# Artificial Intelligence Lecture Notes by Dr. Belal Al-Khateeb

## Week No.: 14

## Lecture Title: Control Strategies for Resolution Inference (Problem Solving).

* **Resolution**

If $C_1$ and $C_2$ in normal form such that $C_1$ contains a literal and $C_2$ contains the negation of that literal. The result is a clause, which consist of the disjunction of all literals in $C_1$ and $C_2$, except the literal and its negation in the two clauses (P, ¬P). The resulting clause is called resolvent.

**Example:**

$C_1$: $p \vee \cancel{q} \vee r \vee s$
$C_2$: $t \vee z \vee \cancel{\neg q}$

$p \vee r \vee s \vee t \vee z$ → resolvent.

**Example:**

$C_1$: $p \vee q$
$C_2$: $\neg p$
$C_3$: $\neg q$

Empty Clause

$C_4$: $C_1$ and $C_2$: q
$C_5$: $C_4$ and $C_3$: $\boxed{\phantom{xx}}$

If the $\boxed{\phantom{xx}}$ is appear in the result it mean that the contradiction in the terms (q,¬q). So to prove the goal, add the negation of the goal and then apply resolution and if $\boxed{\phantom{xx}}$ appears then there is an error in the negation of the goal, which means the goal, is true and vice-versa.

## * Steps of Inference Using Resolution

1- put the axioms in clause normal form.
2- add the negation of the goal to be proved in clause normal form to the set of axioms.
3- produce a contradiction by generating the empty clause using resolution if no contradiction can be generated, the goal cannot be proved.
4- the substitutions that are used to produce the empty clause represent the values of the variables for which the goal is true.

**Example:** given the following:
1- fido is a dog.
2- all dogs are animals.
3- all animals will die.
Prove: "fido will die".

**Solution:**

$C_1$: dog (fido).
$C_2$: $\forall$X1 [dog (X1) $\longrightarrow$ animal (X1)].
$C_3$: $\forall$X2 [animal(X2) $\longrightarrow$ die (X2)].
$C_1'$: dog (fido).
$C_2'$: $\neg$ dog (X1) $\vee$ animal (X1).
$C_3'$: $\neg$ animal (X2) $\vee$ die (X2).
g': $\neg$ die (fido).

#1
g': $\neg$ ~~die (fido)~~.
$C_3'$: $\neg$ ~~animal (X2)~~ $\vee$ ~~die (X2)~~.
$\xrightarrow{\text{(fido,X2)}}$ $C_4$: $\neg$ animal (fido).

#2
$C_4$: $\neg$ ~~animal (fido)~~.
$C_2'$: $\neg$ dog (X1) $\vee$ ~~animal (X1)~~.
$\xrightarrow{\text{(fido,X1)}}$ $C_5$: $\neg$ dog (fido).

#3
$C_5$: $\neg$ ~~dog (fido)~~.
$C_1'$: ~~dog (fido)~~.
$\longrightarrow$ $\square$ Empty clause

Contradiction, which means goal, is true.

**Example:**

“ all people that are not poor and smart are happy. Those people that read are not stupid. Ali can read and is wealthy. Happy people have easy life”.

Prove: can any one be found with easy life.

**<u>Solution:</u>**

$C_1$: $\forall X1\ [\neg\ poor\ (X1) \wedge smart\ (X1) \longrightarrow happy(X1)]$.
$C_2$: $\forall X2\ [read\ (X2) \longrightarrow smart\ (X2)]$.
$C_3$: $read(ali) \wedge \neg\ poor\ (ali)$.
$C_4$: $\forall X3\ [happy\ (X3) \longrightarrow easy\_life\ (X3)]$.
Goal: $\exists Z\ easy\_life(Z)$.

$C_1'$: $poor\ (X1) \vee \neg smart\ (X1) \vee happy\ (X1)$.
$C_2'$: $\neg\ read\ (X2) \vee smart\ (X2)$.
$C_{31}'$: $read\ (ali)$.
$C_{32}'$: $\neg\ poor\ (ali)$.
$C_4'$: $\neg\ happy\ (X3) \vee easy\_life\ (X3)$.
g': $\neg\ \exists Z\ easy\_life(Z) \equiv \forall Z\ \neg\ easy\_life(Z) \equiv \neg\ easy\_life(Z)$ .

<u>#1</u>
g': $\neg$ ~~easy_life~~(Z).
$C_4'$: $\neg$ ~~happy~~ (X3) $\vee$ ~~easy_life~~ (X3).
$\xrightarrow{(Z,X3)}$ $C_5$: $\neg$ happy (Z).

<u>#2</u>
$C_5$: $\neg$ ~~happy~~ (Z).
$C_1'$: poor (X1) $\vee$ $\neg$smart (X1) $\vee$ ~~happy~~ (X1).
$\xrightarrow{(Z,X1)}$ $C_6$: poor (Z) $\vee\neg$smart (Z).

<u>#3</u>
$C_6$: poor ~~(Z)~~ $\vee\neg$smart (Z).
$C_{32}'$: $\neg$ ~~poor~~ (ali).
$\xrightarrow{(ali,Z)}$ $C_7$: $\neg$smart (ali).

<u>#4</u>
$C_7$: $\neg$~~smart~~ (ali).
$C_2'$: $\neg$ read (X2) $\vee$ ~~smart~~ (X2).
$\xrightarrow{(ali,X2)}$ $C_8$: $\neg$read (ali).

<u>#5</u>
$C_8$: $\neg$~~read~~ (ali).
$\longrightarrow$ $\boxed{\phantom{x}}$ Empty clause

$C_{31}$': read (ali).

Contradiction, which means goal, is true.

## * Resolution Control Strategies

1- **Breadth First Strategy**

In this strategy, each clause in the base set (starting set of clauses) is compared for resolution with every other clause on the first round. On the second round, the new clauses produced on the first round plus all the clauses of the base set are compared for resolution. For the $n^{th}$ round all previously generated clauses are added to the base set and all clauses are compared for resolution.

In this strategy, the number of clauses to be compared can become extremely large, since all early rounds are considered makes this approach inefficient for large problems.

**Example:** consider the following clauses:

$C_1$: $\forall X1\ [r\ (X1) \longrightarrow t(X1)]$.
$C_2$: $\forall X2\ [d\ (X2) \longrightarrow \neg t(X2)]$.
$C_3$: $\exists X3\ [d\ (X3) \wedge h\ (X3)]$.

Prove/ disprove the goal $\exists Z\ [h\ (Z) \wedge \neg\ r\ (Z)]$.

**Solution:**

Convert to normal form:

$C_1$': $\neg\ r\ (X1) \vee t\ (X1)$.
$C_2$': $\neg\ d\ (X2) \vee \neg\ t\ (X1)$.
$C_{31}$': $d\ (a)$.
$C_{32}$': $h\ (a)$.

Goal: $\neg\ h\ (Z) \vee r\ (Z)$.

Round 1

1- resolves $C_1$' and $C_2$'
  $C_4$: $\neg\ r\ (X1) \vee \neg\ d\ (X1)$. {(X1,X2)}
2- resolves $C_1$' and goal
  $C_5$: $t\ (X1) \vee \neg\ h\ (X1)$. {(X1,Z)}
3- resolves $C_2$' and $C_{31}$'
  $C_6$: $\neg\ t\ (a)$ {(a,X2)}
4- resolves $C_{32}$' and goal
  $C_7$: $r\ (a)$. {(a,Z)}

Round 2

5- resolves $C_1'$ and $C_6$
$C_8$: $\neg$ r (a). {(a,X1)}
6- resolves $C_1'$ and $C_7$
$C_9$: t (a). {(a,X1)}
7- resolves $C_2'$ and $C_5$
$C_{10}$: $\neg$ d (X1) $\vee$ $\neg$ h (X1). {(X1,X2)}
8- resolves $C_{31}'$ and $C_4$
$C_{11}$: $\neg$ r (a). {(a,X1)}
9- resolves $C_{32}'$ and $C_5$
$C_{12}$: t (a). {(a,X1)}
10- resolves goal and $C_4$
$C_{13}$: $\neg$ d (X1) $\vee$ $\neg$ h (X1). {(X1,Z)}
11- resolves $C_4$ and $C_7$
$C_{14}$: $\neg$ d (a). {(a,X1)}
12- resolves $C_5$ and $C_6$
$C_{15}$: $\neg$ h (a). {(a,X1)}

Round 3

13- resolves $C_2'$ and $C_9$
$C_{16}$: $\neg$ d (a). {(a,X2)}
14- resolves $C_2'$ and $C_{12}$
$C_{17}$: $\neg$ d (a). {(a,X2)}

15- resolves $C_{31}'$ and $C_{10}$
$C_{18}$: $\neg$ h (a). {(a,X1)}
16- resolves $C_{31}'$ and $C_{13}$
$C_{19}$: $\neg$ h (a). {(a,X1)}
17- resolves $C_{31}'$ and $C_{14}$
$C_{20}$: $\square$ empty.