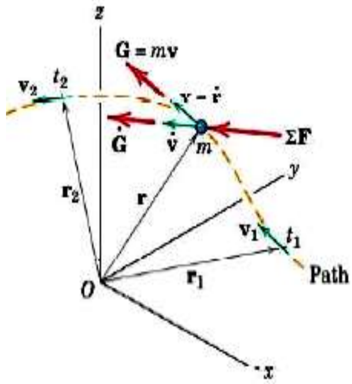
- **Section C: Impulse and Momentum**

Linear Impulse and linear momentum:

The resultant ∑F of all forces on m is in the direction of its acceleration v. We may now write the basic equation of motion for the particle



$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v}) \quad \text{or} \quad \boxed{\Sigma \mathbf{F} = \dot{\mathbf{G}}} \qquad \textbf{(3/25)}$$

where the product of the mass and velocity is defined as the linear momentum *G* = *mv* of the particle.

Equation 3/25 states that the resultant of all forces acting on a particle equals its time rate of change of linear momentum. In SI the units of linear momentum my are seen to be kg.m/s, which also equals N .s. In U.S. customary units, the units of linear momentum *mv* are [lb/(ft/sec^2)][ft.sec]= lb-sec.

Because Eq. 3/25 is a vector equation, we recognize that, in addition to the equality of the magnitudes of ΣF and Ġ, the direction of the resultant force coincides with the direction of the rate of change in linear momentum, which is the direction of the rate of change in velocity. Equation 3/25 is one of the most useful and important relationships in dynamics, and it is valid as long as the mass *m* of the particle is not changing with time. The case where *m* changes with time is discussed in Art. 4/7 of Chapter 4.

We now write the three scalar components of Eq. 3/25 as

$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y \qquad \Sigma F_z = \dot{G}_z \qquad \textbf{(3/26)}$$

These equations may be applied independently of one another.

The Linear Impulse-Momentum Principle

All that we have done so far in this article is to rewrite Newton's second law in an alternative form in terms of momentum. But we are now able to describe the effect of the resultant force $\Sigma \mathbf{F}$ on the linear momentum of the particle over a finite period of time simply by integrating Eq. 3/25 with respect to the time t. Multiplying the equation by dt gives $\Sigma \mathbf{F}\, dt = d\mathbf{G}$, which we integrate from time t_1 to time t_2 to obtain

$$\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 - \mathbf{G}_1 = \Delta \mathbf{G} \qquad \textbf{(3/27)}$$

Here the linear momentum at time t_2 is $G_2 = mv_2$ and the linear momentum at time t_1 is $G_1 = mv_1$. ***The product of force and time is defined as the linear impulse of the force***, and Eq. 3/27 states that the total linear impulse on m equals the corresponding change in linear momentum of m. Alternatively, we may write Eq. 3/27 as

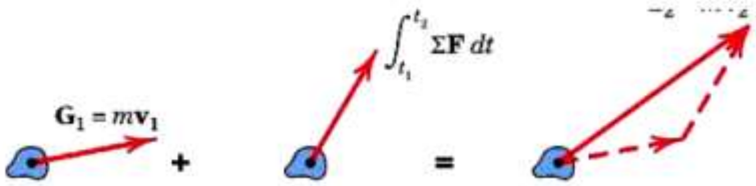
$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 \qquad \textbf{(3/27a)}$$

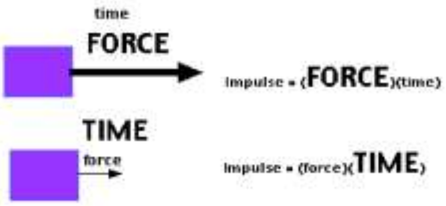
which says that the initial linear momentum of the body plus the linear impulse applied to it equals its final linear momentum.

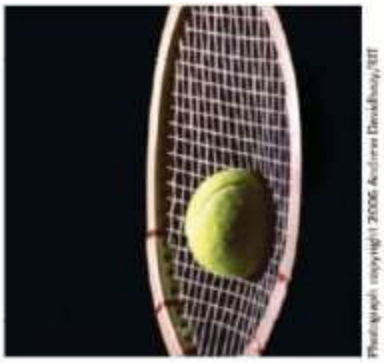
The impulse integral is a vector which, in general, may involve changes in both magnitude and direction during the time interval. Under these conditions, it will be necessary to express ΣF and G in component form and then combine the integrated components. The components of Eq. 3/27*a* are the scalar equations

$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x \, dt = m(v_2)_x$$
$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y \, dt = m(v_2)_y \qquad \textbf{(3/27b)}$$
$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z \, dt = m(v_2)_z$$

We now introduce the concept of the *impulse-momentum diagram*. Once the body to be analyzed has been clearly identified and isolated, we construct three drawings of the body as shown in Fig. 3/12. In the first drawing, we show the initial momentum $m\mathbf{v}_1$, or components thereof. In







The impact force exerted by the racquet on this tennis ball will usually be much larger than the weight of the tennis ball.