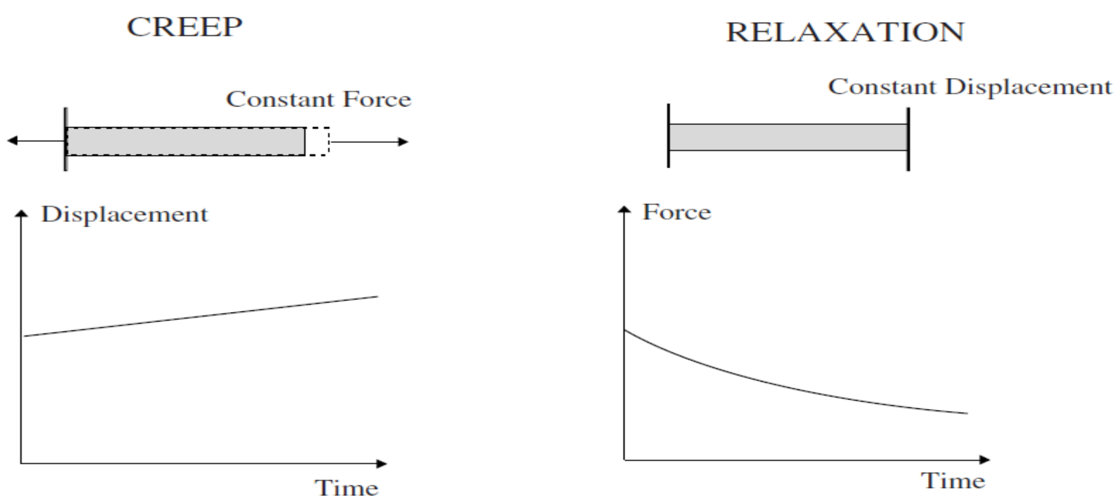

Figure 1.7. *Typical creep and relaxation curves.*