

Semester I (2019-2020)

• **Curvilinear motion**

The choice of an appropriate coordinate system depends on the conditions of the problem and is one of the basic decisions to be made in solving curvilinear motion problems.

Rectangular coordinates

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \end{aligned} \quad (3/6)$$

where $a_x = \ddot{x}$ and $a_y = \ddot{y}$

Normal and tangential coordinates (Art. 2/5, Fig. 2/10)

$$\begin{aligned} \Sigma F_n &= ma_n \\ \Sigma F_t &= ma_t \end{aligned} \quad (3/7)$$

where $a_n = \rho \dot{\beta}^2 = v^2/\rho = v \dot{\beta}$, $a_t = \dot{v}$, and $v = \rho \dot{\beta}$

where $a_n = \rho \dot{\beta}^2 = v^2/\rho = v \dot{\beta}$, $a_t = \dot{v}$, and $v = \rho \dot{\beta}$

Polar coordinates (Art. 2/6, Fig. 2/15)

$$\begin{aligned} \Sigma F_r &= ma_r \\ \Sigma F_\theta &= ma_\theta \end{aligned} \quad (3/8)$$

where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Semester I (2019-2020)

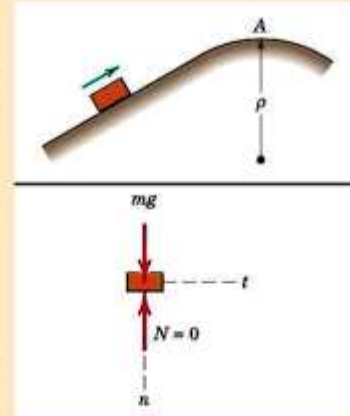
Sample Problem 3/6

Determine the maximum speed v which the sliding block may have as it passes point A without losing contact with the surface.

Solution. The condition for loss of contact is that the normal force N which the surface exerts on the block goes to zero. Summing forces in the normal direction gives

$$[\Sigma F_n = ma_n] \quad mg = m \frac{v^2}{\rho} \quad v = \sqrt{g\rho} \quad \text{Ans.}$$

If the speed at A were less than $\sqrt{g\rho}$, then an upward normal force exerted by the surface on the block would exist. In order for the block to have a speed at A which is greater than $\sqrt{g\rho}$, some type of constraint, such as a second curved surface above the block, would have to be introduced to provide additional downward force.



Sample Problem 3/7

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B . Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.

Solution. The free-body diagram of the object is shown together with the coordinate directions n and t . The normal force N depends on the n -component of the acceleration which, in turn, depends on the velocity. The velocity will be cumulative according to the tangential acceleration a_t . Hence, we will find a_t first for any general position.

$$[\Sigma F_t = ma_t] \quad mg \cos \theta = ma_t \quad a_t = g \cos \theta$$

① Now we can find the velocity by integrating

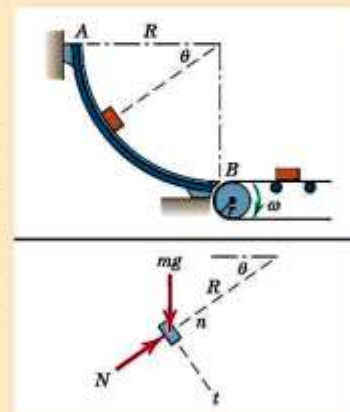
$$[v dv = a_t ds] \quad \int_0^v v dv = \int_0^\theta g \cos \theta d(R\theta) \quad v^2 = 2gR \sin \theta$$

We obtain the normal force by summing forces in the positive n -direction, which is the direction of the n -component of acceleration.

$$[\Sigma F_n = ma_n] \quad N - mg \sin \theta = m \frac{v^2}{R} \quad N = 3mg \sin \theta \quad \text{Ans.}$$

The conveyor pulley must turn at the rate $v = r\omega$ for $\theta = \pi/2$, so that

$$\omega = \sqrt{2gR}/r \quad \text{Ans.}$$



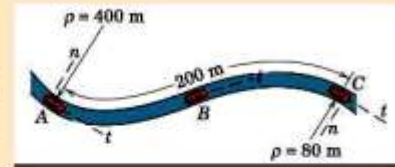
Helpful Hint

① It is essential here that we recognize the need to express the tangential acceleration as a function of position so that v may be found by integrating the kinematical relation $v dv = a_t ds$, in which all quantities are measured along the path.

Semester I (2019-2020)

Sample Problem 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at A to a speed of 50 km/h as it passes C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. Determine the total horizontal force exerted by the road on the tires at positions A, B, and C. Point B is the inflection point where the curvature changes direction.



Solution. The car will be treated as a particle so that the effect of all forces exerted by the road on the tires will be treated as a single force. Since the motion is described along the direction of the road, normal and tangential coordinates will be used to specify the acceleration of the car. We will then determine the forces from the accelerations.

The constant tangential acceleration is in the negative t -direction, and its magnitude is given by

$$① \quad [v_C^2 = v_A^2 + 2a_t \Delta s] \quad a_t = \left| \frac{(50/3.6)^2 - (100/3.6)^2}{2(200)} \right| = 1.447 \text{ m/s}^2$$

The normal components of acceleration at A, B, and C are

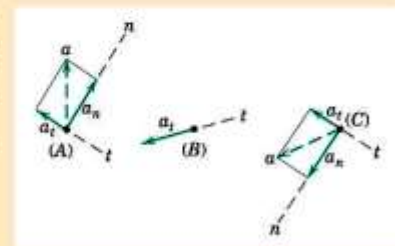
$$② \quad [a_n = v^2/\rho] \quad \begin{array}{ll} \text{At A,} & a_n = \frac{(100/3.6)^2}{400} = 1.929 \text{ m/s}^2 \\ \text{At B,} & a_n = 0 \\ \text{At C,} & a_n = \frac{(50/3.6)^2}{80} = 2.41 \text{ m/s}^2 \end{array}$$

Application of Newton's second law in both the n - and t -directions to the free-body diagrams of the car gives

$$\begin{array}{ll} [\Sigma F_t = ma_t] & F_t = 1500(1.447) = 2170 \text{ N} \\ ③ \quad [\Sigma F_n = ma_n] & \text{At A,} \quad F_n = 1500(1.929) = 2890 \text{ N} \\ & \text{At B,} \quad F_n = 0 \\ & \text{At C,} \quad F_n = 1500(2.41) = 3620 \text{ N} \end{array}$$

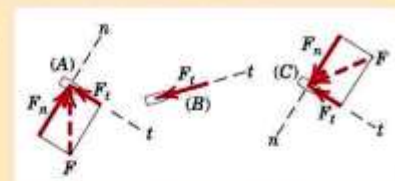
Thus, the total horizontal force acting on the tires becomes

$$\begin{array}{ll} \text{At A,} & F = \sqrt{F_n^2 + F_t^2} = \sqrt{(2890)^2 + (2170)^2} = 3620 \text{ N} \quad \text{Ans.} \\ \text{At B,} & F = F_t = 2170 \text{ N} \quad \text{Ans.} \\ ④ \quad \text{At C,} & F = \sqrt{F_n^2 + F_t^2} = \sqrt{(3620)^2 + (2170)^2} = 4220 \text{ N} \quad \text{Ans.} \end{array}$$



Helpful Hints

- ① Recognize the numerical value of the conversion factor from km/h to m/s as 1000/3600 or 1/3.6.
- ② Note that a_n is always directed toward the center of curvature.



- ③ Note that the direction of F_n must agree with that of a_n .

- ④ The angle made by \mathbf{a} and \mathbf{F} with the direction of the path can be computed if desired.

Semester I (2019-2020)

Sample Problem 3/9

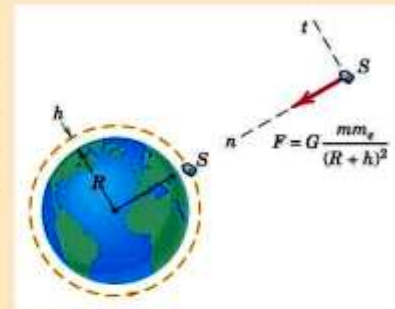
Compute the magnitude v of the velocity required for the spacecraft S to maintain a circular orbit of altitude 200 mi above the surface of the earth.

- ① **Solution.** The only external force acting on the spacecraft is the force of gravitational attraction to the earth (i.e., its weight), as shown in the free-body diagram. Summing forces in the normal direction yields

$$[\Sigma F_n = ma_n] \quad G \frac{mm_e}{(R+h)^2} = m \frac{v^2}{(R+h)}, \quad v = \sqrt{\frac{Gm_e}{(R+h)}} = R \sqrt{\frac{g}{(R+h)}}$$

where the substitution $gR^2 = Gm_e$ has been made. Substitution of numbers gives

$$v = (3959)(5280) \sqrt{\frac{32.234}{(3959 + 200)(5280)}} = 25,326 \text{ ft/sec} \quad \text{Ans.}$$



Helpful Hint

- ① Note that, for observations made within an inertial frame of reference, there is no such quantity as “centrifugal force” acting in the minus n -direction. Note also that neither the spacecraft nor its occupants are “weightless,” because the weight in each case is given by Newton’s law of gravitation. For this altitude, the weights are only about 10 percent less than the earth-surface values. Finally, the term “zero- g ” is also misleading. It is only when we make our observations with respect to a coordinate system which has an acceleration equal to the gravitational acceleration (such as in an orbiting spacecraft) that we appear to be in a “zero- g ” environment. The quantity which does go to zero aboard orbiting spacecraft is the familiar normal force associated with, for example, an object in contact with a horizontal surface within the spacecraft.

Sample Problem 3/10

Tube A rotates about the vertical O -axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b . Determine the tension T in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.

Solution. With r a variable, we use the polar-coordinate form of the equations of motion, Eqs. 3/8. The free-body diagram of B is shown in the horizontal plane and discloses only T and F_θ . The equations of motion are

$$[\Sigma F_r = ma_r] \quad -T = m(\ddot{r} - r\dot{\theta}^2)$$

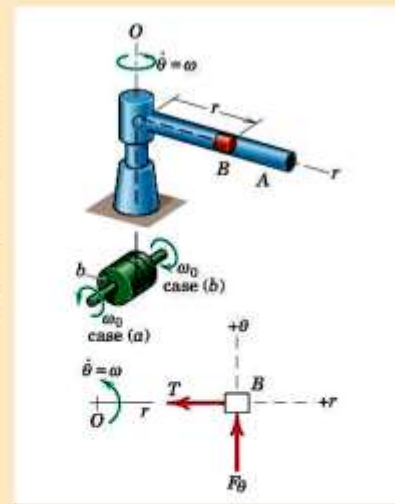
$$[\Sigma F_\theta = ma_\theta] \quad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Case (a). With $\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \quad F_\theta = 2mb\omega_0\omega \quad \text{Ans.}$$

- ① **Case (b).** With $\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \quad F_\theta = -2mb\omega_0\omega \quad \text{Ans.}$$



Helpful Hint

- ① The minus sign shows that F_θ is in the direction opposite to that shown on the free-body diagram.