Lecture-Three
Fluid in Static

1- Introduction.

Many fluid problems do not involve motion. They concern the pressure distribution in a static fluid and its effect on solid surfaces and on floating and sub- merged bodies. When the fluid velocity is zero, denoted as the ***hydrostatic condition,*** the pressure variation is due only to the weight of the fluid. Assuming a known fluid in a given gravity field, the pressure may easily be calculated by integration. Important applications in this chapter are

I. Pressure distribution in the atmosphere and the oceans
II. The design of manometer pressure instruments.
III. Forces on submerged flat and curved surfaces.
IV. Buoyancy on a submerged body.
V. The behavior of floating bodies and its stability with the result of Archimedes principles.
VI. If the fluid is moving in ***rigid-body motion***, such as a tank of liquid which has been spinning for a long time, the pressure also can be easily calculated, because the fluid is free of shear stress. We apple this idea here to simple rigid –body accelerations.

2- Types of Forces on Fluid Elements.

Fluid Element:- is the infinitesimal region of the fluid continuum in isolation from its surroundings.

Types of forces on fluid elements:-

a) **Body Force**: it’s the force which distributed over the entire mass or volume of the element, as the gravitational force, Electromagnetic force fields.
b) **Surface Force**: is the forces exerted on the fluid element by its surroundings through direct contact at the surface.

Surface force has two components

I. Normal Force: along the normal to the area.

II. Shear Force: along the plane of the area.

When the $\left[\lim_{\delta A \to 0} \frac{\delta F}{\delta A}\right] \longrightarrow$ normal & shear stresses

Shear stress $\to 0$ for any fluid at rest, and hence the only surface force on a fluid element is the normal component.

3- *Pressure Force on a Fluid Element.*

Let the pressure vary arbitrarily $p = p(x,y,z,t)$ consider the pressure acting on the two x-faces as in Fig.1 . The net force in the x-direction on the element is given by

$$dF_x = p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz = -\frac{\partial p}{\partial x}\,dx\,dy\,dz \qquad (1)$$

In like manner the net force dF_y involves $-\frac{\partial p}{\partial y}$, and the net force dF_z concerns $-\frac{\partial p}{\partial z}$ the total net –force vector on the element due to pressure is

$$dF_{press} = \left(-i\frac{\partial p}{\partial x} - j\frac{\partial p}{\partial y} - k\frac{\partial p}{\partial z}\right) dx\,dy\,dz \qquad (2)$$

Rewrite Eq. 2 as the net force per unit element volume and is denoted by (f)

$$f_{press} = -\nabla p \qquad (3)$$

This is the pressure gradient causing a net force which must be balanced by gravity or acceleration.

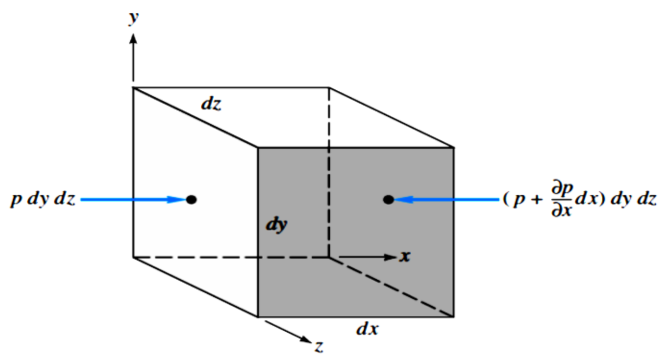

Figure 1: Net *x* force on an element due to pressure variation.

The pressure gradient is a surface force which acts on the sides of the element. Also, may be a body force, due to electromagnetic or gravitational potentials acting on the entire mass of the element. Consider only the gravity force or weight of element

Or
$$\left\{\begin{matrix} dF_{grav} = \rho g\,dx\,dy\,dz \\ f_{grav} = \rho g \end{matrix}\right\} \qquad (4)$$

For an incompressible fluid with constant viscosity the net viscous force is or (viscous stress)

$$f_{vs} = \mu\left(\frac{\partial^2 \vec{V}}{\partial x^2} + \frac{\partial^2 \vec{V}}{\partial y^2} + \frac{\partial^2 \vec{V}}{\partial z^2}\right) = \mu \nabla^2 \vec{V} \qquad (5)$$

The total vector resultant of these three forces which are pressure, gravity, and viscous stress must either keep the element in equilibrium or cause it to move with acceleration (**a)**. Form Newton's law of motion per unit volume

$$\sum f = \rho \mathbf{a} = f_{press} + f_{grav} + f_{vs} = -\nabla p + \rho g + \mu \nabla^2 V \qquad (6)$$

Rewrite Eq. 2.11 as follows

$$\nabla p = \rho(g - a) + \mu \nabla^2 V \qquad (7)$$

Examining Eq. 2.12, we can single out at least four special cases:

1- Flow at rest or at constant velocity: The acceleration and viscous terms vanishes identically, and p depends only upon gravity and density. This is the hydrostatic condition.
2- Rigid – body translation and rotation: The viscous term vanishes identically, and p depends only upon the term ρ(g-a).
3- Irrotational motion $\nabla \times \vec{V} \equiv 0$: The viscous term vanishes identically and exact integral Bernoulli's equation.
4- Arbitrary viscous motion, no general rules apply, but still the integration is quite straight forward.

When the fluid at rest or at constant velocity, ***a*** = 0 and $\nabla^2 V = 0$, Eq. 7 for the pressure distribution reduces to

$$\nabla p = \rho g \qquad (8)$$

This is a hydrostatic distribution formula and is correct for all fluid at rest. Where ***(g)*** is the magnitude of local gravity, Eq.8 has the pressure components are

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad (9)$$

Where the coordinate system ***z*** is up i.e (p) is independent of ***x&y***. Hence $\frac{\partial p}{\partial z}$ can be replaced by the total derivative $\frac{dp}{dz}$ and the hydrostatic condition reduce to

$$\frac{dp}{dz} = -\gamma \qquad (10)$$

This equation indicates that the pressure gradient in the vertical direction is negative; that is, the pressure decrease as we move upward in a fluid at rest.

This leads to the statement,

I. The pressure will be the same at the same level in any connected static fluid and at all points on a given horizontal plane whose density is constant or a function of pressure only.
II. The pressure increases with depth of fluid.
III. The pressure is independent of the shape of the container and the free surface of a liquid will seek a common level in any container, where the free surface is everywhere exposed to the same pressure.

Equation 10 is the solution to the hydrostatic problem. The integration requires an assumption about the density and gravity distribution.

4- *Incompressible Fluid.*

For liquids the variation in density is usually negligible, even over large vertical distances, so that the assumption of constant specific weight when dealing with liquids is a good one. For this instant, Eq. 10 can be directly integrated

$\int_{p1}^{p2} dp = -\int_{z1}^{z2} \gamma \, dz$ To yields $p_2 - p_1 = -\gamma(z_2 - z_1)$

Or $$p_1 - p_2 = \gamma(z_2 - z_1) \qquad (11)$$

Where p_1 and p_2 are pressures at the vertical elevation z_1 and z_2 as is illustrated in Fig. 2. Eq. 11 can be written in compact form $p_1 - p_2 = \gamma * h$

or $$p_1 = p_2 + \gamma * h \qquad (12)$$

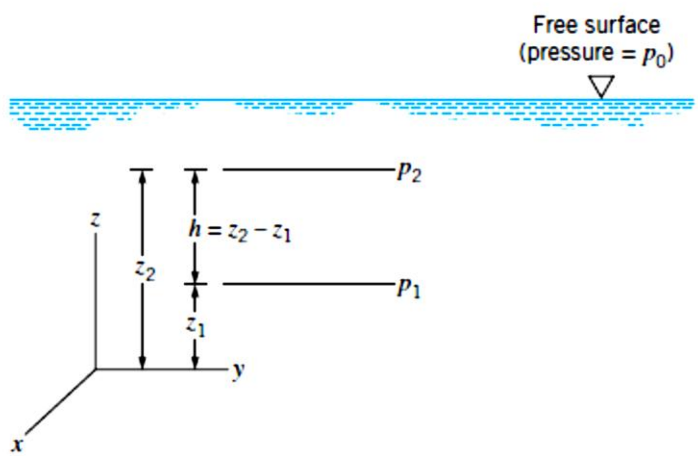

Figure 2: Notation for pressure variation in a fluid at rest.

Where ***h*** is the distance, z_2-z_1. This type of pressure distribution is commonly called a *hydrostatic distribution*. Eq. 12 shows that in an incompressible fluid at rest the pressure varies linearly with depth. It can also be observed from Eq. 12 that the pressure difference between two points can be specified by the distance *h* since

$$h = \frac{p_1 - p_2}{\gamma} \qquad (13)$$

Where ***h*** is called the pressure head and is interpreted as the height of column of fluid of specific weight γ to give a pressure difference ($p_1 - p_2$).]

5- <u>Compressible Fluid.</u>

Due to the specific weights of common gases are small when compared with liquids, it follows from Eq. 10 that the pressure gradient in the vertical direction is correspondingly small, and even over distances of several hundred meter the pressure will remain essentially constant for a gas. This means we can neglect the effect of elevation changes on the pressure in gases in tanks and pipes.

For an ideal gas is $p=\rho RT$ combine with eq.10

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

By separating variables

$$\int_{p1}^{p2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} \qquad (14)$$

Where g and R are assumed to be constant over the elevation change from z_1 to z_2. Before completing the integration, one must specify the nature of the variation of temperature with elevation involved. If we assumed that the temperature has a constant value $\boldsymbol{T_o}$ over the range z_1 to z_2 *(isothermal conditions)*,

$$p_2 = p_1\, exp\left[-\frac{g(z_2 - z_1)}{RT_o}\right] \qquad (15)$$

Eq.15 provides the desired pressure-elevation relationship for an isothermal layer. For non-isothermal condition a similar procedure can be followed if the temperature-elevation relationship is known