University of Anbar
College of Engineering
Dams & Water Resources Eng. Dept.

Calculus I

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Phase: 1

Semester I (2019-2020)

Course Description:

Functions, Limits and continuity, Differentiation, Applications of derivatives, Integration, Applications of the Integral

Recommended Textbook(s):

Calculus, Early Transcendental By James Stewart, 6th Edition, 2008, Brooks/Cole

Course Topics:

1. **Functions and models**: four ways to represent a function , mathematical models: a catalogue of essential functions , new functions from old functions , exponential functions, inverse functions and logarithms
2. **Limits**: the tangent and velocity problems. The limit of a function, calculating limits using the limit laws. Continuity, limits at infinity, horizontal asymptote. Infinite limits, vertical asymptotes. derivatives and rates of change
3. **Differentiation rules:** Differentiation of Polynomials. The Product and Quotient Rules. Derivatives of Trigonometric Functions. The Chain Rule, Implicit Differentiation. Related Rates, Hyperbolic functions.
4. **Applications of differentiation**: maximum and minimum values. The mean value theorem. How derivatives affect the shape of a graph. Summary of curve sketching. Optimization problems. Antiderivatives. Indeterminate forms and l'hospital's rule.
5.**Integrals**: the definite integral. The fundamental theorem of calculus. The indefinite integral and net change theorem. The substitution rule.
6. **Applications of integrals**: areas between curves. volumes. volumes by cylindrical shells.

The goals of this course are to enable students to:

1. Solve problems using the Fundamental Theorem of Calculus.
2. Evaluate Limits of the functions and their continuity.
3. Find the derivative of algebraic, trigonometric, exponential, and logarithmic functions.
4. Sketch the graph of a function using the information for the first and second derivatives
5. Solve problems involving applications of integrals including finding volume of solids of revolution and area between curves

Program and Course Outcomes:

1. Apply arithmetic, algebraic, geometric and logical reasoning to solve problems.
2. Evaluate of basic mathematical and/or logical information numerically, graphically, and symbolically.
3. understand of the concept of limit and rate of change and how to use it to solve real world problem.
4. understand the concept of continues functions and compute instantaneous rate of change.
5. Compute of derivatives of polynomial, logarithmic, trigonometric, hyperbolic and inverse hyperbolic functions in addition to be proficient in techniques of differentiation.
6. Solve related rate and optimization problems using techniques of differentiations.
7. evaluate integrals using basic integration formulas to obtain and method of substitution for more complicated functions.

Selected References

1 – Advanced Engineering Mathematics, Kreyszig

2 – Advanced Engineering Mathematics, Wyle

3 – Further Engineering Mathematics, Stroud.

4 – Engineering Mathematics, Kandasamy.

5 – Advanced Engineering Mathematics, Gustafson

6 – Elementary Differential Equations, Boyce.

7 – Numerical Analysis, Burden.

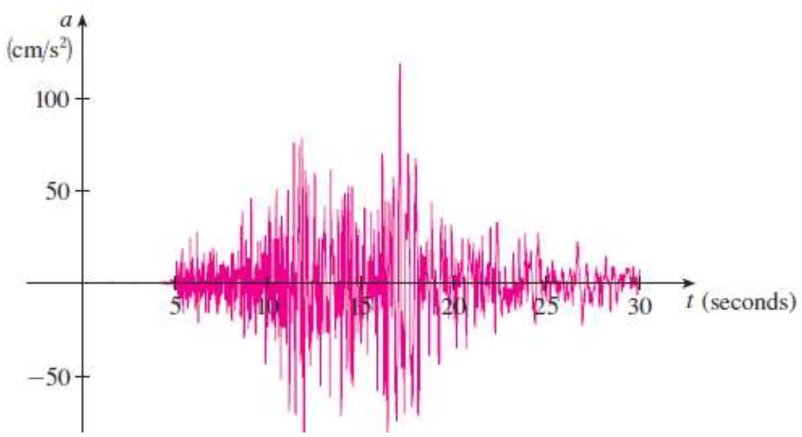
9 - Calculus by Thomas & Finney.

1 Functions

Examples of functions

A. The most useful representation of the area of a circle as a function of its radius is probably the algebraic formula A= πr^2

B. The vertical acceleration of the ground as measured by a seismograph during an earthquake is a function of the elapsed time. Figure below shows a graph generated by seismic activity during the Northridge earthquake that shook Los Angeles in 1994. For a given value of the graph provides a corresponding value of a.



So, function is y = f(x), expressing y as a dependent variable on f and x is an independent variable.

For example f(x) = 2x-1

If x = 1 then 2*1-1 = 1
X= -1 then 2 *-1-1 = -3

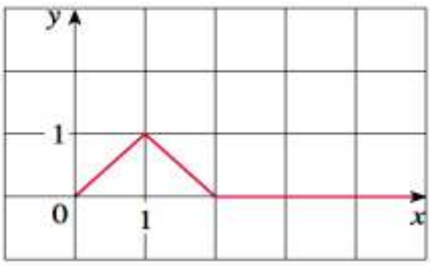
And so on

If an absolute value like f(x) = IxI then $x = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$

Note:

I-aI = IaI
IabI = IaI IbI
Ia/bI = IaI/IbI but b ≠0
Ia+bI = IaI+IbI

Example 1: Find a formula for the function f graphed in Figure ?



Point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

SOLUTION The line through (0, 0) and (1, 1) has slope $m = 1$ and y-intercept $b = 0$, so its equation is $y = x$. Thus, for the part of the graph of f that joins (0, 0) to (1, 1), we have

$$f(x) = x \qquad \text{if } 0 \leqslant x \leqslant 1$$

The line through (1, 1) and (2, 0) has slope $m = -1$, so its point-slope form is

$$y - 0 = (-1)(x - 2) \qquad \text{or} \qquad y = 2 - x$$

So we have $$f(x) = 2 - x \qquad \text{if } 1 < x \leqslant 2$$

We also see that the graph of f coincides with the x-axis for $x > 2$. Putting this information together, we have the following three-piece formula for f:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } 1 < x \leqslant 2 \\ 0 & \text{if } x > 2 \end{cases}$$

■

2. Symmetry

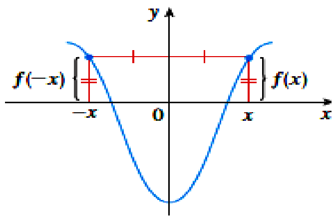
If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called **an even function**. For instance, the function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

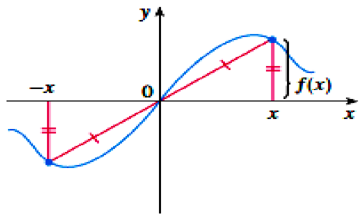
The geometric significance of an even function is that its graph is symmetric with respect to the y-axis (see Figure 19). This means that if we have plotted the graph of f for $x \geqslant 0$, we obtain the entire graph simply by reflecting this portion about the y-axis.

If f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**. For example, the function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$



An even function



An odd function

3. Increasing and Decreasing Functions

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

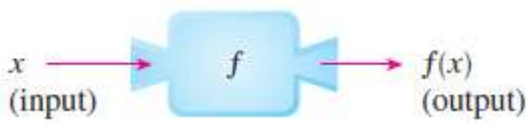
It is called **decreasing** on I if

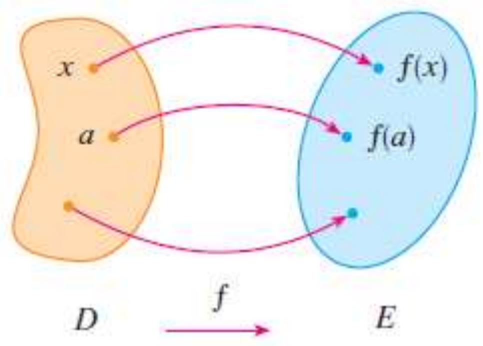
$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

4. Domain and Range

We usually consider functions for which the sets D and E are sets of real numbers. The set D is called the domain of the function. The number f(x) is the value of f at x and is read “f of x” The range of f is the set of all possible values of f(x) as varies throughout the domain.

Then domains and ranges of many functions are intervals of real numbers.





Example 2: Find the domain and range of the following functions:

(a) $f(x)=2x-1$

(b) $f(x)=x^2$

(c) $f(x)=\tan x$

(d) $y=\sqrt{x}$

(e) $y=\dfrac{x-12}{x^2-5x+6}$

Solution

(a) $f(x)=2x-1$

Domain: x = R $-\infty\leq x\leq\infty$

Range f(x) = R R: denotes as all real number

(b) $f(x)=x^2$

Domain: x = R $-\infty\leq x\leq\infty$

Range f(x) = R

(c) $f(x) = \tan x$

Domain: x = R excluding $\pm\frac{\pi}{2}, \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, ...$

(d) $f(x) = \sqrt{x}$

Domain $0 \le x$ and Range $0 \le y$

(e) $y = \frac{x-12}{x^2 - 5x + 6}$

The denominator not equal to zero

$x^2 - 5x + 6 = 0$

(x-3)(x-2) then domain all but $x \neq 3$ and $x \neq 2$

2. Sketch of functions

The points in the plane whose (x,y) are the input and output pairs of a function make up the graph of the function.

Definition:
Even function: if f(x) = f(-x)
Odd function: if f(x) = -f(-x)