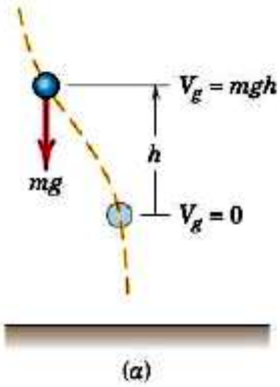
- **Potential energy**

Gravitational Potential Energy

We consider first the motion of a particle of mass *m* in close proximity to the surface of the earth, where the gravitational attraction (weight) mg is essentially constant, Fig. The gravitational potential energy V_g of the particle is defined as the work *mgh* done against the gravitational field to elevate the particle a distance *h* above some arbitrary reference plane (called a datum), where V_s is taken to be zero. Thus, we write the potential energy as



$$V_g = mgh \qquad (3/18)$$

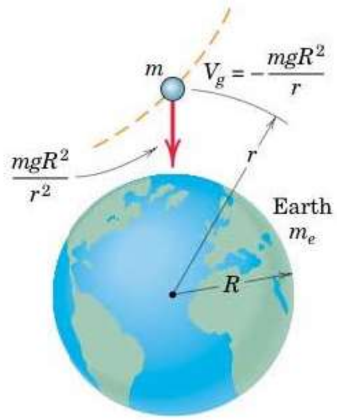
This work is called **potential energy** because it may be converted into energy if the particle is allowed to do work on a supporting body while it returns to its lower original datum plane.

In going from one level at h = h_1 to a higher level at h = h_2, the change in potential energy becomes

$$\Delta V_g = mg(h_2 - h_1) = mg\Delta h$$

The corresponding work done by the gravitational force on the particle is $-mg\Delta h$. Thus, the work done by the gravitational force is the negative of the change in potential energy.

When large changes in altitude in the field of the earth are encountered, Fig., the gravitational force $Gmm_e/r^2 = mgR^2/r^2$ is no longer constant. The work done against this force to change the radial position of the particle from r_1 to r_2 is the change $(V_g)_2 - (V_g)_1$ in gravitational potential energy, which is



(b)

$$\int_{r_1}^{r_2} mgR^2 \frac{dr}{r^2} = mgR^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (V_g)_2 - (V_g)_1$$

It is customary to take $(V_g)_2 = 0$ when $r_2 = \infty$, so that with this datum we have

$$V_g = -\frac{mgR^2}{r} \qquad \textbf{(3/19)}$$

In going from r_1 to r_2, the corresponding change in potential energy is

$$\Delta V_g = mgR^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

<u>Elastic Potential Energy</u>: The second example of potential energy occurs in the deformation of an elastic body, such as a spring. The work which is done on the spring to deform it is stored in the spring and is called its **elastic potential energy** V_e.

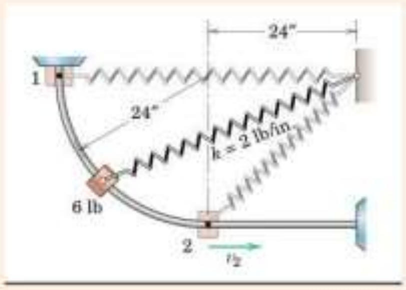
$$V_e = \int_0^x kx\,dx = \frac{1}{2}kx^2 \qquad \textbf{(3/20)}$$

If the deformation, either tensile or compressive, of a spring in-creases from x_1 to x_2 during the motion, then the change in potential en-rgy of the spring is its final value minus its initial value or

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

Sample Problem 3/16

The 6-lb slider is released from rest at position 1 and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 2 lb/in. and has an unstretched length of 24 in. Determine the velocity of the slider as it passes position 2.



Solution. The work done by the weight and the spring force on the slider will be treated using potential-energy methods. The reaction of the rod on the slider is normal to the motion and does no work. Hence, $U'_{1\text{-}2} = 0$. We define the datum to be at the level of position 1, so that the gravitational potential energies are

① $$V_1 = 0$$

$$V_2 = -mgh = -6\left(\frac{24}{12}\right) = -12 \text{ ft-lb}$$

The initial and final elastic (spring) potential energies are

$$V_1 = \frac{1}{2}k{x_1}^2 = \frac{1}{2}(2)(12)\left(\frac{24}{12}\right)^2 = 48 \text{ ft-lb}$$

$$V_2 = \frac{1}{2}k{x_2}^2 = \frac{1}{2}(2)(12)\left(\frac{24\sqrt{2}}{12} - \frac{24}{12}\right)^2 = 8.24 \text{ ft-lb}$$

Substitution into the alternative work-energy equation yields

$$[T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2] \qquad 0 + 48 + 0 = \frac{1}{2}\left(\frac{6}{32.2}\right){v_2}^2 - 12 + 8.24$$

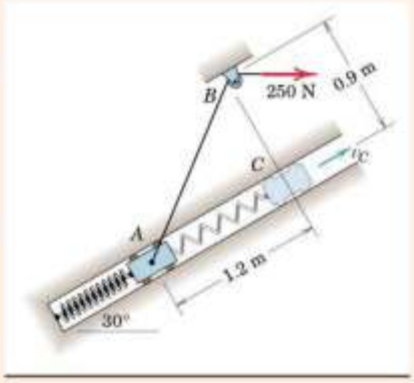
$$v_2 = 23.6 \text{ ft/sec} \qquad \textit{Ans.}$$

Helpful Hint

① Note that if we evaluated the work done by the spring force acting on the slider by means of the integral $\int \mathbf{F}\cdot d\mathbf{r}$, it would necessitate a lengthy computation to account for the change in the magnitude of the force, along with the change in the angle between the force and the tangent to the path. Note further that v_2 depends only on the end conditions of the motion and does not require knowledge of the shape of the path.

Sample Problem 3/17

The 10-kg slider moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position A, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v_C of the slider as it passes point C.



Solution. The slider and inextensible cord together with the attached spring will be analyzed as a system, which permits the use of Eq. 3/21a. The only nonpotential force doing work on this system is the 250-N tension applied to the cord. While the slider moves from A to C, the point of application of the 250-N force moves a distance of $\overline{AB} - \overline{BC}$ or $1.5 - 0.9 = 0.6$ m.

① ②

$$U'_{A\text{-}C} = 250(0.6) = 150 \text{ J}$$

We define a datum at position A so that the initial and final gravitational potential energies are

$$V_A = 0 \qquad V_C = mgh = 10(9.81)(1.2 \sin 30^\circ) = 58.9 \text{ J}$$

The initial and final elastic potential energies are

$$V_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(60)(0.6)^2 = 10.8 \text{ J}$$

$$V_C = \frac{1}{2}kx_B^2 = \frac{1}{2}60(0.6 + 1.2)^2 = 97.2 \text{ J}$$

Substitution into the alternative work-energy equation 3/21a gives

$$[T_A + V_A + U'_{A\text{-}C} = T_C + V_C] \qquad 0 + 0 + 10.8 + 150 = \frac{1}{2}(10)v_C^2 + 58.9 + 97.2$$

$$v_C = 0.974 \text{ m/s} \qquad \textit{Ans.}$$

Helpful Hints

① Do not hesitate to use subscripts tailored to the problem at hand. Here we use A and C rather than 1 and 2.

② The reactions of the guides on the slider are normal to the direction of motion and do no work.