Lecture Fourteen
Flow Measurements Part-1

1- *Measurement of Flow Rate Through Pipe*.

The determination of the flow rate from the measurement of pressure drop depends on the straight forward application of Bernoulli's equation.

Four different flow meters operate on this principle.

- Venturimeter
- Orificemeter
- Flow nozzle
- Pitot tube

A- Venturimeter.

Working:

1- The gradual diverging passage in the direction of flow avoiding the losses of energy due to separation

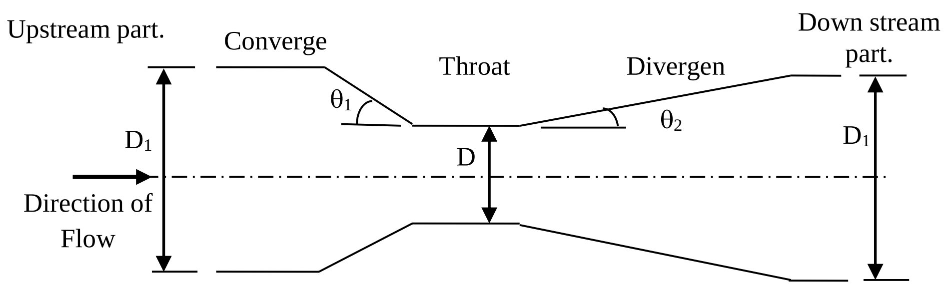


Figure 1: Venturimeter.

2- $\frac{dV}{dx} > 0$, $\frac{dp}{dx} < 0$ → To satisfy continuity equation in convergent part according to Bernoulli's Equation

3- $V_{max.}$, p_{min} → At throat area, demonstrated by Battisla Venturi 1797.

4- $dV/dx<0$, $dp/dx>0$ → In the divergent part.

For measuring the flow rate through pipe, let us consider a steady, ideal and one dimensional, where the flow of fluid, the velocity and pressure at any section will be uniform. Let V_1&p_1 are the velocity and pressure at inlet section (1), while those at throat V_2 & p_2 at section (2) as shown in Fig. 2. Applying Bernoulli's equation between sec.1&2, we get

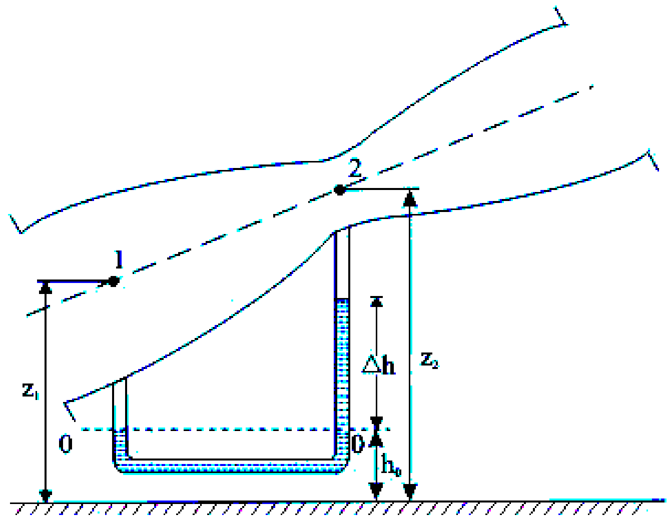


Figure 2: Measurement of Flow by a Venturimeter.

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad (1)$$

$$\frac{V_2^2-V_1^2}{2g}=\frac{p_1-p_2}{\rho g}+z_1-z_2 \quad (2)$$

Where ρ is the density of fluid through the Venturimeter. From continuity

$$A_1\,V_1=V_2A_2 ---\rightarrow V_1=\frac{V_2A_2}{A_1} \quad (3)$$

Subtituting Eq. 3 in Eq. 2

$$\frac{V_2^2}{2g}\left(1-\frac{A_2^2}{A_1^2}\right)=\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)$$

$$V_2=\frac{1}{\sqrt{1-\frac{A_2^2}{A_1^2}}}\ \sqrt{2g(h_1^*-h_2^*)} \quad (4)$$

Where $h_1^*\&h_2^*$ are the piezometric pressure at Sec.1 & Sec.2 and are defined as

$$h_1^*=\frac{p_1}{\rho g}+z_1 \quad (5.a)$$

$$h_2^*=\frac{p_2}{\rho g}+z_2 \quad (5.b)$$

Hence, the volume flow rate through the pipe is given by

$$Q=A_2\ V_{2=}\frac{A_2}{\sqrt{1-\frac{A_2^2}{A_1^2}}}\ \sqrt{2g(h_1^*-h_2^*)} \quad (6)$$

The pressure difference between Sec.1&2 is measured by a manometer as shown in Fig 2, we can write

$$p_1+\ \rho g(\,z_1-\ h_0)=p_2+\rho g(z_2-h_0-\Delta h)+\Delta h\,\rho_m g$$

$$or,(p_1+\rho g z_1)-(p_2+\rho g z_2)=(\rho_m-\rho)g\Delta h$$

$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\left(\frac{\rho_m}{\rho}-1\right)\Delta h$$

$$or\ \ h_1^*-\ h_2^*=\left(\frac{\rho_m}{\rho}-1\right)\Delta h \quad (7)$$

Where ρ_m is the density of the manometric liquid. Eq. 7 shows that a manometer always registers a direct reading of the difference in piezometric pressures. Now, substitution of $(h_1^* - h_2^*)$ from Eq. 7 in Eq. 6 will gives the flow rate through pipe.

$$Q = \frac{A_1 A_2}{\sqrt{A_1 - A_2}} \sqrt{2g(h_1^* - h_2^*)} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \qquad (8)$$

If C the constant of Venturimeter which is equal to $\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g}$ and the pipe along with the Venturimeter is horizontal, then $z_1=z_2$, and hence $h_1^* - h_2^*$ becomes$(h_1 - h_2)$, where h_1 and h_2 are the static pressure heads can be written as $(h_1 = \frac{p_1}{pg}, h_2 = \frac{p_2}{pg})$ then, the manometric Eq. 7 becomes

$$h_1 - h_2 = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$$

Eq.8 gives the flow rate in pipe with the terms of manometer deflection Δh is remain the same irrespective of whether the pipe-line along with the Venturimeter connection is horizontal or not. Eq.8 always overestimates the actual flow rate due to the ideal flow assumption and read fluid measurement *(Δh)*. Multiplying Eq.8 by the factor C_d, called the coefficient of discharge as follows.

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \qquad (9)$$

C_d <1.0 and is defined as

$$C_d = \frac{Actual\ rate\ of\ discharge}{theoretical\ rate\ of\ discharge\ by\ Eq5.8}$$

Value of C_d between (0.95 to 0.98); Cd≈ 0.9858-0.196 $\beta^{4.5}$ where β=(d2/d1)

Ex.1

A Venturimeter is placed at 30° to the horizontal (sloping upwards in the direction of flow) to a pipe line carrying on oil of specific gravity 0.8. A differential with mercury as the manometer fluid is attached to the inlet and throat of the Venturimeter. The manometer shows a deflection of 100 mm. the pipe diameter is 200 mm, while the diameter of Venturi throat is 100 mm.

a) Find the volume flow rate of oil if the coefficient of discharge of the Venturimeter is 0.96.
b) What will be the reading of differential manometer if the Venturimeter is turned horizontal? The length of Venturimeter between the inlet and the throat is 320 mm.

Sol.

$A_1 = 0.0314\ m^2; A_2 = 0.00785\ m^2$

$$Q_{act} = Cd \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\ \left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \qquad \rho = 800\frac{kg}{m^3}\ , \rho_m = 13600\frac{kg}{m^3}$$

$\Delta h = 0.1m, Cd = 0.96$

a) $Q_{aut} = 0.04386\frac{m^3}{s}$

b) $V_1 = \frac{Q}{A_1} = \frac{0.04386}{0.0314} = 1.388\frac{m}{s}$

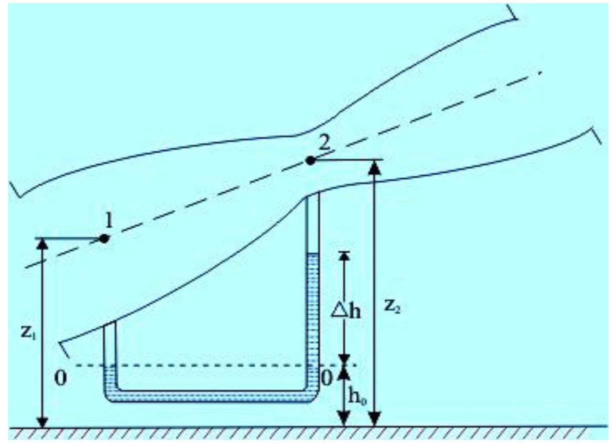
$V_2 = \frac{Q}{A_2} = \frac{0.04381}{0.00785} = 5.58\frac{m}{s}$

$IF\ z_1 = z_2$

$$\frac{p_1 - p_2}{Pg} = \frac{V_2^2 - V_1^2}{2g} = \frac{5.50^2 - 1.382^2}{2*9.81} = \frac{31.38 - 1.96}{2*9.81} = 1.488\ m$$

From Eq.7 $\frac{p_1-p_2}{\rho g} = h_1 - h_2 = (\frac{\rho_m}{\rho} - 1)\Delta h$ where $z_1=z_2$

$\frac{p_1-p_2}{\rho g} = \left(\frac{\rho_m}{\rho} - 1\right)\Delta\overline{h} = 1.488m$ $\quad \Delta\overline{h} = \frac{1.488}{16} = 0.093m$



B- Orificemeter.

1- First Method

Is a cheaper arrangement for the measurement of flow through a pipe, is essentially a thin circular plate with a sharp edged concentric circular hole in it as in Fig. 3.

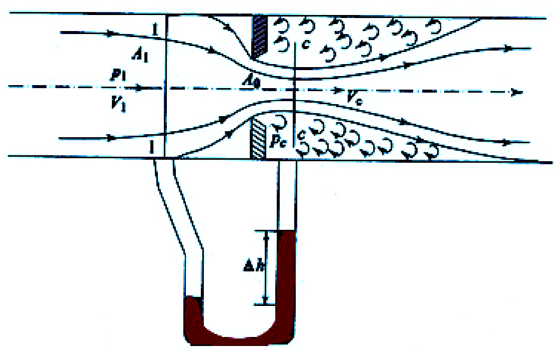


Figure 3: Flow through an Orificemeter.

Consider the fluid to be ideal, by applying Bernoulli's theorem between Sec.1-1 and Sec. c - c

$$\frac{p_1^*}{\rho g} + \frac{V_1^2}{2g} = \frac{p_c^*}{\rho g} + \frac{V_c^2}{2g} \qquad (10)$$

Where p_1^* & p_2^* are the piezometric pressure at Sec. 1-1 & c - c respectively. From continuity equation

$$V_1 A_1 \approx V_c A_c \qquad (11)$$

Where A_c is the area of the *vena contracta* from Eq's. 10 & 11 we can written as,

$$V_c = \sqrt{2(p_1^* - p_c^*)/\rho\left(1 - \frac{A_C^2}{A_1^2}\right)} \qquad (12)$$

The measured value of the piezometric pressure drop for a real fluid is always more due to friction than that assumed in case of an inviscid flow, a coefficient of velocity C_V (always less than 1) has to be introduce to determine the actual velocity V_c when the pressure drop $p_1^* - p_c^*$ in Eq.12 is substituted by its measured value in terms of the monometer deflection Δh.

$\Delta p = (\rho_{merc} - \rho_{water})g\Delta h = \rho g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h.$

Hence,

$$V_c = C_V\sqrt{\frac{2g\left(\frac{\rho_m}{\rho}-1\right)\Delta h}{1-A_c^2/A_1^2}} \qquad (13)$$

Where

Δh is the difference in liquid level.

ρ_m is the density of the manometric liquid.

ρ is the density of the working fluid.

$\therefore$ Volumetric flow rate

$$Q = A_c\,V_c \qquad (14)$$

If a coefficient of contraction C_c is defined as $C_c = \frac{A_c}{A_2}$, $\quad A_c = C_c\,A_2$

A_2 is the area of orifice due to unknown the position of A_c along the flow. Eq.14 can be written with help of Eq. 13.

$$Q = C_c\,A_2\,C_V\sqrt{\frac{2g\left(\frac{\rho_m}{\rho}-1\right)\Delta h}{1-\frac{C_C^2A_2^2}{A_1^2}}}$$

$$Q = C_cA_2C_V\sqrt{\frac{2g}{1-\frac{C_c^2A_2^2}{A_1^2}}}\sqrt{\left(\frac{\rho_m}{\rho}-1\right)\Delta h}$$

$$Q = C_d\sqrt{\left(\frac{\rho_m}{\rho}-1\right)\Delta h} \qquad (15)$$

With, $C_d = C\quad A_2\sqrt{\frac{2g}{1-\frac{C_C^2A_2^2}{A_1^2}}}$, $Where\ (C\quad = C_VC_c)$

Where ***C*** is depends upon the ratio of orifice to duct area, and Reynolds number of flow.

2- ***Orificemetes (Second Method)***

A_1, V_1, p_1 at Sec.1 A_2, V_2, p_2 at Sec.2. Applying ***B.E.*** at Sec.1 & 2 we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Delta h^* = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{V_2^2}{2g} = \Delta h^* + \frac{V_1^2}{2g}$$

$$or\ V_2 = \sqrt{2g\left(\Delta h^* + \frac{V_1^2}{2g}\right)} = \sqrt{2g\,\Delta h^* + V_1^2} - - - - - (a)$$

Section 2 is at *vena contracta* and A2 represents the area of vena contracta, $\boldsymbol{A_o}$ is the area of orifice,

$C_C = \frac{A_2}{A_0}$ Where C_C =Co-efficient of contraction

$\therefore\ A_2 = C_CA_0 - - - - - - (b)$ `

Using C.E. ,we get

$A_1V_1 = A_2V_2 - - - \rightarrow OR\ V_1 = \frac{A_2V_2}{A_1}$

$$Or \quad V_1 = \frac{A_0 C_C V_2}{A_1} --------(c)$$

Substituting the value of V_1 Eq. (a), we get

$$V_2 = \sqrt{2g\,\Delta h^* + A_0^2 C_C^2 . \frac{V_2^2}{A_1^2}}$$

$$Or \quad V_2^2 = 2g\Delta h^* + \left(\frac{A_0}{A_1}\right)^2 . C_C^2 . V_2^2$$

$$V_2^2\left[1 - \left(\frac{A_0}{A_1}\right)^2 C_C^2\right] = 2g\,\Delta h^*$$

$$V_2 = \frac{\sqrt{2g\,\Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}$$

$$\therefore The\ discharge \quad Q = A_2 V_2 = A_0 . C_c V_2 \quad = A_0 C_C \frac{\sqrt{2g\Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}} ----(d)$$

The above expression is simplified by using

$$C_d = C_C \frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}} \qquad C_d = coefficient\ of\ discharge$$

$$C_C = C_d \frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}} \qquad C_d = C_C . C_V$$

Substituting the value of C_C in Eq. d, we get

$$Q = A_0 . C_d \frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}} * \frac{\sqrt{2g\Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_c^2}} \ ; \ Q = \frac{C_d A_0 \sqrt{2g\Delta h}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}} = \frac{C_d A_0 A_1 \sqrt{2g\Delta h^*}}{\sqrt{A_1^2 - A_0^2}}$$

$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = \Delta h^* = h_1^* - h_2^* = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$ is the differential head.

$$Q = C_d \frac{A_0 A_1}{\sqrt{A_1^2 - A_0^2}} \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \qquad (16)$$

Ex.2

The following date related to an orificemeter
Diameter of pipe =240 mm
Diameter of orifice =120mm
Reading of differential manometer =400 mm of mercury Co-efficient of discharge of the meter =0.65. Determine the rate of oil flow

Sol.

$$d_1 = 240mm = 0.24\,m$$

$$\therefore Area\ of\ pipe\ A_1 = \frac{\pi}{4} * 0.24^2 = 0.0452m^2$$

$$orifice\ diameter\ d_o = 120\ mm = 0.12m$$

$$A_0 = \frac{\pi}{4} * 0.12^2 = 0.0113m^2$$

$C_d = 0.65$
$S.G_{oil} = 0.88$
Reading differential h= 400mm = 0.4 m of mercury
$\therefore\ differential\ head = \Delta h^* = \Delta h \left(\frac{\rho_m}{\rho} - 1\right)$
$\therefore\ \ \Delta h^* = 0.4 \left[\frac{13.6}{0.88} - 1\right] = 5.78\ m\ of\ oil$
$Q = C_d\ \frac{A_0 * A_1 \sqrt{2g\ \Delta h^*}}{\sqrt{A_1^2 - A_0^2}}$
$Q = 0.65 * \frac{0.0113 * 0.0452 \sqrt{2 * 9.81 * 5.78}}{\sqrt{0.0452^2 - 0.0113^2}}$
$Q = 0.08 \frac{m^3}{S}$

C- ***Flow Nozzle.***

- The flow nozzle as shown in Fig.4 is essentially a Venturimeter with the divergent part omitted. Therefore the basic equations for calculation of flow rate are the same as those for a Venturimeter.
- The dissipation of energy downstream of the throat due to flow separation is greater than that for a Venturimeter. But this disadvantage is often offset by the lower cost of the nozzle.
- The downstream connection of the manometer may not necessarily be at the throat of the nozzle or at a point sufficiently far from the nozzle.
- The deviations are taken care of in the values of C_d, The coefficient C_d depends on the shape of the nozzle, the ratio of pipe to nozzle diameter and the Reynolds number of flow.

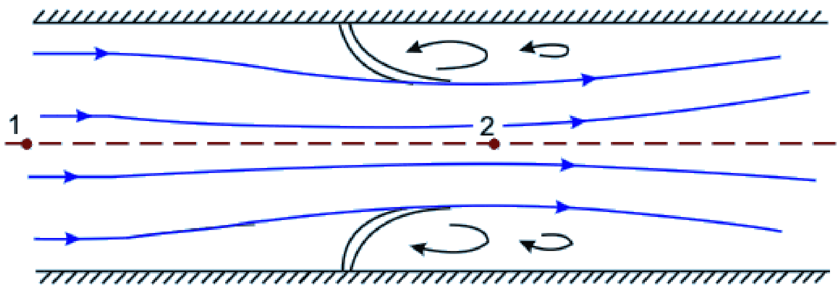


Figure 4: A flow nozzle.

- A comparative picture of the typical values of C_d, accuracy, and the cost of three flow meters (venturimeter, orificemeter and flow nozzle) is given below:

Type of Flowmeter	Accuracy	Cost	Loss of Total Head	Typical Values of C_d
Venturimeter	High	High	Low	0.95 to 0.98
Orificemeter	Low	Low	High	0.60 to 0.65
Flow Nozzle	Intermediate between a venturimeter and an orificemeter			0.70o 0.80

2- ***Orifice in a Reservoir.***

(h) is the head is measured from the center of the orifice to the free surface as in Fig.5. Bernoulli's Eq. applied from a point (1) on the free surface to the center of the *vena contracta* point (c). Neglecting losses, is written

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_c}{\rho g}+\frac{V_c^2}{2g}+z_c$$

$h=\frac{V_c^2}{2g}--\rightarrow\ V_C=\sqrt{2gh}$ also $V_{ci}=\sqrt{2gh}$ where V_{ci} is the theoretical velocity.

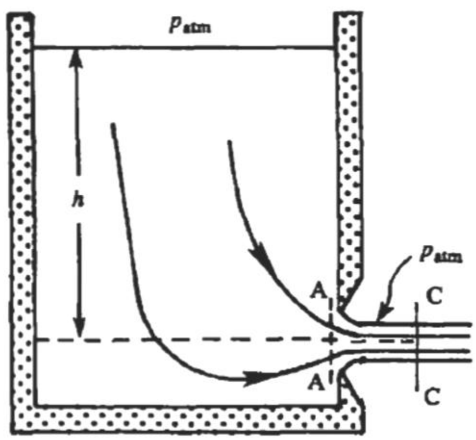


Figure 5: Flow through a sharp edge orifice.

To calculate the discharge from orifice in reservoir, we must find the actual velocity (V_{ca}). C_v is the coefficient of velocity

$$C_V=\frac{V_{ca}}{V_{ci}}---\rightarrow V_{ca}=C_V\quad V_{ci}$$

$$V_{ca}=C_V\sqrt{2gh}$$

To calculate the actual flow rate $A_c=C_cA_2$, where A_2 is the orifice area

$$\therefore Q_{act}=A_cV_{ca}=C_c\,C_VA_2\sqrt{2gh}$$

C_d is the coefficient of discharge

$$C_d=C_cC_V \qquad or\ C_d=\frac{Q_{act}}{Q_i}$$

$Q_{act} = C_d A_2 \sqrt{2gh}$

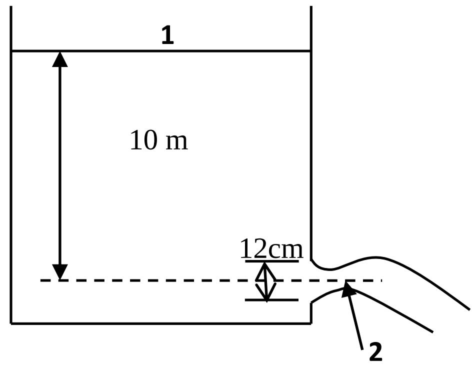
$Q_{act} = C_d\, Q_i$

Ex.3

As in figure the orifice diameter is ***(12 cm)*** in reservoir and the level of water above the orifice is ***(10 m)***. Calculate the actual flow rate when the coefficient of discharge is ***(0.65)***.

Sol.

Applying B.E. between point ***1 & 2***.



$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_{ci}^2}{2g} + z_2$

$10 = \frac{V_{Ci}^2}{2*9.8}$

$V_{ci} = \sqrt{10*2*9.81} = 14.0\,\frac{m}{s}$

$Q_i = A_2 * V_{ci} = \frac{\pi}{4}(0.12)^2 * 14 = 0.15833\,\frac{m^3}{s}$

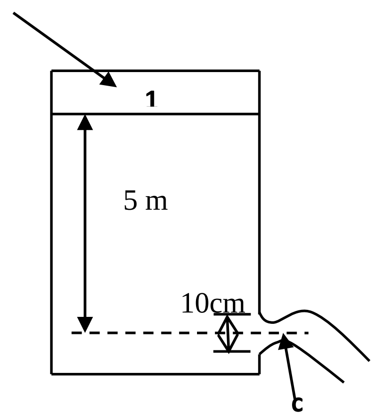
$Q_{act} = C_d Q_i = 0.65 * 0.15833 = 0.1029\,\frac{m^3}{s}$

Ex.4

Calculate the actual flow rate from the orifice diameter ***(10 cm)*** in reservoir forming a *vina contracta* diameter ***(8.5 cm)*** and the $(C_V\ \&\ C_C)$ as in figure. Take the discharge coefficient $C_d = (0.58) \& S.G. = 0.9$

Sol.

Apply B.E. between (1&c)



$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c$

$\frac{35*10^3}{0.9*9810} + 5 = \frac{V_{ci}^2}{2*9.81} ----\rightarrow V_{ci} = 13.26\,\frac{m}{s}$

$V_{2i} = V_{ci} = 13.26\,\frac{m}{s}$

$Q_i = A_2 * V_{2i} = \frac{\pi}{4} * (0.1)^2 * 13.26 = 0.1041\,\frac{m^3}{s}$

$Q_{act} = C_d * Q_i = 0.58 * 0.1041 = 0.0604\,\frac{m^3}{s}$

$C_c = \frac{A_c}{A_2} = \frac{\frac{\pi}{4}(0.085)^2}{\frac{\pi}{4}(0.1)^2} = 0.7225$

$C_d = C_V C_C ----\rightarrow C_V = \frac{C_d}{C_C} = \frac{0.58}{0.7225}$

$C_V = 0.8$