Lecture Title: Control Strategies for Resolution Inference (Problem Solving).

2- **Set of Support Strategy**
For a set of input clauses S, we can specify a subset T of S called the set of support. The strategy requires that at least one of the resolvents in each resolution operation be in the set of support. This strategy is based on the principle that the negation of the goal is going to be responsible for generating empty clause.

The set of support consists initially of the negation of the goal consequently any resolvent whose parent in the set of support become member of the set of support. This strategy is good for dealing with large number of clauses.

**Example:** consider the following clauses:

\[ C_1 : \forall X_1 [r (X_1) \rightarrow t(X_1)]. \]
\[ C_2 : \forall X_2 [d (X_2) \rightarrow \neg t(X_2)]. \]
\[ C_3 : \exists X_3 [d (X_3) \land h (X_3)]. \]

Prove/ disprove the goal \( \exists Z [h (Z) \land \neg r (Z)] \).

**Solution:** Convert to normal form:

\[ C_1' : \neg r (X_1) \lor t (X_1). \]
\[ C_2' : \neg d (X_2) \lor \neg t (X_1). \]
\[ C_3_{1}': d (a). \]
\[ C_3_{2}': h (a). \]
\[ \text{Goal: } \neg h (Z) \lor r (Z). \]
\[ S = \{ \neg h (Z) \lor r (Z) \}. \]

1- resolves goal and \( C_1' \)
C₄: ¬h (Z) ∨ t (Z₁).  \{(Z,X₁)\}
S = {¬h (Z) ∨ r (Z), C₄}.

2- resolves C₄ and C₂'
C₅: ¬h (Z) ∨ ¬d (Z).  \{(Z,X₂)\}
S = {¬h (Z) ∨ r (Z), C₄, C₅}.

3- resolves C₅ and C₃₁'
C₆: ¬h (a)  \{(a,Z)\}
S = {¬h (Z) ∨ r (Z), C₄, C₅, C₆}.

4- resolves C₆ and C₃₂'
C₇: empty.

3- **Unit Preference Strategy**

In this strategy, clauses are chosen for resolution, with as fewer literals as possible so that the empty clause can be produced with fewer number of resolutions.

**Example:** consider the following clauses:

C₁: ∀X₁ [r (X₁) → t(X₁)].
C₂: ∀X₂ [d (X₂) → ¬t(X₂)].
C₃: ∃X₃ [d (X₃) ∧ h (X₃)].
Prove/ disprove the goal \( \exists Z \, [h(Z) \land \neg r(Z)] \).

**Solution:** Convert to normal form:

\[
C_1': \neg r(X_1) \lor t(X_1). \\
C_2': \neg d(X_2) \lor \neg t(X_1). \\
C_{31}': d(a). \\
C_{32}': h(a). \\
\text{Goal:} \, \neg h(Z) \lor r(Z).
\]

1- Resolves goal and \( C_{32}' \)
   \[ C_4: r(a). \quad \{(a,Z)\} \]

2- Resolves \( C_4 \) and \( C_1' \)
   \[ C_5: t(a). \quad \{(a,X_1)\} \]

3- Resolves \( C_5 \) and \( C_2' \)
   \[ C_6: \neg d(a) \quad \{(a,X_2)\} \]

4- Resolves \( C_6 \) and \( C_{31}' \)
   \[ C_7: \text{empty.} \]

4- **Linear Input Format Strategy**
   In this strategy, the negated goal is resolved with one of the original input clauses. The resulting clause is resolved with one of the original clauses and so on until the empty clause is generated or there are no other resolutions. This strategy is not complete.
**Example:** consider the following clauses:

\[ C_1: \forall X_1 [r (X_1) \rightarrow t(X_1)]. \]
\[ C_2: \forall X_2 [d (X_2) \rightarrow \neg t(X_2)]. \]
\[ C_3: \exists X_3 [d (X_3) \land h (X_3)]. \]

Prove/disprove the goal \( \exists Z [h (Z) \land \neg r (Z)]. \)

**Solution:** Convert to normal form:

\[ C_1': \neg r (X_1) \lor t (X_1). \]
\[ C_2': \neg d (X_2) \lor \neg t (X_1). \]
\[ C_{31}': d (a). \]
\[ C_{32}': h (a). \]

Goal: \( \neg h (Z) \lor r (Z). \)

1- resolves goal and \( C_{32}' \)
   \[ C_4: r (a). \] \( \{(a,Z)\} \)

2- resolves \( C_4 \) and \( C_1' \)
   \[ C_5: t (a). \] \( \{(a,X_1)\} \)

3- resolves \( C_5 \) and \( C_2' \)
   \[ C_6: \neg d (a) \] \( \{(a,X_2)\} \)

4- resolves \( C_6 \) and \( C_{31}' \)
   \[ C_7: \] empty.