Artificial Intelligence Lecture Notes by Dr. Belal Al-Khateeb

Week No.: 11

Lecture Title: Horn Clauses; Unification and Skolemization

* **Horn Clause**

A Horn clause has the following form:

$b_1(X,Y) \wedge b_2(X,Y) \wedge \ldots\ldots \wedge b_n(X,Y) \longrightarrow a(X,Y)$. Where the literals a, $b_1, \ldots, b_n$ are all positive.

The a (X,Y) is called the head of the clause and $b_1(X,Y) \wedge \ldots\ldots \wedge b_n(X,Y)$ is called the body of the clause.

These are three cases to consider:

1- The original clause has no head $b_1(X,Y) \wedge b_2(X,Y) \wedge \ldots\ldots \wedge b_n(X,Y)$. This represents a goal to be proved.
2- The clause has no body: a (X,Y). This represents a fact.
3- The clause has a body and a head: $b_1(X,Y) \wedge \ldots\ldots \wedge b_n(X,Y) \longrightarrow a(X,Y)$. In this case the clause is called a rule.

A Horn clause may be written in the form: $\neg b_1(X,Y) \vee \neg b_2(X,Y) \vee \ldots \vee \neg b_n(X,Y) \vee a(X,Y)$. Thus a Horn can be defined as a clause with at most one positive literal.

* **Common Identities**

1- $\neg \exists X\, p(X) \equiv \forall X\, \neg p(X)$.
2- $\neg \forall X\, p(X) \equiv \exists X\, \neg p(X)$.
3- $\exists X\, p(X) \equiv \exists Y\, p(Y)$.
4- $\forall X\, p(X) \equiv \forall Y\, p(Y)$.
5- $\forall X\, [p(X) \wedge q(X)] \equiv \forall X\, p(X) \wedge \forall Y\, q(Y)$.

6- $\exists X\,[p(X) \vee q(X)] \equiv \exists X\, p(X) \vee \exists Y\, q(Y)$.

Note that $\forall X\,[p(X) \vee q(\cancel{X})] \equiv \forall X\, p(X) \vee \forall Y\, q(Y)$.

* **Examples:**

1- If it does not rain tomorrow, Zeki will go to the lake.

Solution\ $\neg$ weather (tomorrow,rain) $\longrightarrow$ go (Zeki,lake).

2- All basketball players are tall.

Solution\ $\forall X$ tall (X). $X \in$ {basketball player}.

$\forall X$ [basketball_player (X) $\longrightarrow$ tall (X). $X \in$ {players}.

$\forall X$ [player (X) $\wedge$ play (X,basketball) $\longrightarrow$ tall (X). $X \in$ {all people}.

3- Some students like artificial intelligence, where $X \in$ {set of all things}.

Solution\ $\exists X$ [student (X) $\wedge$ like (X,artificial_intelligence)].

4- Nobody like taxes.

Solution\ $\neg \exists X$ [like (X,taxes)] $X \in$ {set of people}.

$\neg \exists X$ [person (X) $\wedge$ like (X,taxes)] $X \in$ {set of all things}.

* **Unification**

Unification is the process of making two literals look alike.

Assume that we have a set of literals to be unified: {L1, L2, L3, …., Lk}, we seek a substitution, such that: F= {(t1,v1), (t2,v2), ……, (tn,vn)} such that: L1F=L2F=……=Lkf.

Example:

1- Assume that L = p (X,Y,f(Y),b)

F = {(a,X),(f(Z),Y)}

LF = p (a,f(Z),f(f(Z)),b).

2- L1 = father (X,Y).

L2 = father (X1,Y1).

F = {(X,X1),(Y,Y1)}.

L1F = father (X,Y).

L2F = father (X,Y).

Q = {(ali,X),(ahmed,Y),(ali,X1),(ahmed,Y1)}.

L1Q = father (ali,ahmed).

L2Q = father (ali,ahmed).

R = {(ali,X),(ali,X1),(Y,Y1)}.

L1R = father (ali,Y).

L2R = father (ali,Y).

Let F be a substitution, then F is mgu (most general unifier) of s = {L1, L2, …., Lk} provided that for any other unifier Q of {L1, L2, ….., Lk} there exist a unifier R of {L1, L2, …., Lk} such that: Q(S) = R(F(S)).

Unification is done by replacing a variable by a term (important).

* **Procedure mgu**

Begin

- Generate the first disagreement set D.
- Repeat

While (D is disagreement) do

If non of the terms in D consists of a variable by itself then

- stop and report failure becomes the set of literals can not be unified by any substitutions.

Else

- Convert variable into a term by adding a pair of the form (t,v) and simultaneously perform the substitution in the literals.

If v is not in F as a second argument of a pair, then

- add a pair (tp,vp) to F.
Else
- stop and report failure.
End if
End if
End while
- generate the next disagreement set D;
Until (no D remain)
Return (F).
End.

Example:

Let L1 = p (X,f(Y)).
L2 = p(a,f(g(Z))).
Find the mgu.

Solution\

F = {∅}.

Step1:

D = {X,a}.
F = {(a,X)}.
L1′ = L1F = p(a,f(Y)).
L2′ = L2F = p(a,f(g(Z))).

Step2:

D = {f(Y),f(g(Z))}.
F′ = {(a,X),(g(Z),Y)}.
L1″ = L1′F′ = p(a,f(g(Z))).
L2″ = L2′F′ = p(a,f(g(Z))).

Stop.

* **Skolemization**

Skolemization is the process of replacing existentially quantifier (∃) variables by a constant (skolen constant) or a function (skolen function) of universally quantified (∀) variables under whose scope.

Example:

1- $\exists X$ father (X,ahmed)   X=ali
father (ali,ahmed).

2- $\forall Y \exists X$ father (X,Y)
$\forall Y$ father (f(Y),Y).
f (Y) is a skolen function.

3- $\exists X \ \forall Y$ p (X,Y). Replace X by a constant $\forall Y$ p (a,Y).

4- $\forall X \ \forall Y \exists Z \ \forall W$ p (X,Y,Z,W).
$\forall X \ \forall Y \ \forall W$ p (X,Y,f(X,Y),W).

5- $\forall X \ \forall Y \exists Z \ \exists W$ p (X,Y,Z,W).
$\forall X \ \forall Y$ p (X,Y,f(X,Y),g(X,Y)).