1. Power

The capacity of a machine is measured by the time rate at which it can do work or deliver energy. The total work or energy output is not a measure of this capacity since a motor, no matter how small, can deliver a large amount of energy if given sufficient time. On the other hand, a large and powerful machine is required to deliver a large amount of energy in a short period of time. Thus, the capacity of a machine is rated by its **power**, **which is defined as the time rate of doing work.**

Accordingly, the power P developed by a force $\mathbf{F}$ which does an amount of work U is $P = dU/dt = \mathbf{F} \cdot d\mathbf{r}/dt$. Because $d\mathbf{r}/dt$ is the velocity $\mathbf{v}$ of the point of application of the force, we have

$$P = \mathbf{F} \cdot \mathbf{v} \qquad \textbf{(3/16)}$$

Power is clearly a scalar quantity, and in SI it has the units of N·m/s = J/s. The special unit for power is the *watt* (W), which equals one joule per second (J/s). In U.S. customary units, the unit for mechanical power is the *horsepower* (hp). These units and their numerical equivalences are

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ hp} = 550 \text{ ft-lb/sec} = 33{,}000 \text{ ft-lb/min}$$

$$1 \text{ hp} = 746 \text{ W} = 0.746 \text{ kW}$$

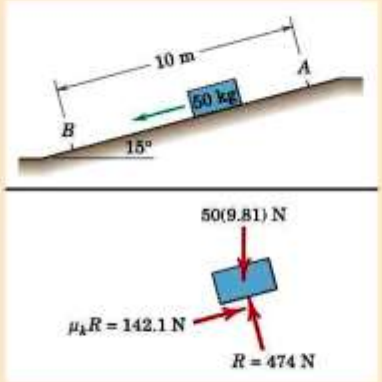
2. Efficiency

The ratio of the work done by a machine to the work done on. the machine during the same time interval is called the mechanical efficiency of the machine.

$$e_m = \frac{P_{output}}{P_{input}} \qquad (3/17)$$

Sample Problem 3/11

Calculate the velocity v of the 50-kg crate when it reaches the bottom of the chute at B if it is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.30.



Solution. The free-body diagram of the crate is drawn and includes the normal force R and the kinetic friction force F calculated in the usual manner. The work done by the weight is positive, whereas that done by the friction force is negative. The total work done on the crate during the motion is

① $[U = Fs]$ $\qquad U_{1\text{-}2} = 50(9.81)(10 \sin 15°) - 142.1(10) = -151.9 \text{ J}$

The work-energy equation gives

$[T_1 + U_{1\text{-}2} = T_2]$ $\qquad \frac{1}{2}mv_1^2 + U_{1\text{-}2} = \frac{1}{2}mv_2^2$

$$\frac{1}{2}(50)(4)^2 - 151.9 = \frac{1}{2}(50)v_2^2$$

$$v_2 = 3.15 \text{ m/s} \qquad \textit{Ans.}$$

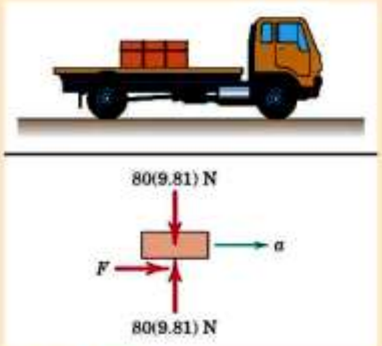
Since the net work done is negative, we obtain a decrease in the kinetic energy.

Helpful Hint

① The work due to the weight depends only on the *vertical* distance traveled.

Sample Problem 3/12

The flatbed truck, which carries an 80-kg crate, starts from rest and attains a speed of 72 km/h in a distance of 75 m on a level road with constant acceleration. Calculate the work done by the friction force acting on the crate during this interval if the static and kinetic coefficients of friction between the crate and the truck bed are (*a*) 0.30 and 0.28, respectively, or (*b*) 0.25 and 0.20, respectively.



Solution. If the crate does not slip on the bed, its acceleration will be that of the truck, which is

$[v^2 = 2as]$ $\qquad a = \frac{v^2}{2s} = \frac{(72/3.6)^2}{2(75)} = 2.67 \text{ m/s}^2$

Case (a). This acceleration requires a friction force on the block of

$[F = ma]$ $\qquad F = 80(2.67) = 213 \text{ N}$

which is less than the maximum possible value of $\mu_s N = 0.30(80)(9.81) = 235$ N. Therefore, the crate does not slip and the work done by the actual static friction force of 213 N is

① $[U = Fs]$ $\qquad U_{1\text{-}2} = 213(75) = 16\,000 \text{ J} \quad$ or $\quad 16 \text{ kJ} \qquad$ *Ans.*

Case (b). For $\mu_s = 0.25$, the maximum possible friction force is $0.25(80)(9.81) = 196.2$ N, which is slightly less than the value of 213 N required for no slipping. Therefore, we conclude that the crate slips, and the friction force is governed by the kinetic coefficient and is $F = 0.20(80)(9.81) = 157.0$ N. The acceleration becomes

$[F = ma]$ $\qquad a = F/m = 157.0/80 = 1.962 \text{ m/s}^2$

The distances traveled by the crate and the truck are in proportion to their accelerations. Thus, the crate has a displacement of $(1.962/2.67)75 = 55.2$ m, and the work done by kinetic friction is

② $[U = Fs]$ $\qquad U_{1\text{-}2} = 157.0(55.2) = 8660 \text{ J} \quad$ or $\quad 8.66 \text{ kJ} \qquad$ *Ans.*

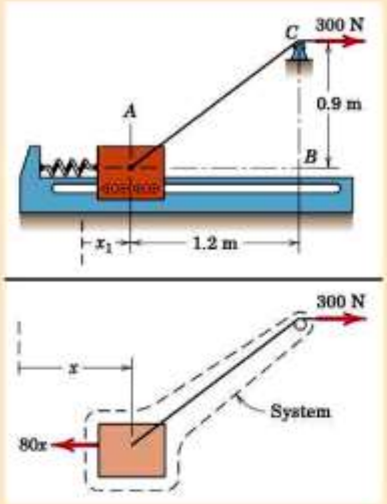
Helpful Hints

① We note that static friction forces do no work when the contacting surfaces are both at rest. When they are in motion, however, as in this problem, the static friction force acting on the crate does positive work and that acting on the truck bed does negative work.

② This problem shows that a kinetic friction force can do positive work when the surface which supports the object and generates the friction force is in motion. If the supporting surface is at rest, then the kinetic friction force acting on the moving part always does negative work.

Sample Problem 3/13

The 50-kg block at A is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300-N force in the cable. The block is released from rest at A, with the spring to which it is attached extended an initial amount $x_1 = 0.233$ m. The spring has a stiffness $k = 80$ N/m. Calculate the velocity v of the block as it reaches position B.



Solution. It will be assumed initially that the stiffness of the spring is small enough to allow the block to reach position B. The active-force diagram for the system composed of both block and cable is shown for a general position. The spring force $80x$ and the 300-N tension are the only forces external to this system which do work on the system. The force exerted on the block by the rail, the weight of the block, and the reaction of the small pulley on the cable do no work on the system and are not included on the active-force diagram.

As the block moves from $x_1 = 0.233$ m to $x_2 = 0.233 + 1.2 = 1.433$ m, the work done by the spring force acting on the block is

① $[U_{1\text{-}2} = \frac{1}{2}k(x_1{}^2 - x_2{}^2)]$ $\quad U_{1\text{-}2} = \frac{1}{2}80[0.233^2 - (0.233 + 1.2)^2]$

$$= -80.0 \text{ J}$$

The work done on the system by the constant 300-N force in the cable is the force times the net horizontal movement of the cable over pulley C, which is $\sqrt{(1.2)^2 + (0.9)^2} - 0.9 = 0.6$ m. Thus, the work done is $300(0.6) = 180$ J. We now apply the work-energy equation to the system and get

$[T_1 + U_{1\text{-}2} = T_2]$ $\quad 0 - 80.0 + 180 = \frac{1}{2}(50)v^2 \qquad v = 2.00$ m/s $\qquad$ *Ans.*

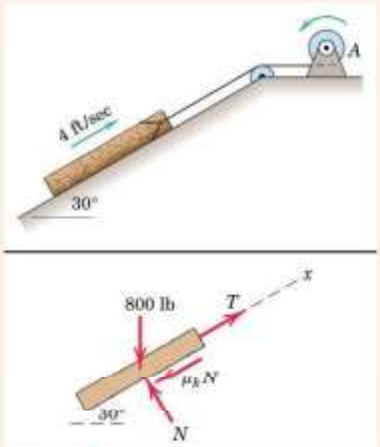
We take special note of the advantage to our choice of system. If the block alone had constituted the system, the horizontal component of the 300-N cable tension on the block would have to be integrated over the 1.2-m displacement. This step would require considerably more effort than was needed in the solution as presented. If there had been appreciable friction between the block and its guiding rail, we would have found it necessary to isolate the block alone in order to compute the variable normal force and, hence, the variable friction force. Integration of the friction force over the displacement would then be required to evaluate the negative work which it would do.

Helpful Hint

① Recall that this general formula is valid for any initial and final spring deflections x_1 and x_2, positive (spring in tension) or negative (spring in compression). In deriving the spring-work formula, we assumed the spring to be linear, which is the case here.

Sample Problem 3/14

The power winch A hoists the 800-lb log up the 30° incline at a constant speed of 4 ft/sec. If the power output of the winch is 6 hp, compute the coefficient of kinetic friction μ_k between the log and the incline. If the power is suddenly increased to 8 hp, what is the corresponding instantaneous acceleration a of the log?



Solution. From the free-body diagram of the log, we get $N = 800 \cos 30° = 693$ lb, and the kinetic friction force becomes $693\mu_k$. For constant speed, the forces are in equilibrium so that

$[\Sigma F_x = 0]$ $\quad T - 693\mu_k - 800 \sin 30° = 0 \qquad T = 693\mu_k + 400$

The power output of the winch gives the tension in the cable

① $[P = Tv]$ $\qquad T = P/v = 6(550)/4 = 825$ lb

Substituting T gives

$825 = 693\mu_k + 400 \qquad \mu_k = 0.613$ $\qquad$ *Ans.*

When the power is increased, the tension momentarily becomes

$[P = Tv]$ $\qquad T = P/v = 8(550)/4 = 1100$ lb

and the corresponding acceleration is given by

$[\Sigma F_x = ma_x]$ $\qquad 1100 - 693(0.613) - 800 \sin 30° = \frac{800}{32.2}a$

② $\qquad a = 11.07 \text{ ft/sec}^2$ $\qquad$ *Ans.*

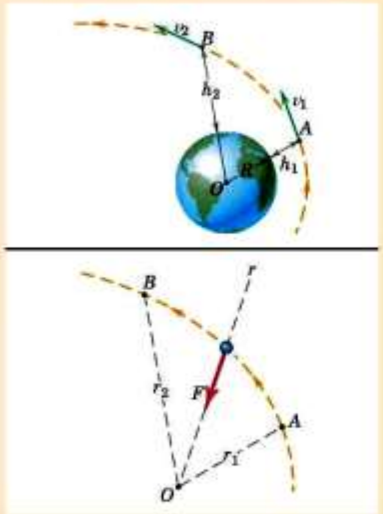
Helpful Hints

① Note the conversion from horsepower to ft-lb/sec.

② As the speed increases, the acceleration will drop until the speed stabilizes at a value higher than 4 ft/sec.

Sample Problem 3/15

A satellite of mass m is put into an elliptical orbit around the earth. At point A, its distance from the earth is $h_1 = 500$ km and it has a velocity $v_1 = 30\,000$ km/h. Determine the velocity v_2 of the satellite as it reaches point B, a distance $h_2 = 1200$ km from the earth.



Solution. The satellite is moving outside of the earth's atmosphere so that the only force acting on it is the gravitational attraction of the earth. For the large change in altitude of this problem, we cannot assume that the acceleration due to gravity is constant. Rather, we must use the work expression, derived in this article, which accounts for variation in the gravitational acceleration with altitude. Put another way, the work expression accounts for the variation of the weight $F = \frac{Gmm_e}{r^2}$ with altitude. This work expression is

$$U_{1\text{-}2} = mgR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

The work-energy equation $T_1 + U_{1\text{-}2} = T_2$ gives

① ② $$\frac{1}{2}mv_1^2 + mgR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}mv_2^2 \qquad v_2^2 = v_1^2 + 2gR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Substituting the numerical values gives

$$v_2^2 = \left(\frac{30\,000}{3.6}\right)^2 + 2(9.81)[(6371)(10^3)]^2\left(\frac{10^{-3}}{6371 + 1200} - \frac{10^{-3}}{6371 + 500}\right)$$

$$= 69.44(10^6) - 10.72(10^6) = 58.73(10^6)\ (\text{m/s})^2$$

$$v_2 = 7663 \text{ m/s} \qquad \text{or} \qquad v_2 = 7663(3.6) = 27\,590 \text{ km/h}$$ *Ans.*

Helpful Hints

① Note that the result is independent of the mass m of the satellite.

② Consult Table D/2, Appendix D, to find the radius R of the earth.