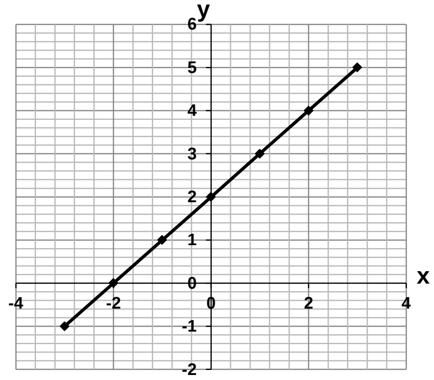
Example3: sketch the function y= x+2

Solution:

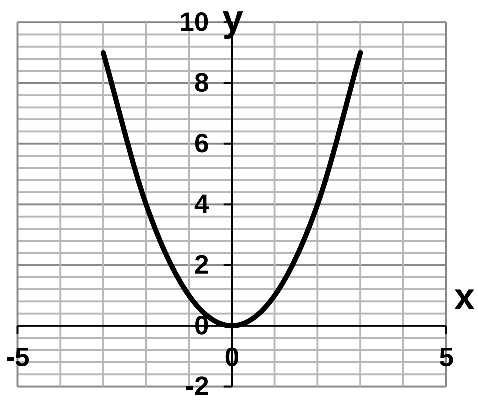
x	0	1	2	…..	-1	-2
y	2	3	4	….	1	0



Example4: sketch the power function y= x^2

Solution:

x	0	1	2	…..	-1	-2
y	0	1	4	….	1	4

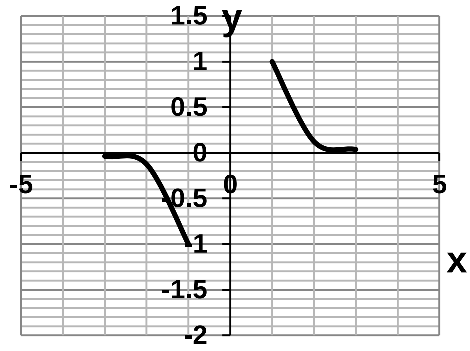


Notes: if n is odd then symmetric about origin and pass through (1,1) and (-1,-1)

if n is even then symmetric about y axis and pass through (1,1) and (-1,1)

if the power is negative then y=1/x^n

like y= 1/x^3, 1/x …..



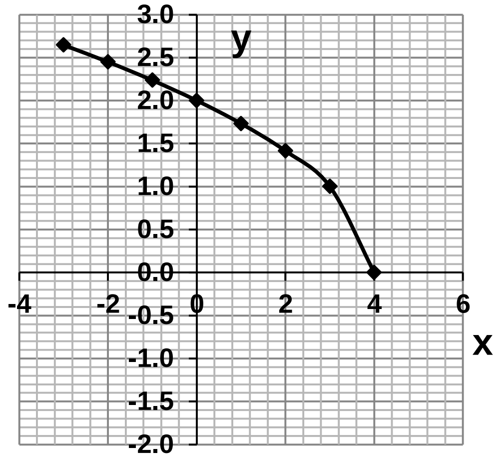
Graph of $y= 1/x^3$

Example 5: Find the domain and range then sketch the function $y=\sqrt{4-x}$

Solution

Domain : $4-x \geq 0$ then $x \leq 4$

Range $y \geq 0$



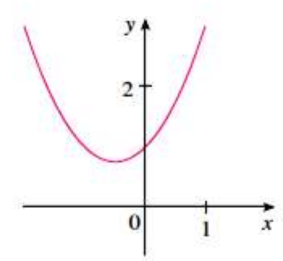
2. Polynomials

The general form is

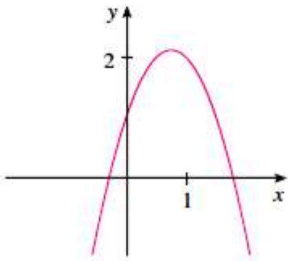
$K_n x^n + K_{n-1} x^{n-1} + K_{n-2} x^{n-2} + \ldots\ldots + K_1 x + K_0$

Ex: $x^3 + 5x^2 + 3$
$x^5 - x^3 + x^{0.5}$ ………….. etc.

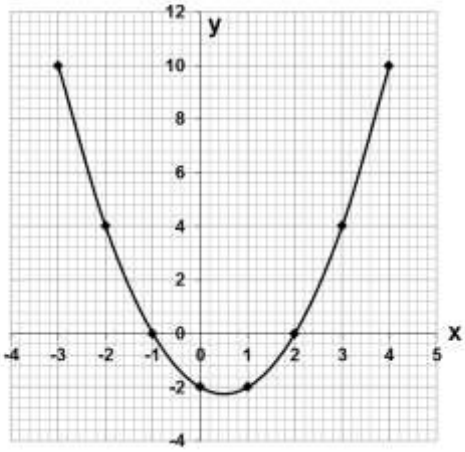
The graph is

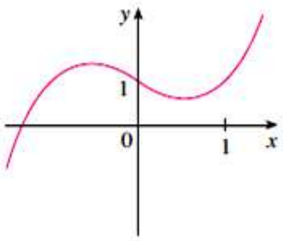


(a) $y = x^2 + x + 1$

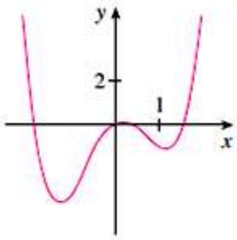


(b) $y = -2x^2 + 3x + 1$





(a) $y = x^3 - x + 1$



(b) $y = x^4 - 3x^2 + x$

Example 6: sketch the function $f_{(x)} = (x-2)(x+1)$

Solution

$$f_{(x)} = y = x^2 + x - 2x - 2$$

$y = x^2 - x - 2$ $\qquad\qquad$ $y = ax^2 + bx + c$

The vertex = x= -b/2a , $\qquad$ x = -(-1)/2*1 = ½
y = $(1/2)^2$ -1/2 -2 = -9/4

vertex (1/2, -9/4)

points of intercept

at x= 0 y = -2
at y= 0 0 = (x-2)(x-1)

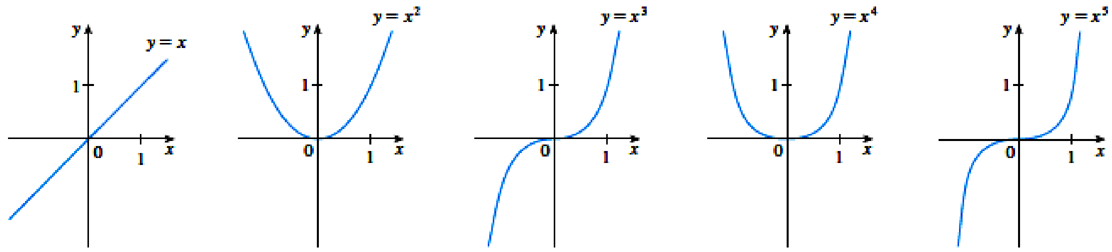
x= 2   (2,0)
x = -1   (-1,0)

2. Power functions

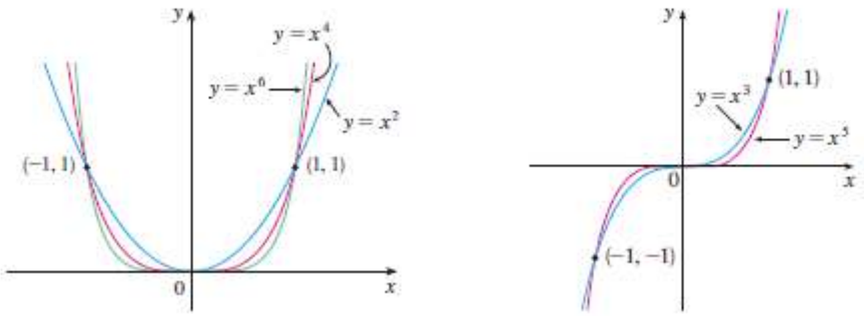
A function of the form f(x) = x^a , where a is constant, is called a power function. We consider several cases.

(I) $a = n$, where n is a positive integer

The graphs of $f(x) = x^n$ for $n = 1, 2, 3, 4$, and 5 are shown in Figure 11. (These are polynomials with only one term.) We already know the shape of the graphs of $y = x$ (a line through the origin with slope 1) and $y = x^2$ [a parabola, see Example 1.1.2(b)].

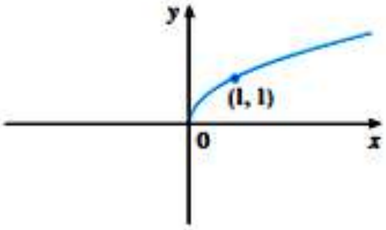


I Graphs of $f(x) = x^n$ for $n = 1, 2, 3, 4, 5$

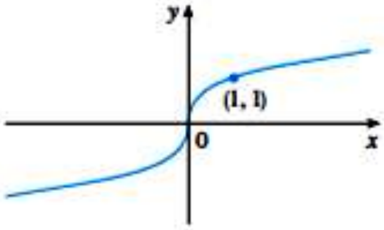


(II) $a = 1/n$, where n is a positive integer

The function $f(x) = x^{1/n} = \sqrt[n]{x}$ is a root function. For $n = 2$ it is the square root function $f(x) = \sqrt{x}$, whose domain is $[0, \infty)$ and whose graph is the upper half of the parabola $x = y^2$. [See Figure 13(a).] For other even values of n, the graph of $y = \sqrt[n]{x}$ is similar to that of $y = \sqrt{x}$. For $n = 3$ we have the cube root function $f(x) = \sqrt[3]{x}$ whose domain is $\mathbb{R}$ (recall that every real number has a cube root) and whose graph is shown in Figure 13(b). The graph of $y = \sqrt[n]{x}$ for n odd ($n > 3$) is similar to that of $y = \sqrt[3]{x}$.



(a) $f(x)=\sqrt{x}$



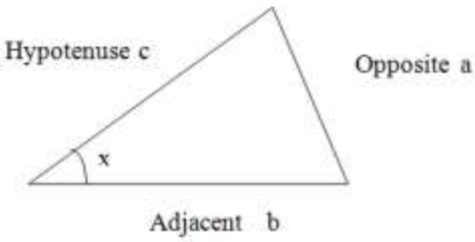
(b) $f(x)=\sqrt[3]{x}$

3. Algebraic Functions

A function f is called an algebraic function if it can be constructed using algebraic operations (such as addition, subtraction, multiplication, division, and taking roots) starting with polynomials. Any rational function is automatically an algebraic function. Here are two more examples.

4. Trigonometric functions

Sine	sinx = a/c
Cosine	cosx = b/c
Tangent	tanx = a/b = sinx/cosx
Cotangent	cotx = b/a = cosx/sinx
Secant	secx = c/a = 1/cosx
Cosecant	csecx = c/a = 1/sinx



$$f(x) = \sqrt{x^2 + 1} \qquad g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$$

Identities

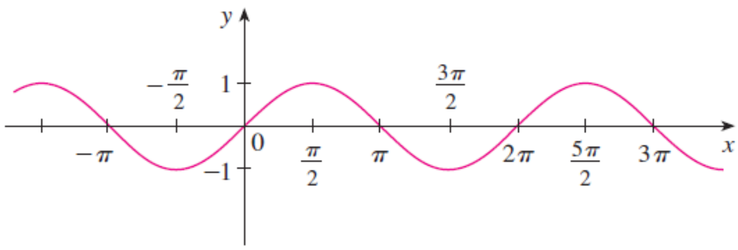
Trigonometric Identities – part 1 www.GiMathS.Com

Reciprocal Identities		Half Angle Identities	Double Angle Identities	Pythagoras Identities
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin^2\theta + \cos^2\theta = 1$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	$1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$	$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$	

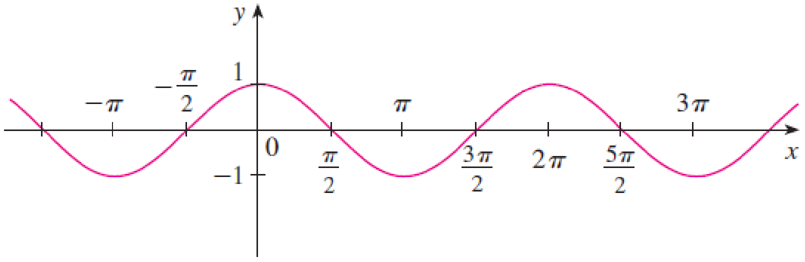
Sum to Product Identities	Product to Sum Identities
$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$
$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Even/Odd Identities
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$
$\sec(-\theta) = \sec\theta$
$\cot(-\theta) = -\cot\theta$

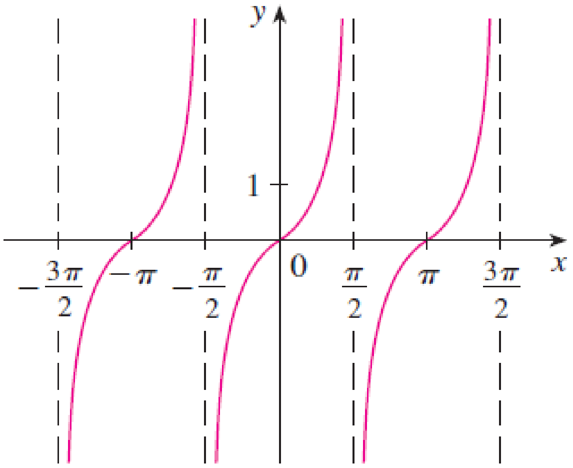
Graphs:



(a) $f(x) = \sin x$



(b) $g(x) = \cos x$



$y = \tan x$

Notice that for both the sine and cosine functions the domain is $(-\infty, \infty)$ and the range is the closed interval $[-1, 1]$. Thus, for all values of x, we have

$$-1 \leqslant \sin x \leqslant 1 \qquad -1 \leqslant \cos x \leqslant 1$$

or, in terms of absolute values,

$$|\sin x| \leqslant 1 \qquad |\cos x| \leqslant 1$$

Also, the zeros of the sine function occur at the integer multiples of π; that is,

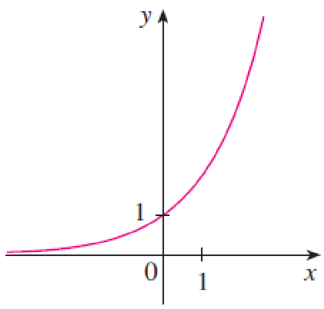
$$\sin x = 0 \quad \text{when} \quad x = n\pi \quad n \text{ an integer}$$

An important property of the sine and cosine functions is that they are periodic functions and have period 2π. This means that, for all values of x,

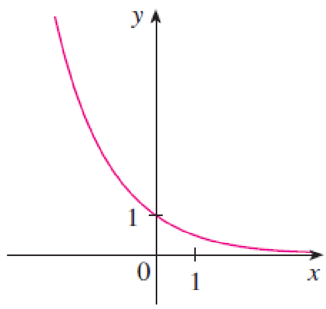
$$\sin(x + 2\pi) = \sin x \qquad \cos(x + 2\pi) = \cos x$$

5. Exponential functions

The exponential functions are the functions of the form $f(x) = a^x$, where the base a is a positive constant.

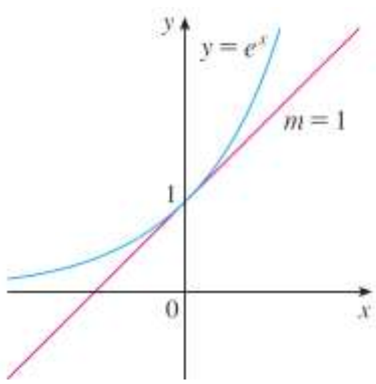


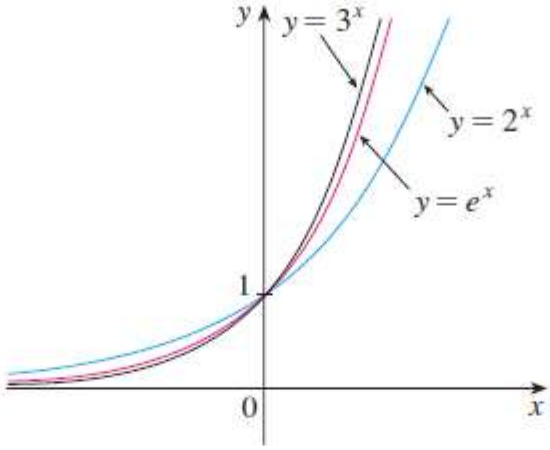
(a) $y = 2^x$



(b) $y = (0.5)^x$

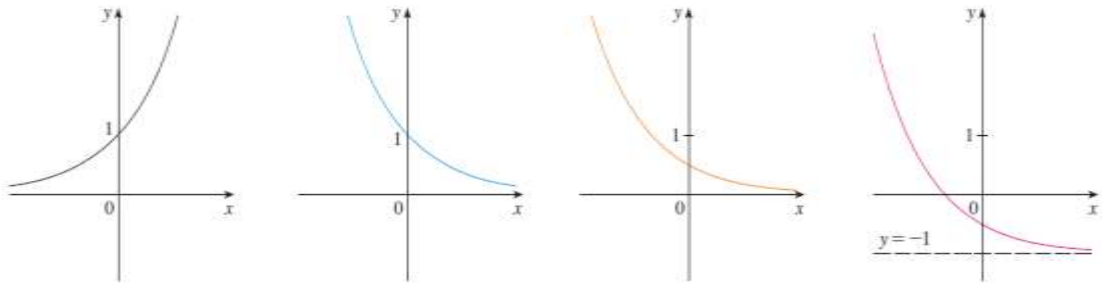
If we choose the base a so that the slope of the tangent line tangent line to the y = ax at (0,1) is exactly. In fact, there is such a number and it is denoted by the letter e. e =2.71828





Illustrative example: Graph the function $y = \frac{1}{2}e^{-x} - 1$ and state the domain and range.

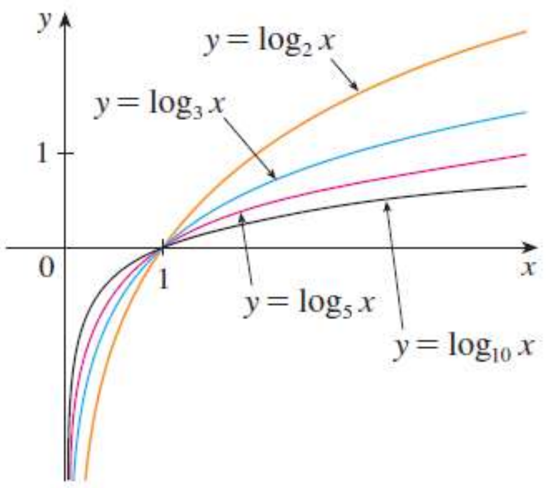
Solution We start with the graph of $y = e^x$ from Figure below, and reflect about the y-axis to get the graph of $y = e^{-x}$ in . Figure (b). (Notice that the graph crosses the y-axis with a slope of -1). Then we compress the graph vertically by a factor of 2 to obtain the graph of $y = \frac{1}{2}e^{-x}$ in Figure (c). . Finally, we shift the graph downward one unit to get the desired graph in Figure (d). The domain is $\mathbb{R}$ and the range is $(-1, \infty)$.



(a) $y = e^x$ (b) $y = e^{-x}$ (c) $y = \frac{1}{2}e^{-x}$ (d) $y = \frac{1}{2}e^{-x} - 1$

6. Logarithmic functions

The logarithmic function f(x) = $\log_a$, where a is a positive constant, are the inverse function of the exponential functions. In each case the domain is(0, ∞) and the range is (-∞, ∞) and the function increases slowly when x>1.



Example 7: Classify the following functions as one of the types of functions that we
have discussed.

(a) $f(x) = 5^x$

(b) $g(x) = x^5$

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$

(d) $u(t) = 1 - t + 5t^4$

Solution

(a) $f(x) = 5^x$ is an exponential function. (The x is the exponent.)

(b) $g(x) = x^5$ is a power function. (The x is the base.) We could also consider it to be a polynomial of degree 5.

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ is an algebraic function.

(d) $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4.

Laws of Logarithms If x and y are positive numbers, then

1. $\log_b(xy) = \log_b x + \log_b y$

2. $\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

3. $\log_b(x^r) = r\log_b x$ (where r is any real number)