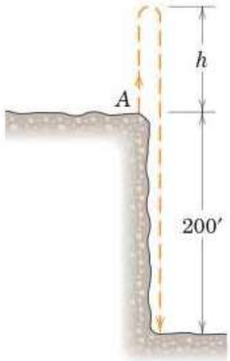
Semester I (2019-2020)

PROBLEMS

2/10: A ball is thrown vertically up with a velocity of 80 ft/sec at the edge of a 200 ft cliff. Calculate the height h to which the ball rises and the total time t after release for the ball to reach the bottom of the cliff. Neglect air resistance and take the downward acceleration to be 32.2 ft/see.



Problem 2/10

Solution:

$y = v_0 t + \frac{1}{2} a t^2$

$\uparrow + y \quad \therefore \quad y = 80t - \frac{1}{2} 32.2\, t^2$

if $y = -200$ ft

$-200 = 80t - 16.1t^2$

$16.1t^2 - 80t - 200 = 0$

$$t = \frac{80 \mp \sqrt{(80)^2 + 4*(16.1)(200)}}{2*16.1}$$

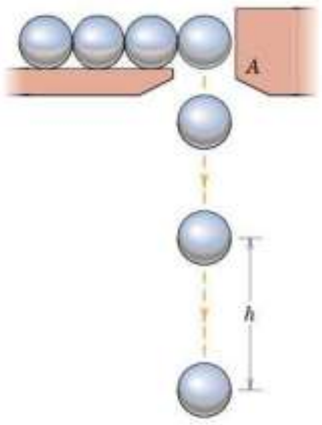
To reach the bottom of the Clif

$v^2 = v_0^2 + 2ay \longrightarrow y = h = \frac{0}{-}$

$\therefore h = 99.4$ ft

<u>*Semester I (2019-2020)*</u>

2/19: Small steel balls falls from through the opening at A at the steady rate of two per second. Find the vertical separation h of two consecutive balls when the lower one has dropped 3 meters. Neglect air resistance.



Problem 2/23

Solution

$S = S_0 + v_0 t + \frac{1}{2} g t^2$

Sphere ①: $H = \frac{1}{2} g t_2^2$

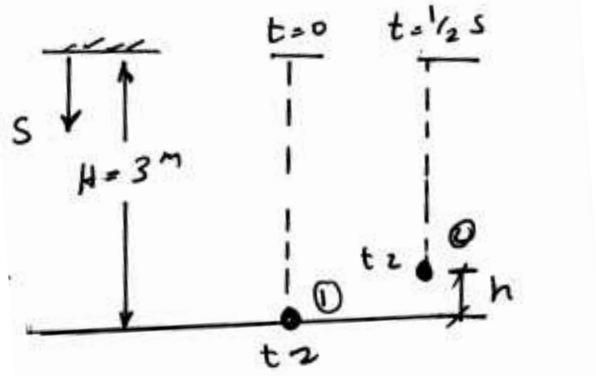
Sphere ②:

$H - h = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$

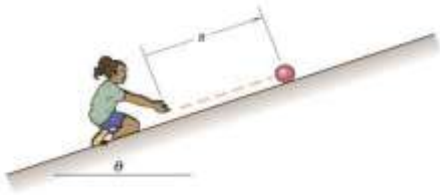
if $H = 3\,m$ & $g = 9.81\ m/s^2$

we solve 2-equations:

$h = 2.61\ m$



2/25: A girl rolls a ball up an incline and allows it to return to her. For the angle θ and ball involved, the acceleration of the ball along the incline is constant at 0.25g, directed down the incline. If the ball is released with a speed of 4 m/s, determine the distance s it moves up the incline before reversing its direction and the total time t required for the ball to return to the child's hand.



Problem 2/25

$v^2 = v_0^2 + 2a(s - s_0)$

$0 = 4^2 + 2 \times (-9.81 \times 0.25) \times S$

$\therefore S = 3.26$ m

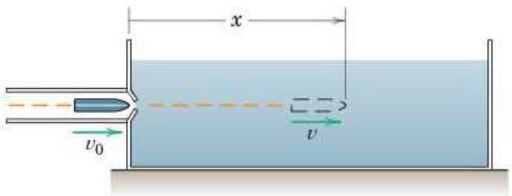
$v = v_0 + at$

$0 = 4 + (-9.81 \times 0.25) \times t_{up}$

$\therefore t_{up} = 1.631$ sec.

$\therefore t_{total} = 2 \times t_{up} = 2 \times 1.631 = 3.26$ S

2/47: A test projectile is fired horizontally into a viscous liquid with a velocity of v_0. The retarding force is proportional to the square of the velocity, so that the acceleration becomes a = $-kv^2$. Derive expressions for the distance D traveled in the liquid and the corresponding time t required to reduce the velocity to $v_0/2$. Neglect any vertical motion.



Problem 2/47

Solution

$a = -k v^2$

$v\,dv = a\,dx \quad , \quad \int_{v_0}^{v} \frac{v\,dv}{-k v^2} = \int_0^x dx$

$\therefore \; x = -\frac{1}{k} \ln v \Big]_{v_0}^{v}$

$x = \frac{1}{k} \ln (v_0/v)$

if $v = v_0/2 \rightarrow x = D = \frac{1}{k} \ln 2 = \frac{0.693}{k}$

$v = \frac{dx}{dt} \rightarrow kx = \ln \frac{v_0}{v} \rightarrow v = v_0 e^{-kx}$

$\therefore \; \frac{dx}{v_0 e^{-kx}} = dt \rightarrow \int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx} \cdot dx$

$\therefore \; t = \frac{1}{v_0} \cdot \frac{1}{k} e^{kx} \Big]_0^x \rightarrow t = \frac{1}{v_0 k} \left[e^{kx} - 1 \right]$

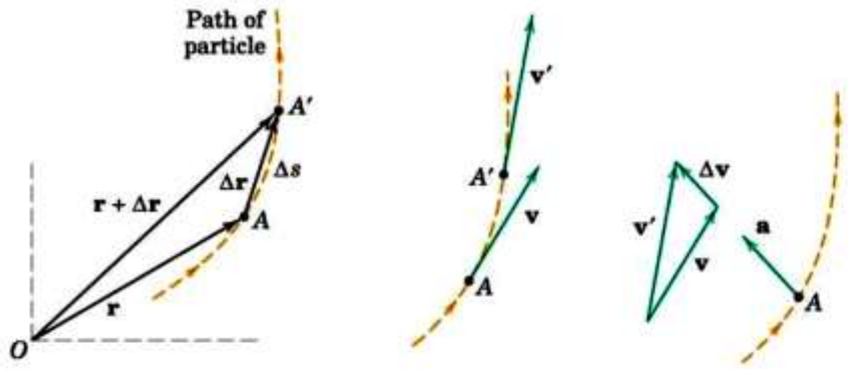
If $x = D$, $e^{kx} = 2 \quad \therefore \; t = \frac{1}{k v_0} [2-1] = \frac{1}{k v_0}$

1. Plane Curvilinear Motion

As mentioned previously, the vast majority of the motions of points or particles encountered in engineering practice can be represented as plane motion. What follows in this article constitutes one of the most basic concepts in dynamics, namely, *the time derivative of a vector*.

For a short time during take-off and landing, planes generally follow plane curvilinear motion

Consider now the continuous motion of a particle along a plane curve as represented in Fig. below. At time t the particle is at position A, which is located by the position vector r measured from some convenient fixed origin O. If both the magnitude and direction of r are known at time t, then the position of the particle is completely specified. At time t + Δt, the particle is at A', located by the position vector r + Δr. We note, of course, that this combination is vector addition and not scalar addition. The displacement of the particle during time At is the vector Δr which represents the vector change of position and is clearly independent of the choice of origin. if an origin were chosen at some different location, the position vector r would be changed, but Δr would be unchanged. The distance actually traveled by the particle as it moves along the path from A to A' is the scalar length As measured along the path. Thus, we distinguish between the vector displacement Δr and the scalar distance Δs.



- **Velocity**

During the time Δt, the average velocity of the particle is

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

The instantaneous velocity is determined from this equation by letting Δt → 0, and consequently the direction of Δr approaches the tangent to the curve. Hence,

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

We observe that the direction of Δr approaches that the tangent to the path as Δt approaches zero and, thus, the velocity v is always a vector tangent to the path:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

The vector is itself a vector having both a magnitude and a direction. The magnitude of v is called the *speed* and is the scalar.

- **Acceleration**

The average acceleration of the particle between A and A' is defined as Δv/Δt, which is a vector whose direction is that of Δv. The magnitude of this average acceleration is the magnitude of Δv divided by Δt.

The instantaneous acceleration a of the particle is defined as the limiting value of the average acceleration as the time interval approaches zero. Thus,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

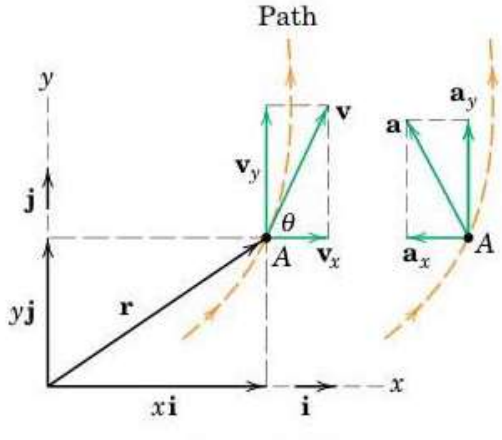
By definition of the derivative, then, we write

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$

2. **Rectangular coordinates**

Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its x, y, z coordinates.

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$
$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$



As we differentiate with respect to time, we observe that the time derivatives of the unit vectors are zero because their magnitudes and directions remain constant. The scalar values of the components of **v** and **a** are merely $v_x = \dot{x}$, $v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x}$, $a_y = \dot{v}_y = \ddot{y}$. (As drawn in Fig. a_x is in the negative x-direction, so that $\ddot{x}$ would be a negative number.)

As observed previously, the direction of the velocity is always tangent to the path, and from the figure it is clear that

$$v^2 = v_x^2 + v_y^2 \qquad v = \sqrt{v_x^2 + v_y^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = a_x^2 + a_y^2 \qquad a = \sqrt{a_x^2 + a_y^2}$$

Important Points

- Curvilinear motion can cause changes in *both* the magnitude and direction of the position, velocity, and acceleration vectors.
- The velocity vector is always directed *tangent* to the path.
- In general, the acceleration vector is *not* tangent to the path, but rather, it is tangent to the hodograph.
- If the motion is described using rectangular coordinates, then the components along each of the axes do not change direction, only their magnitude and sense (algebraic sign) will change.
- By considering the component motions, the change in magnitude and direction of the particle's position and velocity are automatically taken into account.

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. (See Appendix A for a full explanation.)

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6° \qquad \textit{Ans.}$$

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90° \qquad \textit{Ans.}$$

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (8t)^2/10 = 6.4t^2$ and then taking successive time derivatives.

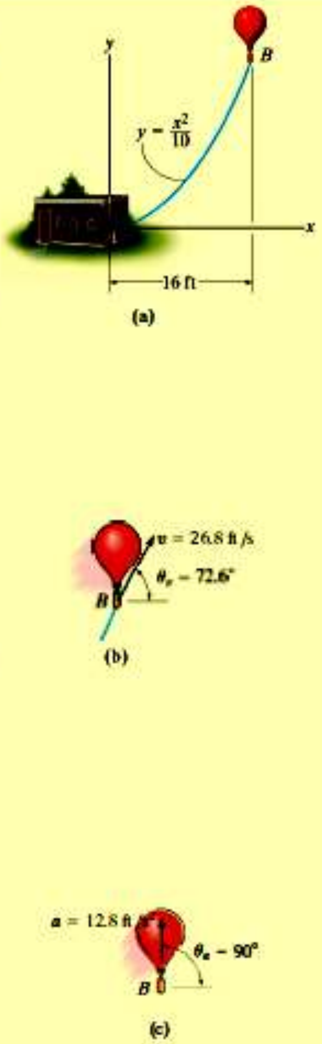


Fig. 12–18