Lecture Six
Floating Bodies.

1- Buoyancy and Stability of Floating Body.
1.1 Buoyancy Force.

The principle of Archimedes is states that, any floating or immersed body in a fluid experiences a vertical bouyant force equal to the weight of the fluid it displaces. The derivation of above principle as follows,

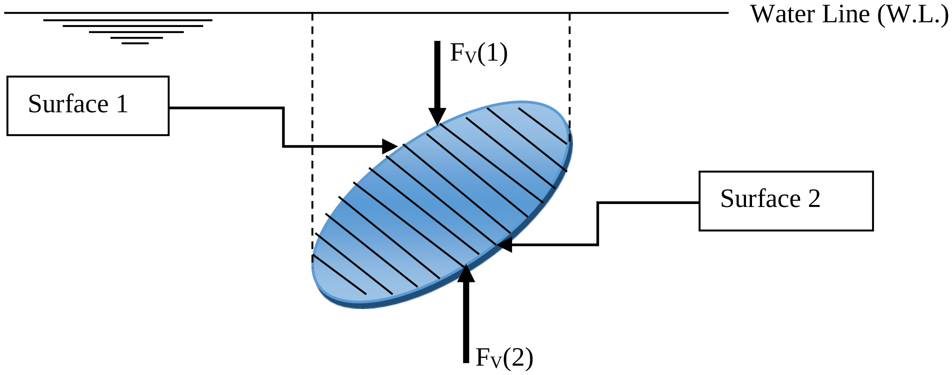


Figure 1: Forces on upper and lower curved surface

From Fig. 1 the body lies between an upper curved surface (1) and lower surface (2),

- $F_V(1)$= The vertical force of the fluid weight above the surface(1).
- $F_V(2)$= The vertical force of the fluid weight above the surface(2).
- F_B = buoyant force.
- $F_B = F_V(2) - F_V(1)$ = weight of fluid equivalent to body volume.

Now, how to find the vertical force on body?, from Fig 2, the sum of vertical forces on elemental vertical slices of immersed body, that can be derived as follows,

$$F_B = \int_{body} (p_2 - p_1) dA_H$$

$$F_B = \gamma \int_{body} (z_2 - z_1) dA_H = \gamma \forall_{body} \qquad (1)$$

F_B acts at the point is called the center of buoyancy.

Since, $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ are the pressure due to weight of fluid on upper and lower horizontal surface of elemental area

$\forall_{body}$ is the body volume.

$\boldsymbol{p = \gamma z}$.

$\boldsymbol{z_1}$ and $\boldsymbol{z_2}$ are the distances from water line to upper and lower horizontal surface of elemental area.

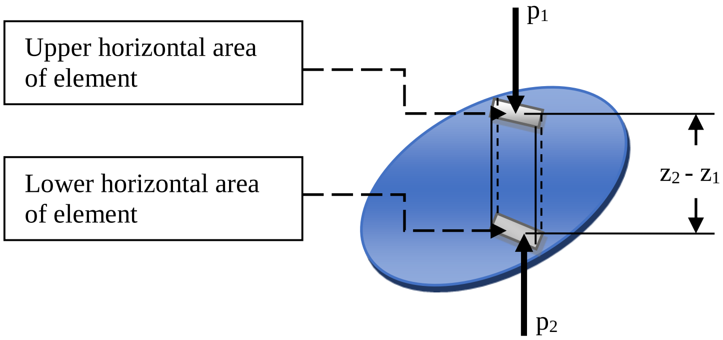


Figure 2: Pressures on upper and lower horizontal surface of elemental area

Ex.1

A body is weight ***400N*** in air and its weight ***222N*** in water. Calculate its volume.

Sol.

The summation of forces is

$F_B + T - W = 0$; where T is the tension in cable.

$\therefore F_B = W - T = 400 - 222 = \mathbf{178N}$ weight of displaced fluid.

$F_B = \gamma \times \forall = 9810 \times \forall = 178$ N

$\therefore \forall = \mathbf{0.018\ m^3}$

Ex.2

A spar buoy is a rod weighted to float vertically as in figure. Let the buoy be maple wood (***S.G.=0.6***), its dimension are (***2 in × 2 in × 10 ft***), floating in seawater (***S.G.=1.025***) how many pounds of steel (***S.G.=7.85***) should be added at the bottom so that (***h=18 in***).

Sol.

Let $\forall_{sp.} = wood\ spar\ volume; \forall_{Imm.sp.} = Immersed\ spar\ volume;$

$W_{st.} = steel\ weight;\ \forall_{st.} = steel\ volume;\ W_{sp} = Wood\ spar\ weight$

$\forall_{sp.} = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(10) = 0.273 ft^3$

$W_{st.} = m_{st.} \times g = \rho_{st.} \times \forall_{st.} \times g$

$SG_{st.} = \frac{\rho_{st}}{\rho_w} \quad \rightarrow \ \rho_{st.} = SG_{st.} \times \rho_w \ \ where\ \rho_w = water\ density$

$\therefore \forall_{st.} = \frac{W_{st.}}{(SG_{st.})(\gamma_w)} = \frac{W_{st}}{(7.85)(62.4)}$ (a)

$\forall_{Imm.Sp.} = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(8.5) = 0.236 ft^3$

From the below figure the buoyant vertical force F_B balances the weights of wood and steel as follows:

$F_B = W_{sp.} + W_{st.} = (\rho \forall g)_{sp} + W_{st.} = \left(SG\gamma_w \forall_{sp.}\right) + (W_{st.})$ (b)

Also, F_B equal to the weight of water displaced by immersed volume

$F_B = W_{Imm.sp.} + W_{st.} = \left(SG \times \gamma_w \times \forall_{Imm.Sp.}\right) + (SG \times \gamma_w \times \forall_{st.})$ (c)

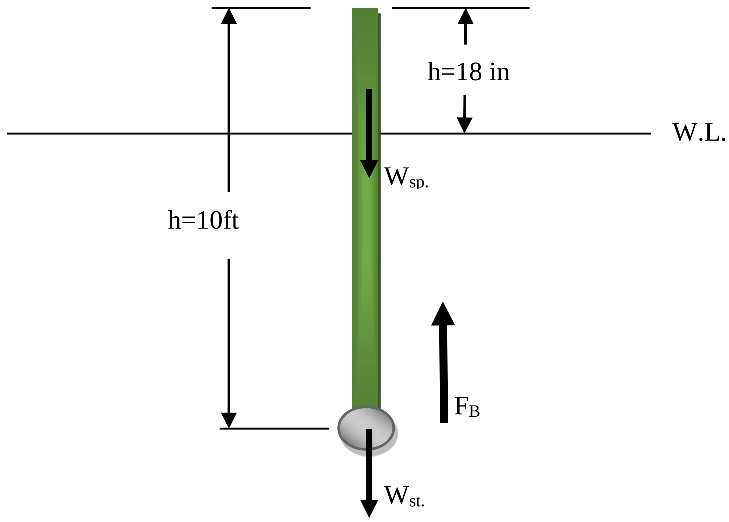
Equating relation ***(b and c)*** and substituting ***Eq. a*** will be given us the following,

$SG \times \gamma_w\left(\forall_{Imm.Sp.} + \forall_{st.}\right) = \left(SG\gamma_w \forall_{sp.}\right) + (W_{st.})$

$(1.025)(62.4)\left[0.236 + \frac{W_{st}}{(7.85)(62.4)}\right] = 0.6 \times 62.4 \times 0.278 + (W_{st.})$

$15.09 + 0.1306 W_{st.} = 10.4 + W_{st.}$ Solving for $W_{st.}$

$\therefore W_{st.} = 5.4\ lb_f$



*1.2 **Stability of floating and submerged bodies.***

Engineer must design to avoid floating instability; there are three possible situations for a body when immersed in a fluid.

I. If the weight of the body is **greater** than the weight of the liquid of equal volume then the **body will sink into** the liquid (to keep it floating additional upward force is required).
II. If the weight of the body **equals** the weight of equal volume of liquid, then the body will submerge and may **stay at any location** below the surface.
III. If the weight of the body is **less** than the weight of equal volume of liquid, then the body will be partly submerged and **will float in** the liquid.

A ship or a boat should not overturn due to small disturbances but should be stable and return to its original position. Equilibrium of a body exists when there is no resultant force or moment on the body. A body can stay in three states of equilibrium.

i) **Stable equilibrium:** Small disturbances will create a correcting couple and the body will go back to its original position prior to the disturbance.

ii) **Neutral equilibrium:** Small disturbances do not create any additional force and so the body remains in the disturbed position. No further change in position occurs in this case.

iii) **Unstable equilibrium:** A small disturbance creates a couple which acts to increase the disturbance and the body may tilt over completely.

Under equilibrium conditions, two forces of equal magnitude acting along the same line of action, but in the opposite directions exist on a floating/submerged body. These are the gravitational force on the body (weight) acting downward along the centroid of the body and buoyant force acting upward along the centroid of the displaced liquid. Whether floating or submerged, under equilibrium conditions these two forces are equal and opposite and act along the same line.

Fig.3 illustrates the computation for the usual case of a symmetric floating body. The steps are as follows;

1- The basic floating position is calculated from
$F_B = \gamma \forall_{body}$= floating body weight
The body's center of mass at point ***G*** and center of buoyancy ***B*** are computed.

2- After tilted the body at $\Delta\theta$, new position B' of the center of buoyancy, a vertical line drawn upward from B' intersects the line of symmetry at point ***M*****,** called the metacenter**.** The point about which the body starts oscillating, is called metacenter.
3- If ***M*** is above center of mass where point ***G*** as in figure, the metacentric height $\overline{MG}$ is positive, a restoring moment is present and the original is stable as in Fig. 2.16.b. If ***M*** is below ***G***, the height $\overline{MG}$ is negative, the body is unstable and the body will overturn as in Fig. 2.16.c. Stable increase with increasing $\overline{MG}$.

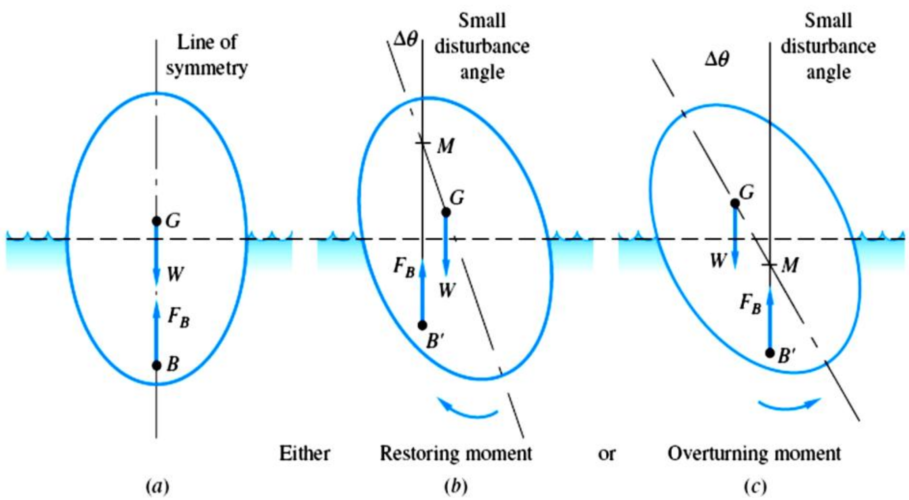
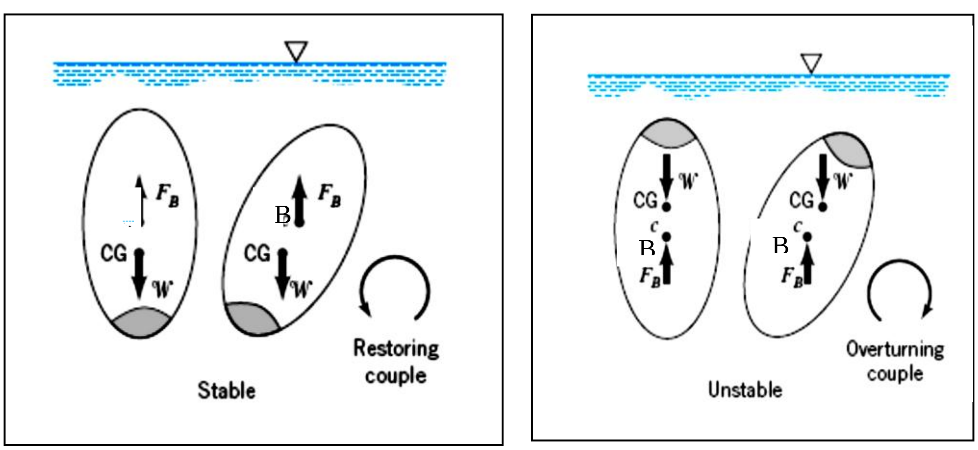


Figure 3: The metacenter ***M*** of the floating body.

Fig.4 below shows the body for completely submerged, which has a center of gravity below the center of buoyancy as in Fig. 4.a. For this configuration the body is stable with respect to small rotation. If the center of gravity is above the center of buoyancy as in Fig.4.b, the resulting couple formed by the weight and the buoyant force will cause the body to overturn and to move to a new equilibrium position. Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.



(a) (b)

Figure 4: Stability of a completely immersed body (a) ***CG*** below ***B***, (b) ***CG*** above ***B***

2- *Stability Related to Waterline Area.*

$\overline{MG} = \frac{I_o}{\forall_{Imm.}} \pm \overline{GB}$; (-) is used if ***G*** above ***B***; (+) is used if ***G*** below ***B***

Where I_o is the area moment of inertia of the waterline footprint of the body about its tilt ***O.*** The computation procedure as follows,

- Firstly determine the distance from ***G*** to ***B***.
- Then make the calculation of I_o, and the submerged volume$\forall_{Imm.}$.
- If metacentric height ***MG*** is positive, the body is stable for small disturbances.
- If $\overline{MG}$ negative then the body is unstable.

Ex.3

Consider a wooden cylinder ***S.G.=0.6***, ***1m*** in diameter and ***0.8m*** long as in Fig.5. Would this cylinder be stable if placed to float with its axis vertical in oil ***S.G.=0.85.***

Sol.

A vertical force balance gives

$F_B = W_{wood}$

$\gamma_{oil}\forall_{Imm.} = \gamma_{wood}\forall_{wood}$

$0.85 \times 1000 \times 9.81 \times \pi R^2 h = 0.6 \times 1000 \times 9.81 \times \pi R^2 \times 0.8$

$0.85 \times \pi R^2 h = 0.6 \times \pi R^2 \times 0.8 \qquad R = 0.5\ m$

$\therefore h = 0.565\ m$

The point ***B*** is at ***h/2 = 0.282 m*** above the bottom, to predict the metacenter location

$$MB = {I_o}/{\forall_{Imm.}} = \left[{\pi(0.5)^4}/{4}\right]/[\pi \times 0.5^2 \times 0.565] = 0.111m$$

$MB = MG + GB$

Now, GB=0.4-0.282=0.118m from figure.

Hence, MG = 0.111-0.118 = - 0.007m

This float position is thus slightly unstable. The cylinder would turn over.

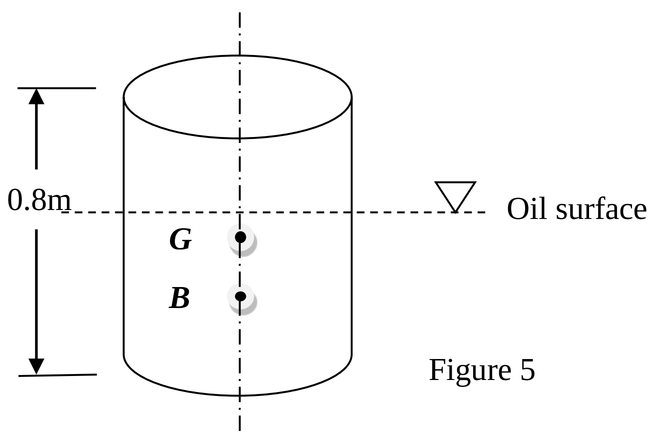


Figure 5