Semester I (2019-2020)

Chapter Three
Kinetics of Particles

1. Introduction

According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces. Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion. In Chapter 3 we will study the kinetics of particles. This topic requires that we combine our knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion just covered in Chapter 2. With the aid of Newton's second law, we can combine these two topics and solve engineering problems involving force, mass, and motion. The three general approaches to the solution of kinetics problems are: (A) direct application of Newton's second law (called the force-mass-acceleration method), (B) use of work and energy principles, and (C) solution by impulse and momentum methods. Each approach has its special characteristics and advantages, and Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution. In addition, a fourth section, Section D, treats special applications and combinations of the three basic approaches. Before proceeding, you should review carefully the definitions and concepts of Chapter 1, because they are fundamental to the developments which follow.

2. Section A: Force, mass and acceleration

- **Newton's second law:**

The basic relation between force and acceleration is found in Newton's second law. The verification of which is entirely experimental. We subject a mass particle to the action of a single force F1, and we measure the acceleration al of the particle in the primary inertial system. We then repeat the experiment by subjecting the same particle to a different force F2 and measuring the corresponding acceleration a2. The experiment is repeated as many times as desired:

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C, \qquad \text{a constant}$$

We conclude that the constant C is a measure of some invariable property of the particle. This property is the ***inertia of the particle***, which is its resistance to rate of change of velocity.

Then:

$$C = km$$

where k is a constant introduced to account for the units used. Thus, we may express the relation obtained from the experiments as

$$\mathbf{F} = km\mathbf{a} \qquad \textbf{(3/2)}$$

If k = 1

F= m a

The acceleration is always in the direction of the force.

- **Inertial system**

Although the results of the ideal experiment are obtained for measurements made relative to the "fixed" primary inertial system, they are equally valid for measurements made with respect to any nonrotating reference system which translates with a constant velocity with respect to the primary system. From our study of relative motion, we know that the acceleration measured in a system translating with no acceleration is the same as that measured in the primary system. Thus, Newton's second law holds equally well in a nonaccelerating system, so that we may define an inertial system as any system in which Eq. 3/2 is valid. If the ideal experiment described were performed on the surface of the earth and all measurements were made relative to a reference system attached to the earth, the measured results would show a slight discrepancy from those predicted by Eq. 3/2, because the measured acceleration would not be the correct absolute acceleration. The discrepancy would disappear when we introduced the correction duo to the acceleration components of the earth. These corrections are negligible for most engineering problems which involve the motions of structures and machines on the surface of the earth. In such cases, the accelerations measured with respect to reference axes attached to the surface of the earth may be treated as "absolute," and Eq. 3/2 may be applied with negligible error to experiments made on the surface of the earth.

*As an example of the magnitude of the error introduced by neglect of the motion of the earth, consider a particle which is allowed to fall from rest (relative to earth) at a height h above the ground. We can show that the rotation of the earth gives rise to an eastward acceleration (Coriolis acceleration) relative to the earth and, neglecting air resistance, that the particle falls to the ground a distance

$$x = \frac{2}{3}\omega\sqrt{\frac{2h^3}{g}}\cos\gamma$$

east of the point on the ground directly under that from which it was dropped. The angular velocity of the earth is $\omega = 0.729(10^{-4})$ rad/s, and the latitude, north or south, is γ. At a latitude of 45° and from a height of 200 m, this eastward deflection would be $x = 43.9$ mm.

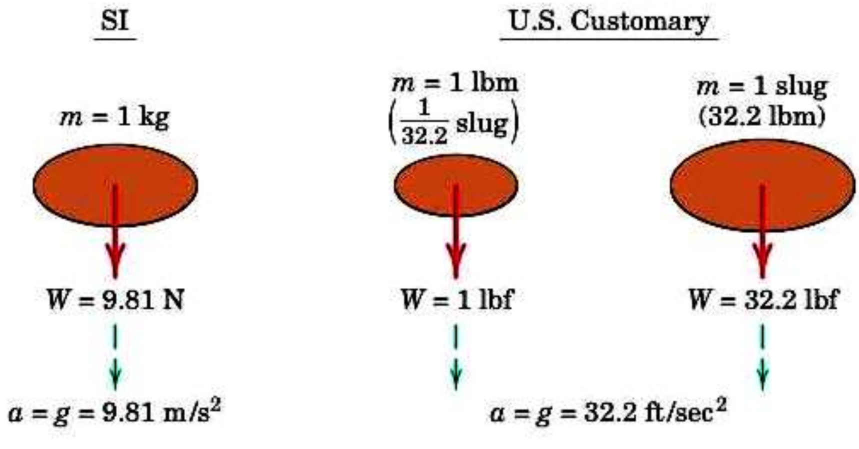
An increasing number of problems occur, particularly in the fields of rocket and spacecraft design, where the acceleration components of the earth are of primary concern. For this work it is essential that the fundamental basis of Newton's second law by thoroughly understood and that the appropriate absolute acceleration components must be employed.

The concept of time, considered an absolute quantity in the Newtonian theory, received a basically different interpretation in the theory of relativity announced by Einstein in 1905. The new concept called for a complete reformulation of the accepted laws of mechanics. The theory of relativity was subjected to early ridicule, but has been verified by experiment and is now universally accepted by scientists. Although the difference between the mechanics of Newton and that of Einstein is basic, there is a practical difference in the results given by the two theories only when velocities of the order of the speed of light ($300 \text{ X } 10^6$ m/s) are encountered*.

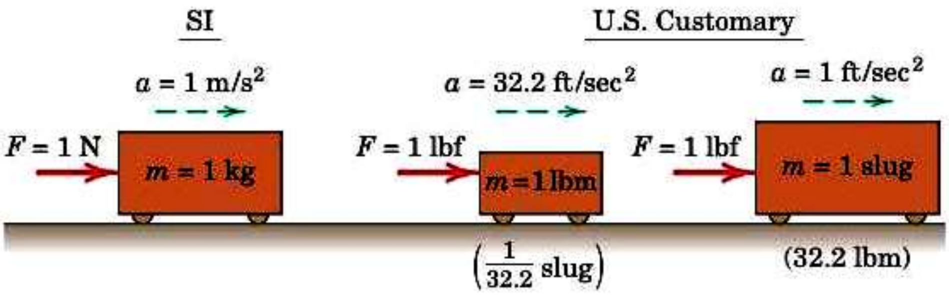
The theory of relativity demonstrates that there is no such thing as a preferred primary inertial system and that measurements of time made in two coordinate systems which have a velocity relative to one another are different. On this basis, for example, the principles of relativity show that a clock carried by the pilot of a spacecraft traveling around the earth in a circular polar orbit of 644 km altitude at a velocity of 27 080 km/h would be slow compared with a clock at the pole by 0.000 001 85 sec for each orbit.

Important problems dealing with atomic and nuclear particles, for example, require calculations based on the theory of relativity.

Force and Mass Units



(a) Gravitational Free-Fall



(b) Newton's Second Law

- **Rectilinear Motion**

We will analyze the motions of bodies which can be treated as particles. This simplification is possible as long as we are interested only in the motion of the mass center of the body. In this case we may treat the forces as concurrent through the mass center.

If we choose the x-direction, for example, as the direction of the rectilinear motion of a particle of mass *m*, the acceleration in the y- and z-directions will be zero and the scalar components of Newton's eq. become

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = 0 \qquad \textbf{(3/4)}$$
$$\Sigma F_z = 0$$

For cases where we are not free to choose a coordinate direction along the motion, we would have in the general case all three component equations

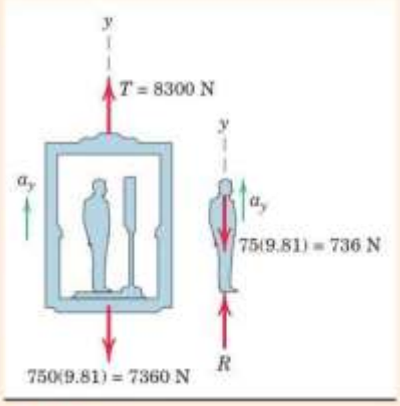
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y \qquad \textbf{(3/5)}$$
$$\Sigma F_z = ma_z$$

Where:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$
$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}}$$
$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$
$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

Sample Problem 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.



Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$[\Sigma F_y = ma_y] \qquad 8300 - 7360 = 750a_y \qquad a_y = 1.257 \text{ m/s}^2$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

① $[\Sigma F_y = ma_y] \qquad R - 736 = 75(1.257) \qquad R = 830 \text{ N}$ *Ans.*

The velocity reached at the end of the 3 seconds is

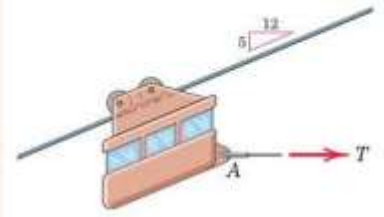
$[\Delta v = \int a\,dt] \qquad v - 0 = \int_0^3 1.257\,dt \qquad v = 3.77 \text{ m/s}$ *Ans.*

Helpful Hint

① If the scale were calibrated in kilograms it would read 830/9.81 = 84.6 kg which, of course, is not his true mass since the measurement was made in a noninertial (accelerating) system. *Suggestion:* Rework this problem in U.S. customary units.

Sample Problem 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.



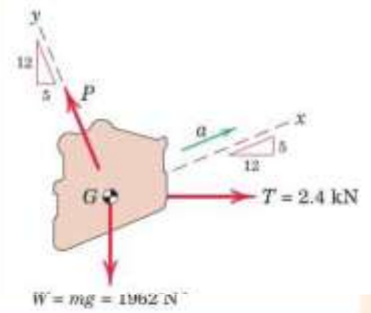
Solution. The free-body diagram of the car and wheels taken together and treated as a particle discloses the 2.4-kN tension T, the weight $W = mg = 200(9.81) = 1962$ N, and the force P exerted on the wheel assembly by the cable.

The car is in equilibrium in the y-direction since there is no acceleration in this direction. Thus,

$[\Sigma F_y = 0] \qquad P - 2.4(\frac{5}{13}) - 1.962(\frac{12}{13}) = 0 \qquad P = 2.73 \text{ kN}$ *Ans.*

① In the x-direction the equation of motion gives

$[\Sigma F_x = ma_x] \qquad 2400(\frac{12}{13}) - 1962(\frac{5}{13}) = 200a \qquad a = 7.30 \text{ m/s}^2$ *Ans.*

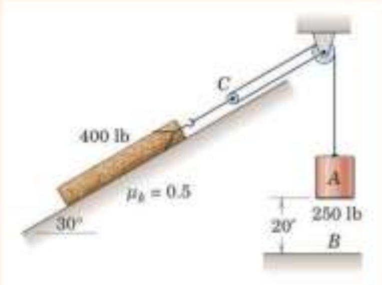


Helpful Hint

① By choosing our coordinate axes along and normal to the direction of the acceleration, we are able to solve the two equations independently. Would this be so if x and y were chosen as horizontal and vertical?

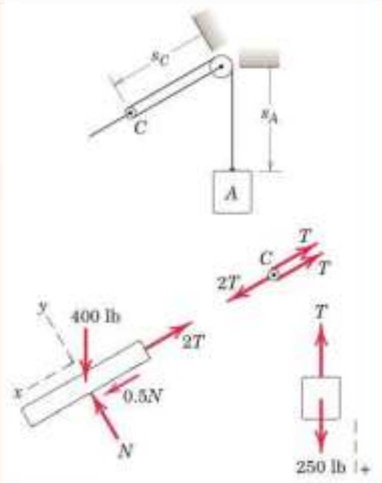
Sample Problem 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.



Solution. The motions of the log and the block A are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of A, we may prove it formally. The constant total length of the cable is $L = 2s_C + s_A +$ constant, where the constant accounts
① for the cable portions wrapped around the pulleys. Differentiating twice with respect to time gives $0 = 2\ddot{s}_C + \ddot{s}_A$, or

$$0 = 2a_C + a_A$$



We assume here that the masses of the pulleys are negligible and that they turn with negligible friction. With these assumptions the free-body diagram of the pulley C discloses force and moment equilibrium. Thus, the tension in the cable attached to the log is twice that applied to the block. Note that the accelerations of the log and the center of pulley C are identical.

The free-body diagram of the log shows the friction force $\mu_k N$ for motion up the plane. Equilibrium of the log in the y-direction gives

② $[\Sigma F_y = 0]$ $\qquad N - 400 \cos 30° = 0 \qquad N = 346$ lb

and its equation of motion in the x-direction gives

$[\Sigma F_x = ma_x]$ $\qquad 0.5(346) - 2T + 400 \sin 30° = \frac{400}{32.2} a_C$

For the block in the positive downward direction, we have

③ $[+\downarrow \Sigma F = ma]$ $\qquad 250 - T = \frac{250}{32.2} a_A$

Solving the three equations in a_C, a_A, and T gives us

$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

④ For the 20-ft drop with constant acceleration, the block acquires a velocity

$[v^2 = 2ax]$ $\qquad v_A = \sqrt{2(5.83)(20)} = 15.27$ ft/sec $\qquad$ *Ans.*

Helpful Hints

① The coordinates used in expressing the final kinematic constraint relationship must be consistent with those used for the kinetic equations of motion.

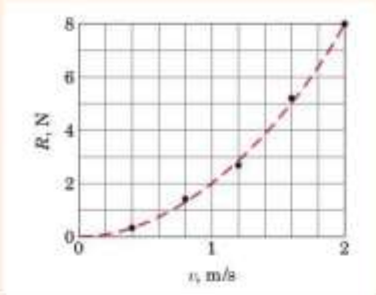
② We can verify that the log will indeed move up the ramp by calculating the force in the cable necessary to initiate motion from the equilibrium condition. This force is $2T = 0.5N + 400 \sin 30° = 373$ lb or $T = 186.5$ lb, which is less than the 250-lb weight of block A. Hence, the log will move up.

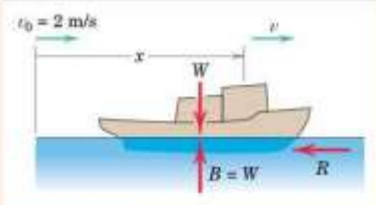
③ Note the serious error in assuming that $T = 250$ lb, in which case, block A would not accelerate.

④ Because the forces on this system remain constant, the resulting accelerations also remain constant.

Sample Problem 3/4

The design model for a new ship has a mass of 10 kg and is tested in an experimental towing tank to determine its resistance to motion through the water at various speeds. The test results are plotted on the accompanying graph, and the resistance R may be closely approximated by the dashed parabolic curve shown. If the model is released when it has a speed of 2 m/s, determine the time t required for it to reduce its speed to 1 m/s and the corresponding travel distance x.





Solution. We approximate the resistance-velocity relation by $R = kv^2$ and find k by substituting $R = 8$ N and $v = 2$ m/s into the equation, which gives $k = 8/2^2 = 2\ \text{N}\cdot\text{s}^2/\text{m}^2$. Thus, $R = 2v^2$.

The only horizontal force on the model is R, so that

① $[\Sigma F_x = ma_x]$ $\qquad -R = ma_x \quad$ or $\quad -2v^2 = 10\dfrac{dv}{dt}$

We separate the variables and integrate to obtain

$$\int_0^t dt = -5\int_2^v \frac{dv}{v^2} \qquad t = 5\left(\frac{1}{v} - \frac{1}{2}\right)\ \text{s}$$

Thus, when $v = v_0/2 = 1$ m/s, the time is $t = 5(\frac{1}{1} - \frac{1}{2}) = 2.5$ s. *Ans.*

The distance traveled during the 2.5 seconds is obtained by integrating $v = dx/dt$. Thus, $v = 10/(5 + 2t)$ so that

② $$\int_0^x dx = \int_0^{2.5} \frac{10}{5 + 2t}\,dt \qquad x = \frac{10}{2}\ln(5 + 2t)\Big|_0^{2.5} = 3.47\ \text{m}$$ *Ans.*

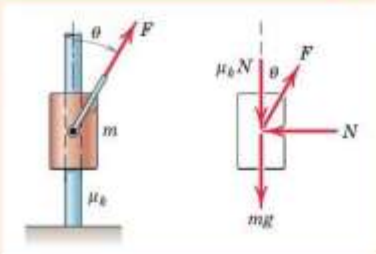
Helpful Hints

① Be careful to observe the minus sign for R.

② *Suggestion:* Express the distance x after release in terms of the velocity v and see if you agree with the resulting relation $x = 5\ln(v_0/v)$.

Sample Problem 3/5

The collar of mass m slides up the vertical shaft under the action of a force F of constant magnitude but variable direction. If $\theta = kt$ where k is a constant and if the collar starts from rest with $\theta = 0$, determine the magnitude F of the force which will result in the collar coming to rest as θ reaches $\pi/2$. The coefficient of kinetic friction between the collar and shaft is μ_k.



Solution. After drawing the free-body diagram, we apply the equation of motion in the y-direction to get

① $[\Sigma F_y = ma_y]$ $\qquad F\cos\theta - \mu_k N - mg = m\dfrac{dv}{dt}$

where equilibrium in the horizontal direction requires $N = F\sin\theta$. Substituting $\theta = kt$ and integrating first between general limits give

$$\int_0^t (F\cos kt - \mu_k F\sin kt - mg)\,dt = m\int_0^v dv$$

which becomes

$$\frac{F}{k}[\sin kt + \mu_k(\cos kt - 1)] - mgt = mv$$

For $\theta = \pi/2$ the time becomes $t = \pi/2k$, and $v = 0$ so that

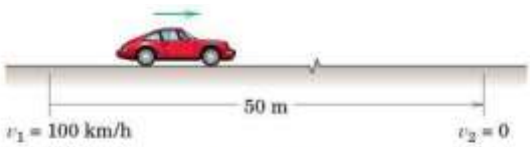
② $$\frac{F}{k}[1 + \mu_k(0 - 1)] - \frac{mg\pi}{2k} = 0 \qquad \text{and} \qquad F = \frac{mg\pi}{2(1 - \mu_k)}$$ *Ans.*

Helpful Hints

① If θ were expressed as a function of the vertical displacement y instead of the time t, the acceleration would become a function of the displacement and we would use $v\,dv = a\,dy$.

② We see that the results do not depend on k, the rate at which the force changes direction.

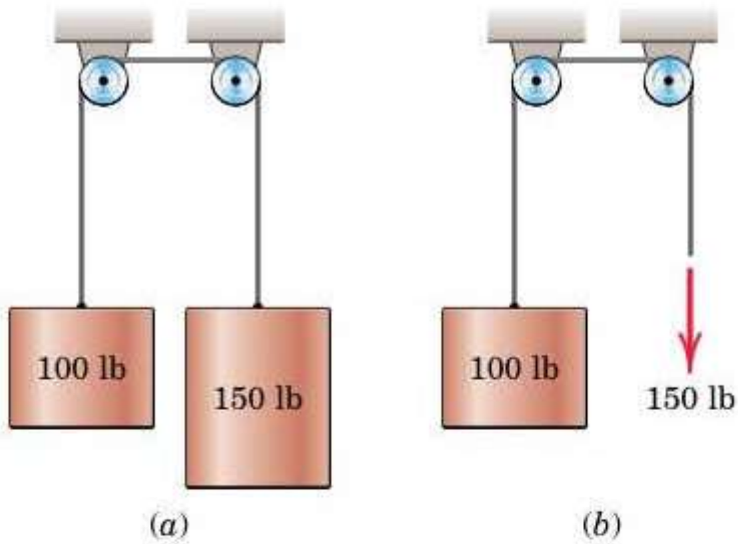
3/1: During a brake test, the rear-engine car is stopped from an initial speed of 100 km/h in a distance of 50 m. If it is known that all four wheels contribute equally to the braking force, determine the braking force F at each wheel. Assume a constant deceleration for the 1500-kg car. Ans. F = 2890 N



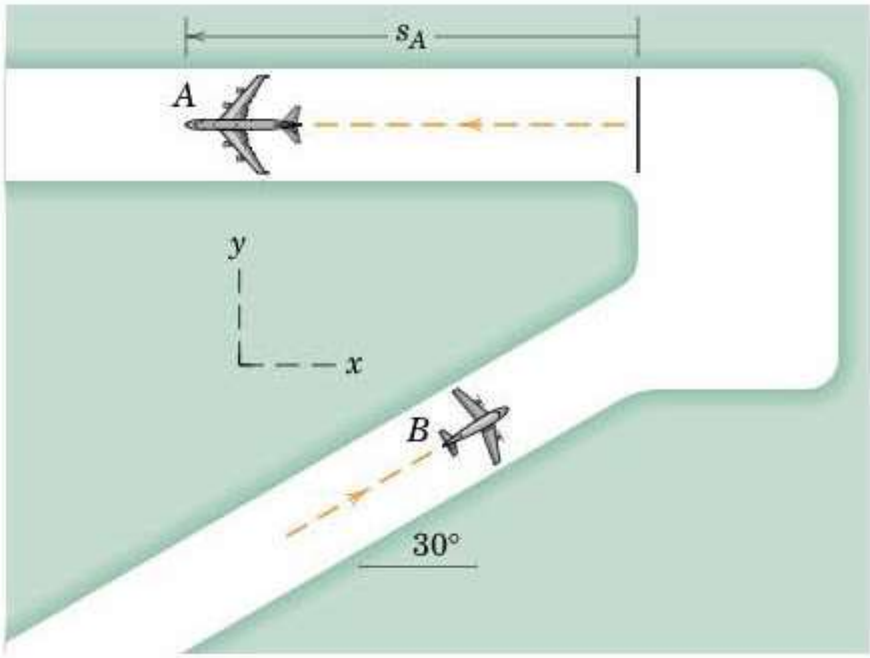
Problem 3/1

3/2: The 50-kg crate is stationary when the force P is applied. Determine the resulting acceleration of the crate if (a) P = 0, (b) P = 150 N, and (c) P = 300 N.

Problem 3/2

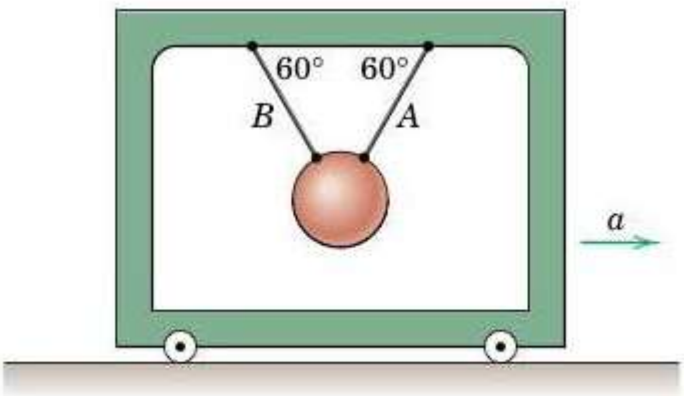


Problem 3/7



Problem 3/10

Ans. $a =$ $3\sqrt{}$



Problem 3/17