Lecture-Four
Pressure Measurements.

1- Introduction.

Since pressure is a very important characteristic of a fluid field, it is defined as the force acting along the normal direction on unit area. A more precise mathematical definition of pressure as

$$p = \lim_{A \to a} \left(\frac{\Delta F}{\Delta A}\right) = \frac{dF}{dA} \qquad (1)$$

This explicitly means that the pressure is the ratio of the element force to the elemental area (a) normal to it.

The unit of pressure in the SI system is (N/m^2) also called Pascal (Pa). The atmospheric pressure is approximately (10^5 N/m^2) is and designated as "bar". From above definition the pressure at a point within a fluid mass will be designated as either an *absolute* pressure or a *gage* pressure.

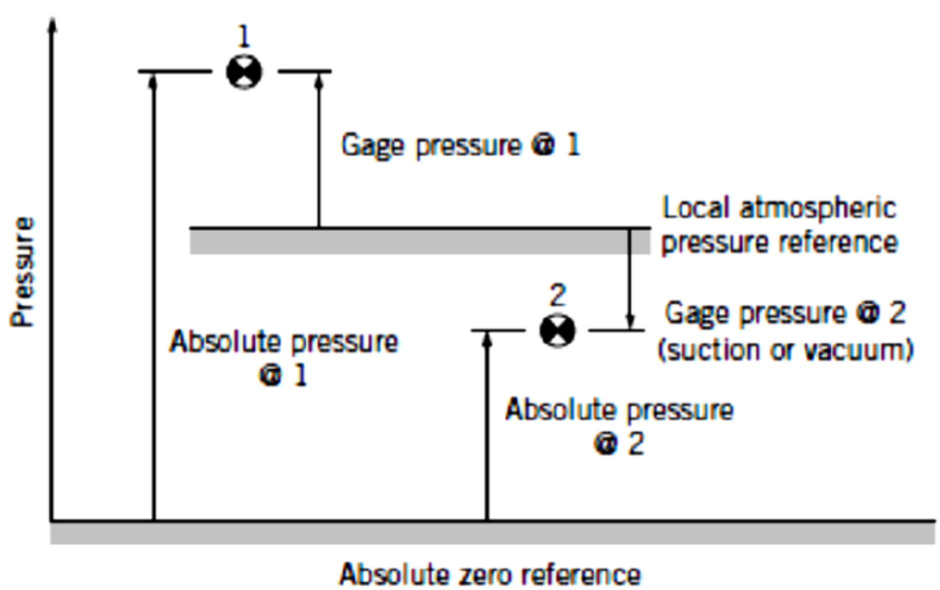

Figure 1 : Graphical representation of gage and absolute pressure.

Absolute pressure is measured relative to a perfect vacuum (*absolute zero pressure*), where as gage pressure is measured relative to the local *atmospheric pressure*. Thus, a gage pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressure can be either positive or negative depending on whether the pressure is above or below atmospheric pressure. A negative gage pressure is also referred to as a *suction* or *vacuum* pressure. The concept of gage and absolute pressure is illustrated graphically in Fig. 2.5 for two typical pressures located at points 1 and 2. Gage pressure is the difference between the value of the pressure and the local atmospheric pressure ($p_{atm.}$)

$$p_{gage} = p - p_{atm.}$$

At sea – level, the international standard atmosphere has been chosen as $p_{atm.}$=101.32 (kN/m

The measurement of atmospheric pressure is usually accomplished with a mercury *barometer*, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in Fig. 2. The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down) with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. Thus,

$$\boldsymbol{p_{atm.} = \gamma h + p_{vapor}} \qquad \text{-------------------------------------(2)}$$

The vapor pressure p_{vapor} can be neglected in most practical cases in comparison to $p_{atm.}$, since its very small for mercury, $p_{vapor}= 0.16*p_{atm.}$. So that,

$$\boldsymbol{p_{atm.} = \gamma h}$$

$$\therefore\ h = \frac{p_{atm.}}{\rho * g} = \frac{1.0132*10^5 \left(N/_{m^2}\right)}{13560\left(kg/_{m^3}\right)*9.81\left(N/_{kg}\right)} = 0.761\ m\ of\ (Hg)$$

If water was used the value of ***h*** will be equal to ***10.32 m***

Ex.1

What will be the (a) the gauge pressure , (b) the absolute pressure of water at depth 12m below the surface ? ρ_w=1000kg/m^3, p_{atm}=101 kN/m^2.

Sol.

(a) $p_{gage} = \rho g h = 1000 * 9.81 * 12 = 117720 \frac{N}{m^2}, (\boldsymbol{Pa})$

(b) $p_{abs.} = p_{gage} + p_{atm.} = (117720 + 101 * 10^3) = 218720 \frac{N}{m^2} = \qquad 218.72 \frac{kN}{m^2}, (kPa)$

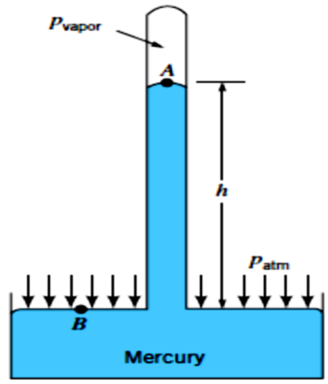

Figure 2: Mercury barometer.

2- *Manometers.*

The manometers are the standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called *manometers*. Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

2.1 Piezometer Tube.

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Fig.3. Since manometers involve columns of fluids at rest, the fundamental equation describing their use is hydrostatic Eq.

$$p = p_0 + \gamma h$$

This gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure p_0 and the vertical distance h between p and p_0. Remember that in a fluid at rest pressure will increase as we move downward and will decrease as we move upward. Application of this equation to the piezometer tube of Fig. 2.7 indicates that the pressure p_A can be determined by a measurement of h through the relationship

$$p_A = \gamma_1 h_1$$

The tube is open at the top, the pressure p_0 can be set equal to zero as using a gage pressure, with the height h_1 measured from the meniscus at the upper surface to point (1) then

$$h_1 = \frac{p_A}{\rho g} \quad (3)$$

ρ is the working fluid density.

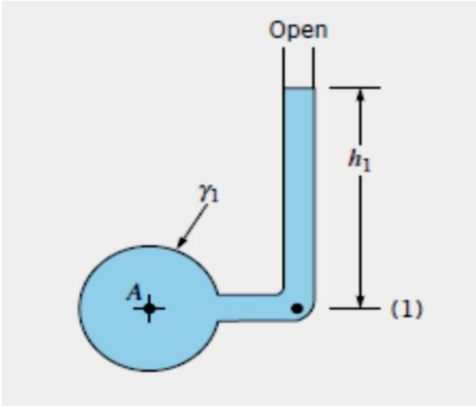

Figure 3 : Piezometer tube.

2.2 U-Tube Manometer.

Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere. Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube. A common type manometer is like a transparent "U-tube" as shown in Fig. 4.

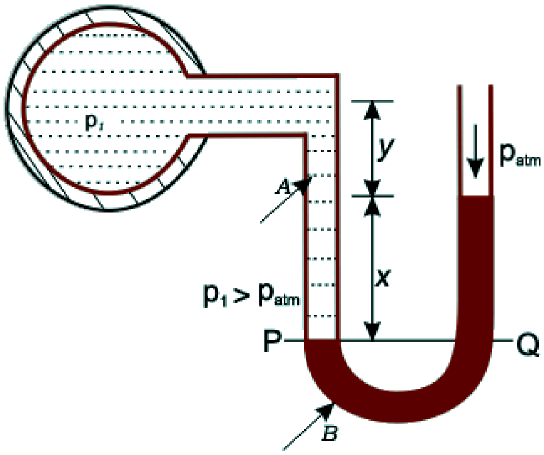

Figure 4: A simple manometer to measure gauge pressure.

One of the ends is connected to a pipe or a container having a fluid (A) whose pressure is to be measured while the other end is open to atmosphere. The lower part of the U-tube contains a liquid immiscible with the fluid A and is of greater density than that of A. This fluid is called the manometric fluid. The pressures at two points ***P*** and ***Q*** in a horizontal plane as shown in Fig. 4 within the continuous expanse of same fluid (the liquid B in this case) must be equal. Then equating the pressures at ***P*** and ***Q*** in terms of the heights of the fluids above those points, with the aid of the fundamental equation of hydrostatics Eq., we have

$p_1 + \rho_A g(y + x) = p_{atm} + \rho_B g x$

Hence $\quad p_1 - p_{atm} = (\rho_B - \rho_A) g x - \rho_A g y \qquad (4)$

Where p_1 is the absolute pressure of the fluid ***A*** in the pipe or container at its centre line, and p_{atm} is the local atmospheric pressure.

2.3 Manometers for Measuring Gauge and Vacuum Pressure.

When the pressure of the fluid in the container is lower than the atmospheric pressure, the liquid levels in the manometer would be adjusted as shown in Fig. 5. Hence it becomes,

$$p_1 + \rho_A g y + \rho_B g x = p_{atm}$$

$$p_{atm} - p_1 = (\rho_A y + \rho_B x) * g \qquad (5)$$

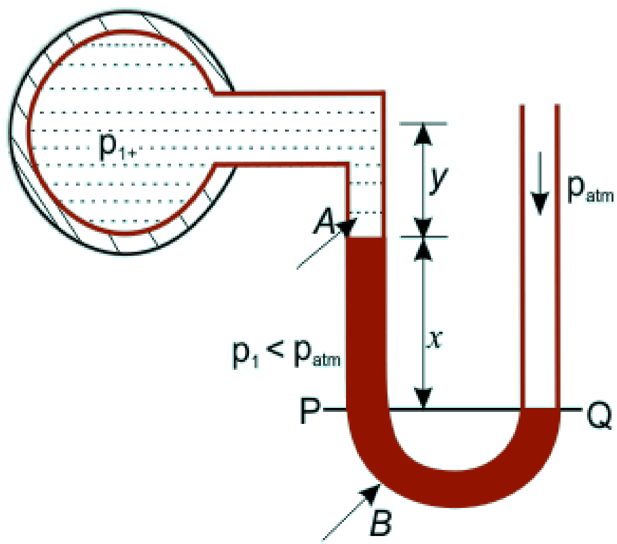

Figure 5: A simple manometer to measure vacuum pressure.

2.4 Manometers to Measure Pressure Difference.

Another type of manometer is also frequently used to measure the pressure difference, in course of flow, across a restriction in a horizontal pipe as shown in Fig. 6. The axis of each connecting tube at A and B should be perpendicular to the direction of flow and also for the edges of the connections to be smooth. Applying the principle of hydrostatics at P and Q we have,

$$p_1 + (y + x)\rho_w g = p_2 + y\rho_w g + \rho_m g x$$
$$p_1 - p_2 = (\rho_m - \rho_w) g x \qquad (6)$$

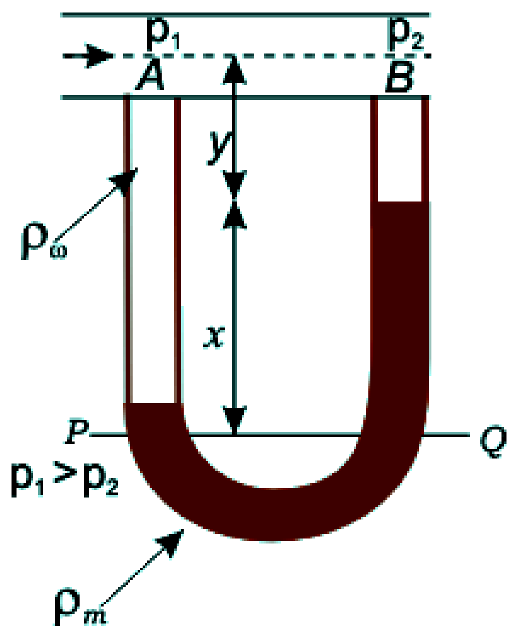

Figure 6: Manometer measuring pressure difference.

Where, ρ_m is the density of manometric fluid and $\boldsymbol{\rho_w}$ is the density of the working fluid flowing through the pipe.

We can express the difference of pressure in terms of the difference of heads (height of the working fluid at equilibrium).

$$h_1 - h_2 = \frac{p_1 - p_2}{\rho_w g} = \left(\frac{\rho_m}{\rho_w} - 1\right) x \qquad (7)$$

Ex.2

A closed tank contains oil and compressed air (***S.G.$_{oil}$ = 0.9***) as is shown in the following figure, a U-tube manometer using mercury (***S.G.$_{Hg}$ = 13.6***) is connected to a tank as shown. For column heights ***h_1*****=914.5** mm, ***h_2*****=152.4** mm and ***h_3*****= 228.6** mm. Determine the pressure reading in ***Pa*** of the gage.

Sol.

The pressure at level (1) is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. The pressure at level (1) is

$$p_1 = p_{air} + \gamma_{oil}(h_1 + h_2)$$

The pressure at level (2) is

$$p_2 = \gamma_{Hg}\, h_3$$

Thus, the manometer equation can be expressed as

$$\therefore \quad p_{air} + \gamma_{oil}(h_1 + h_2) - \gamma_{Hg}\, h_3 = 0$$

Or

$$p_{air} + S.G_{oil}\, \gamma_{H_2o}(h_1 + h_2) - S.G_{Hg}\gamma_{H_2o}\, h_3 = 0$$

$$p_{air} = -0.9 * 1000 * 9.81 * (0.9145 + 0.1524) + 13.6 * 1000 * 9.81 * 0.2286$$

$$= 21079.23\ N/m^2 (Pa)$$ ***Ans.***

This is the pressure read by the gage, since the specific weight of the air above the oil is much smaller than the specific weight of the oil.