Relative velocity

The second approach to rigid-body kinematics is to use the principles of relative motion.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_{A/B} = r\omega \qquad (5/5)$$

$$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r} \qquad (5/6)$$

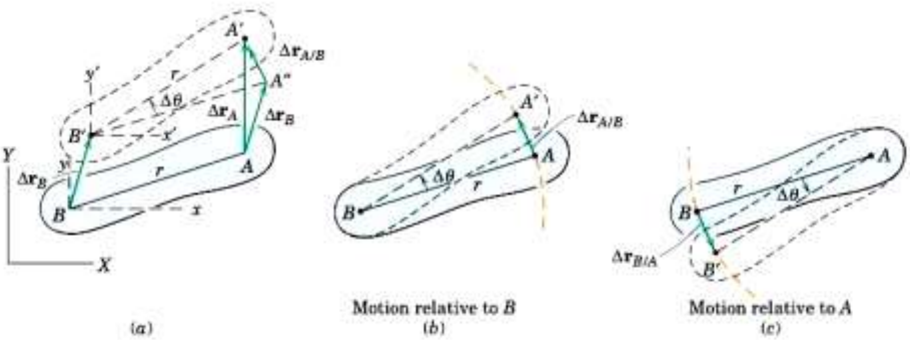
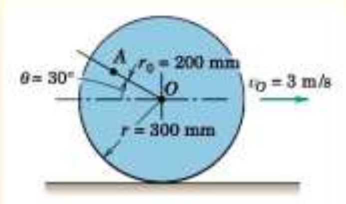
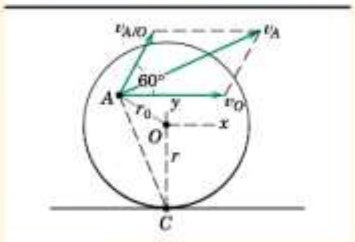


Figure 5/5

Sample Problem 5/7

The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.





Solution I (Scalar-Geometric). The center O is chosen as the reference point for the relative-velocity equation since its motion is given. We therefore write

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

where the relative-velocity term is observed from the translating axes x-y attached to O. The angular velocity of AO is the same as that of the wheel which, from Sample Problem 5/4, is $\omega = v_O/r = 3/0.3 = 10$ rad/s. Thus, from Eq. 5/5 we have

$[v_{A/O} = r_0\dot{\theta}]$ $\qquad v_{A/O} = 0.2(10) = 2$ m/s

① which is normal to AO as shown. The vector sum $\mathbf{v}_A$ is shown on the diagram and may be calculated from the law of cosines. Thus,

② $$v_A{}^2 = 3^2 + 2^2 + 2(3)(2)\cos 60° = 19\ (\text{m/s})^2 \qquad v_A = 4.36 \text{ m/s} \qquad \textit{Ans.}$$

The contact point C momentarily has zero velocity and can be used alternatively as the reference point, in which case, the relative-velocity equation becomes $\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C} = \mathbf{v}_{A/C}$ where

$$v_{A/C} = \overline{AC}\omega = \frac{\overline{AC}}{\overline{OC}} v_O = \frac{0.436}{0.300}(3) = 4.36 \text{ m/s} \qquad v_A = v_{A/C} = 4.36 \text{ m/s}$$

The distance $\overline{AC} = 436$ mm is calculated separately. We see that $\mathbf{v}_A$ is normal to
③ AC since A is momentarily rotating about point C.

Solution II (Vector). We will now use Eq. 5/6 and write

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O} = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_0$$

where

④ $$\boldsymbol{\omega} = -10\mathbf{k} \text{ rad/s}$$

$$\mathbf{r}_0 = 0.2(-\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = -0.1732\mathbf{i} + 0.1\mathbf{j} \text{ m}$$

$$\mathbf{v}_O = 3\mathbf{i} \text{ m/s}$$

We now solve the vector equation

$$\mathbf{v}_A = 3\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -0.1732 & 0.1 & 0 \end{vmatrix} = 3\mathbf{i} + 1.732\mathbf{j} + \mathbf{i}$$

$$= 4\mathbf{i} + 1.732\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

The magnitude $v_A = \sqrt{4^2 + (1.732)^2} = \sqrt{19} = 4.36$ m/s and direction agree with the previous solution.

Helpful Hints

① Be sure to visualize $v_{A/O}$ as the velocity which A appears to have in its circular motion relative to O.

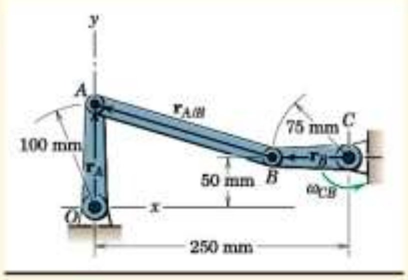
② The vectors may also be laid off to scale graphically and the magnitude and direction of v_A measured directly from the diagram.

③ The velocity of any point on the wheel is easily determined by using the contact point C as the reference point. You should construct the velocity vectors for a number of points on the wheel for practice.

④ The vector $\boldsymbol{\omega}$ is directed into the paper by the right-hand rule, whereas the positive z-direction is out from the paper; hence, the minus sign.

Sample Problem 5/8

Crank CB oscillates about C through a limited arc, causing crank OA to oscillate about O. When the linkage passes the position shown with CB horizontal and OA vertical, the angular velocity of CB is 2 rad/s counterclockwise. For this instant, determine the angular velocities of OA and AB.



Solution I (Vector). The relative-velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ is rewritten as

① $$\boldsymbol{\omega}_{OA} \times \mathbf{r}_A = \boldsymbol{\omega}_{CB} \times \mathbf{r}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

where

$$\boldsymbol{\omega}_{OA} = \omega_{OA}\mathbf{k} \qquad \boldsymbol{\omega}_{CB} = 2\mathbf{k} \text{ rad/s} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$$

$$\mathbf{r}_A = 100\mathbf{j} \text{ mm} \qquad \mathbf{r}_B = -75\mathbf{i} \text{ mm} \qquad \mathbf{r}_{A/B} = -175\mathbf{i} + 50\mathbf{j} \text{ mm}$$

Substitution gives

$$\omega_{OA}\mathbf{k} \times 100\mathbf{j} = 2\mathbf{k} \times (-75\mathbf{i}) + \omega_{AB}\mathbf{k} \times (-175\mathbf{i} + 50\mathbf{j})$$

$$-100\omega_{OA}\mathbf{i} = -150\mathbf{j} - 175\omega_{AB}\mathbf{j} - 50\omega_{AB}\mathbf{i}$$

Matching coefficients of the respective **i**- and **j**-terms gives

$$-100\omega_{OA} + 50\omega_{AB} = 0 \qquad 25(6 + 7\omega_{AB}) = 0$$

the solutions of which are

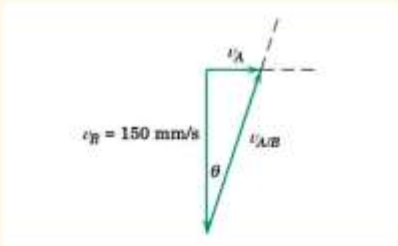
② $$\omega_{AB} = -6/7 \text{ rad/s} \quad \text{and} \quad \omega_{OA} = -3/7 \text{ rad/s}$$ *Ans.*

Solution II (Scalar-Geometric). Solution by the scalar geometry of the vector triangle is particularly simple here since $\mathbf{v}_A$ and $\mathbf{v}_B$ are at right angles for this special position of the linkages. First, we compute v_B, which is

$[v = r\omega]$ $$v_B = 0.075(2) = 0.150 \text{ m/s}$$

and represent it in its correct direction as shown. The vector $\mathbf{v}_{A/B}$ must be perpendicular to AB, and the angle θ between $\mathbf{v}_{A/B}$ and $\mathbf{v}_B$ is also the angle made by AB with the horizontal direction. This angle is given by

$$\tan\theta = \frac{100 - 50}{250 - 75} = \frac{2}{7}$$



③ The horizontal vector $\mathbf{v}_A$ completes the triangle for which we have

$$v_{A/B} = v_B/\cos\theta = 0.150/\cos\theta$$

$$v_A = v_B \tan\theta = 0.150(2/7) = 0.30/7 \text{ m/s}$$

The angular velocities become

$[\omega = v/r]$ $$\omega_{AB} = \frac{v_{A/B}}{AB} = \frac{0.150}{\cos\theta}\frac{\cos\theta}{0.250 - 0.075} = 6/7 \text{ rad/s CW}$$ *Ans.*

$[\omega = v/r]$ $$\omega_{OA} = \frac{v_A}{OA} = \frac{0.30}{7}\frac{1}{0.100} = 3/7 \text{ rad/s CW}$$ *Ans.*

Helpful Hints

① We are using here the first of Eqs. 5/3 and Eq. 5/6.

② The minus signs in the answers indicate that the vectors $\boldsymbol{\omega}_{AB}$ and $\boldsymbol{\omega}_{OA}$ are in the negative **k**-direction. Hence, the angular velocities are clockwise.

③ Always make certain that the sequence of vectors in the vector polygon agrees with the equality of vectors specified by the vector equation.