Lecture Fifteen
Flow Measurements Part-2

1- *Pitot Tube for Flow Measurements*.

Definitions of Hydrostatic, Hydrodynamic, Static and Total Pressure.

The points ***A&B*** are at a height z_A&z_B respectively from the datum & consider a fluid flow through pipe of varying cross section area as in Fig. 1.

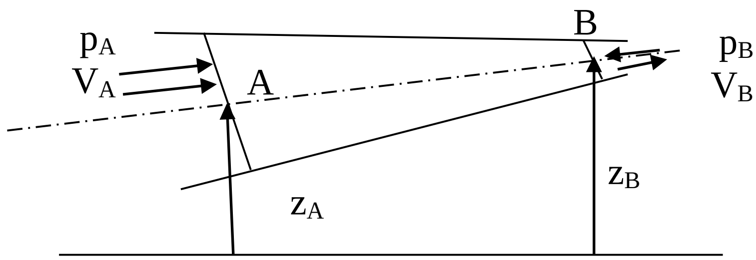


Figure 1: Static and Total Pressure.

- If the fluid is to be stationary , then $(\frac{\partial p}{\partial z})_{hs} = -\rho g$

(hs) Represent the hydrostatic case.

So, $p_{Ahs} - p_{Bhs} = \rho g(z_B - z_A)$

From above equation, the hydrostatic pressure at a point in a fluid is the pressure acting at the point when the fluid is at rest or pressure at the point due to weight of the fluid above it.

- Now, if the fluid to be moving, the pressure at a point can be written as a sum of two components, hydrodynamic & hydrostatic

$p_A = p_{Ahs} + p_{Ahd}$ (1)

- Using Eq. 1 in Bernoulli's equation between ***A&B***

$\frac{p_{Ahd}-p_{Bhd}}{\rho g} + \left[\frac{p_{Ahs}-p_{Bhs}}{\rho g} + (z_A - z_B)\right] = \frac{V_B^2-V_A^2}{2g}$ (2)

From Eq. 2, the terms within the square bracket cancel each other, hence

$\frac{p_{Ahd}-p_{Bhd}}{\rho g} = \frac{V_B^2-V_A^2}{2g}$ (3)

$p_{Ahd} + \frac{\rho V_A^2}{2} = p_{Bhd} + \frac{\rho V_B^2}{2} = C = p_O$ (4)

Eq's (3 &4) convey the flowing

The pressure at a location has → Hydrostatic component
→ Hydrodynamic component

The difference in kinetic energy due to hydrodynamic components only.

Note.

1- The hydrodynamic component is often called static pressure.
2- The velocity term is the dynamic pressure.
The sum of two components is (p_O) is known as total pressure.

$p_O = p + \frac{\rho V^2}{2}$ (5)

Is known as stagnation pressure

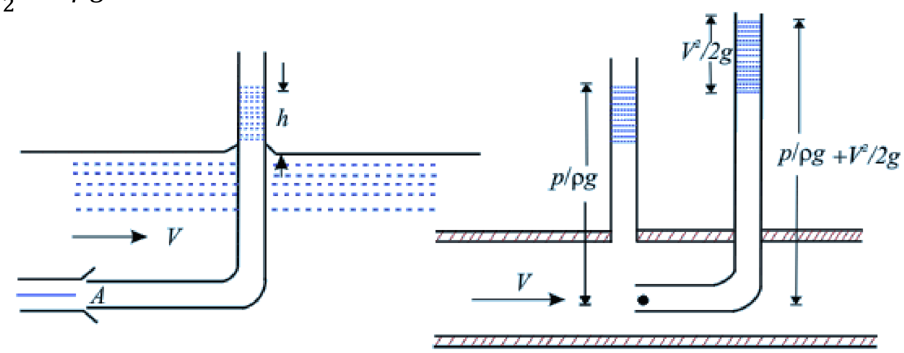
$V = \sqrt{2\frac{p_O - p}{\rho}}$ (6)

2- Pitot Tube Device.

Firstly at 1732 by Henri Pitot, was used a right angled glass tube, one end of the tube face the flow while the other end is open to atmosphere. The difference in level between the liquid in the glass tube and the free surface becomes the measure of dynamic pressures neglecting friction as in Fig. 2.a.

$p_O - p = \frac{\rho V^2}{2} = \rho g\ h$

$V = \sqrt{2gh}$



(a) (b)

Figure 2: Simple Pitot tube (a) Tube for measuring the stagnation pressure.
(b) Static and stagnation tubes together.

For a free stream a single tube is sufficient to determine the velocity. In closed duct the Pitot tube measures only the stagnation pressure and so the static pressure must be measured separately as shown in Fig. 2-b. Applying B.E. between stagnations s & p in horizontal pipe

$\frac{p_O}{\rho g} + \frac{V^2}{2g} = \frac{p_S}{\rho g} \quad or \quad h_o + \frac{V^2}{\rho g} = h_s$

$V = \sqrt{2g(h_s - h_o)} = \sqrt{2g\Delta h}$

Where:

p_o= Pressure at point p. i.e. static pressure.

V= Velocity at point (p) i.e free flow velocity

p_s= Stagnation pressure at point s

Δh= Dynamic pressure

= Difference between stagnation pressure head (h_s) and static pressure (h_0)

If a differential manometer is connected to the tube of a Pitot static tube as in Fig. 5.8 it will measure the dynamic pressure head. The following figure shows the static pressure and stagnation pressure tube are combined into one instrument known as Pitot static tube. If y is the manometric difference, then

$\Delta h = y(\frac{\rho_m}{\rho} - 1)$

$\rho_m = density\ of\ manometeric\ liquid$

ρ = density of the liquid flowing through the pipe

$\therefore V = C\sqrt{2g\Delta h}$ or

$$V = C\sqrt{2\left(\frac{\Delta p}{\rho}\right)}$$

Where *Δp* is the difference between stagnation and static pressure. The value of *C* is usually determine from calibration test of the Pitot tube

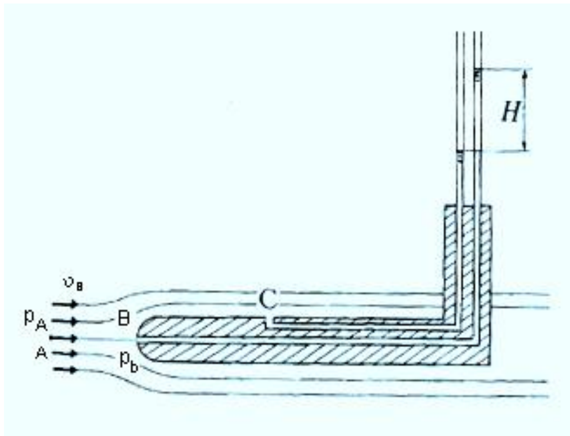


Figure 3: Pitot static tube.

Ex.1 A submarine fitted with a Pitot tube moves horizontally in sea. Its axis is ***12 m*** below the surface of water. The Pitot tube fixed in front of the submarine and along it axis is connected to the two limbs of a u-tube containing mercury, the reading of which is found to be ***200 mm***. Find the speed of the submarine.

Sol.

$\rho_{sea} = 1025\ kg/m^3$, $\rho_{mer} = 13600\ kg/m^3$

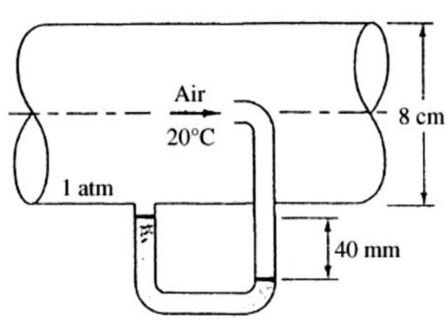
To find the head $\Delta h = y\left(\frac{\rho_m}{\rho} - 1\right) = 0.2(\frac{13600}{1025} - 1)$

$\Delta h = 2.45\ m$

$\therefore Velocity\ V = \sqrt{2g\Delta h} = \sqrt{2 * 9.81 * 2.45} = 6.94\ m/s$

$= 24.97\ km/hr$

Ex.2 For the Pitot-static pressure arrangement of the following figure, the manometer fluid is (colored) water at 20C°. Estimate (a) The centerline velocity, (b) The pipe volume flow rate, and (c) The smooth wall shear stress.



Sol.

$\rho = 1.2\frac{kg}{m^3}$, $\mu = 1.8 * 10^{-5}\frac{kg}{m.s}$ $for\ air.$

$\rho = 998\frac{kg}{m^3}$, $\mu = 0.001\ \frac{kg}{m.s}$ $for\ water.$

The manometer reads

$p_O - p = (\rho_{water} - \rho_{air})gh = (998 - 1.2)(9.81)(0.04)$

$p_O - p = 391\ Pa$

Therefore $V_{cl} = \left[\frac{2\Delta p}{\rho}\right]^{0.5} = \left[\frac{2(391)}{1.2}\right]^{0.5} = 25.5\frac{m}{s}$

$Guess\ V_{av.} \approx 0.85\ V_{CL}\quad \approx 21.7\frac{m}{s}$

$the\ volume\ flow\ rate\ is\ Q = \left(\frac{\pi}{4}\right)(0.08)^2\ (21.7) \approx 0.109\frac{m^3}{s}$

$then\ \ Re = \frac{\rho VD}{\mu} = \frac{1.2(21.7)(0.08)}{1.8*10^{-5}}$

$Re = 115700$

$then\ f\ smoth\ \approx 0.0175$

$finally\ \tau_w = \frac{f}{8}\ \rho V^2 = \frac{0.0175}{8}(1.2)(21.7) \approx 1.23Pas$

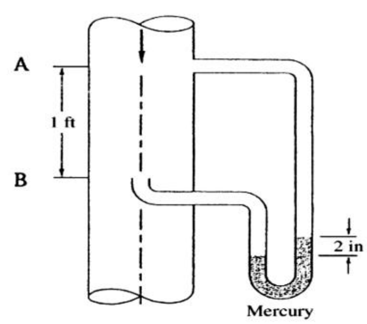
Ex.3

For the water flow of figure use the Pitot- static arrangement to estimate

a) The center line velocity
b) The volume flow in 5 in diameter smooth pipe
c) What error in flow rate is caused by neglecting the *(1 ft)* elevation difference?

Take:- $\rho = 1.94\ slug/\ ft^3$; $\mu = 2.09 * 10^{-5}\frac{slug}{ft.s}$

h = 2 in.



Sol.

For the manometer reading take the equal pressure at 0-0

$p_{OB} + (h + R)\rho_w g = p_A + h\,\rho_m g + R\rho_w g + 1 * \rho_w g$

$p_{OB} - p_A = h\rho_m g - h\rho_w g + \rho_w g = (\rho_m - \rho_w)hg + \ \rho_w g - - - - (a)$

Where R is the vertical distance between point B and the top level of mercury in right leg. From energy equation,

$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_{f-AB}$

$p_A - p_B = \rho g h_{f-AB} - \rho g(1\ ft)$ Static pressure difference ----------(b)

Therefor by summation Eq's (a) & (b)

$p_{OB} - p_A + p_A - p_B = (\rho_m - \rho_w)hg + \rho_w g + \rho g h_{f-AB} - \rho g$

$p_{oB} - P_B = (\rho_m - \rho_w)hg + \rho g\ h_{f-AB}$ Where h_{f-AB} friction losses.

$(p_{OB} - p_B) = (SG - 1)\rho g h = (13.56 - 1)(62.4)\left(\frac{2}{12}\right) \approx \frac{130.6 lb_f}{ft^2}$

$V_{CL} = \left(\frac{2\Delta p}{\rho}\right)^{0.5} = \left(2 * \frac{130.6}{1.94}\right)^{0.5} = 11.6\ \frac{ft}{s}$

$Q = AV_{CL} = \frac{\pi}{4}\left(\frac{5}{12}\right)^2 * 11.6 = 1.58 \frac{ft^3}{s}$

$\Delta p_{friction} = \frac{f\left(\frac{l}{d}\right)\rho V^2}{2} \approx 3.2$ $3\%\ is\ neglecting$

$\Delta p\ pitot = 130.6 + 3.2 = 133.8\ psf$