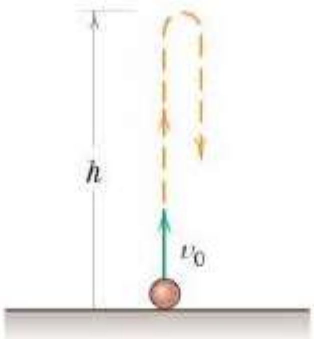
Semester I (2019-2020)

Use the work-energy method to develop an expression for the maximum height attained by a projectile which is launched with initial speed v_0 from ground level. Evaluate your expression for $v_0 = 50$ m/s. Assume a constant gravitational acceleration and neglect air resistance.

Ans. $h = \dfrac{v_0^{\,2}}{2g}$, $h = 127.4$ m



State ① : launch; State ②: apex

$T_1 + U_{1-2} = T_2 : \frac{1}{2}mv_0^2 - mgh = 0$

$\Rightarrow h = \dfrac{v_1^2}{2g}$

For $v_0 = 50$ m/s: $h = \dfrac{50^2}{2(9.81)} = 127.4$ m

3/105 The small cart has a speed $v_A = 4$ m/s as it passes point A. It moves without appreciable friction and passes over the top hump of the track. Determine the cart speed as it passes point B. Is knowledge of the shape of the track necessary?

Ans. $v_B = 7.16$ m/s

$T_A + U_{A-B} = T_B$

$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$

$v_B^2 = v_A^2 + 2gh = 4^2 + 2(9.81)(1.8)$

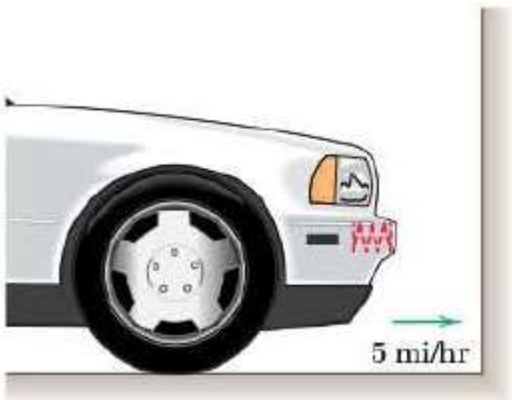
$v_B = 7.16$ m/s

Knowledge of the shape of the track is unnecessary, as long as it is known that the cart passes the highest point.

Semester I (2019-2020)

In the design of a spring bumper for a 3500-lb car, it is desired to bring the car to a stop from a speed of 5 mi/hr in a distance equal to 6 in. of spring deformation. Specify the required stiffness k for each of the two springs behind the bumper. The springs are undeformed at the start of impact.

Ans. $k = 974$ lb/in.

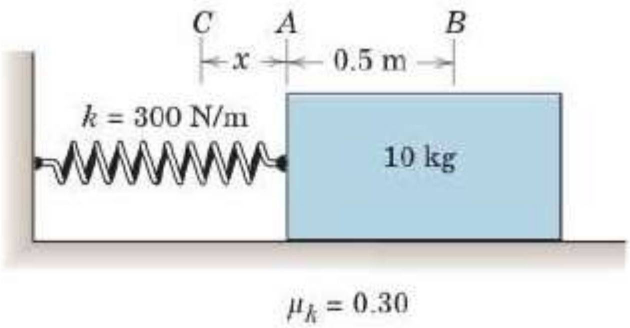


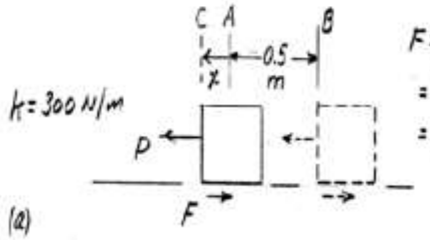
$$U_{1-2}=\Delta T;\quad 2\left(\tfrac{1}{2}kx^2\right)=\tfrac{1}{2}mv^2-0$$

$$k=\frac{1}{2}\frac{mv^2}{x^2}=\frac{1}{2}\,\frac{3500}{32.2}\left(\frac{5}{30}44\right)^2\frac{1}{(6/12)^2}\,\frac{1}{12}=\underline{974\ \text{lb/in.}}$$

The 10-kg block is released from rest on the horizontal surface at point B, where the spring has been stretched a distance of 0.5 m from its neutral position A. The coefficient of kinetic friction between the block and the plane is 0.30. Calculate (a) the velocity v of the block as it passes point A and (b) the maximum distance x to the left of A which the block goes.

Ans. (a) $v = 2.13$ m/s, (b) $x = 0.304$ m





$F=\mu_k N$
$=0.30(10)(9.81)$
$=29.43\ N$

(a)

From B to A: $U_{1-2}=\Delta T$

$$\tfrac{1}{2}(300)(0.5)^2-29.43(0.5)=\tfrac{1}{2}(10)v^2$$

$$v^2=4.557\ (m/s)^2,\quad \underline{v=2.13\ m/s}$$

(b) From A to C; $U_{1-2}=\Delta T$

$$-\tfrac{1}{2}(300)x^2-29.43x=0-\tfrac{1}{2}(10)(4.557)$$

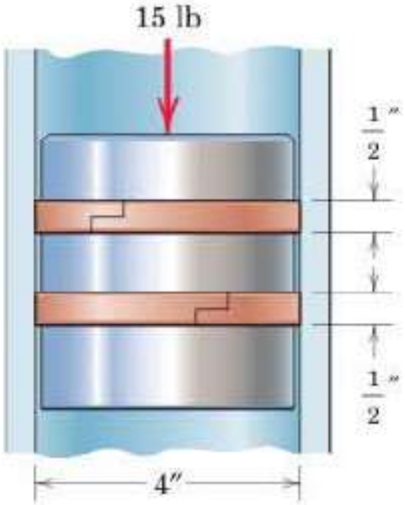
$$x^2+0.1962x-0.1519=0$$

$$x=\frac{-0.1962}{2}\pm\frac{1}{2}\sqrt{(0.1962)^2+4(0.1519)}$$

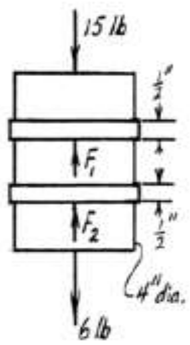
$$=-0.0981\pm0.4019,\quad \underline{x=0.304\ m}\quad (x=-0.48)$$

In a design test of piston-ring pressure, the special 4-in.-diameter aluminum piston weighing 6 lb is released from rest in the vertical cylinder under the action of the constant 15-lb force. The piston reaches a velocity of 8 ft/sec in 10 in. of travel. The coefficient of kinetic friction between the cast-iron rings and the cylinder is 0.15. The piston diameter is slightly smaller than the cylinder diameter so that all frictional resistance to motion is due to piston-ring friction. Calculate the average pressure p between the rings and the cylinder wall. Each of the two rings of $\frac{1}{2}$-in. width is free to expand in its piston groove.

Ans. $p = 7.34 \text{ lb/in.}^2$



$$F_1 + F_2 = \mu_k (\text{Area})\, p$$
$$= 0.15 \times 2 \times 4\pi \times \tfrac{1}{2} p$$
$$= 1.885 p \text{ lb}$$



$$U = \Delta T: \ (15 + 6 - 1.885p)\frac{10}{12} = \frac{1}{2}\frac{6}{32.2}(8^2 - 0^2)$$

Solve & get $p = \underline{7.34 \text{ lb/in.}^2}$