Interpolation

Given (n+ 1) distinct points:

x_i	x_0	x_1	.....	x_n
y_i	y_0	y_1	.....	y_n

Required: Equation of a polynomial **passing** through these points

Direct Substitution Method

- Substitute the known points into the polynomial equation (no. of a's = no. of known points)

$$P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

- Solve the resulting linear algebraic system of equations

- To find the y value for any x, substitute the x value in the polynomial equation and find the corresponding y value.

Example 1: For the following data, use the **Direct Substitution Method** (**DSM**) to find y = f(x) and y at x = 3.2

x	1	2	4	5
y	4	9	61	120

Solution

There are 4 points and thus the polynomial will contain 4 coefficients (4 unknowns) or the polynomial is of degree 3

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Substitute the given points into the polynomial equation

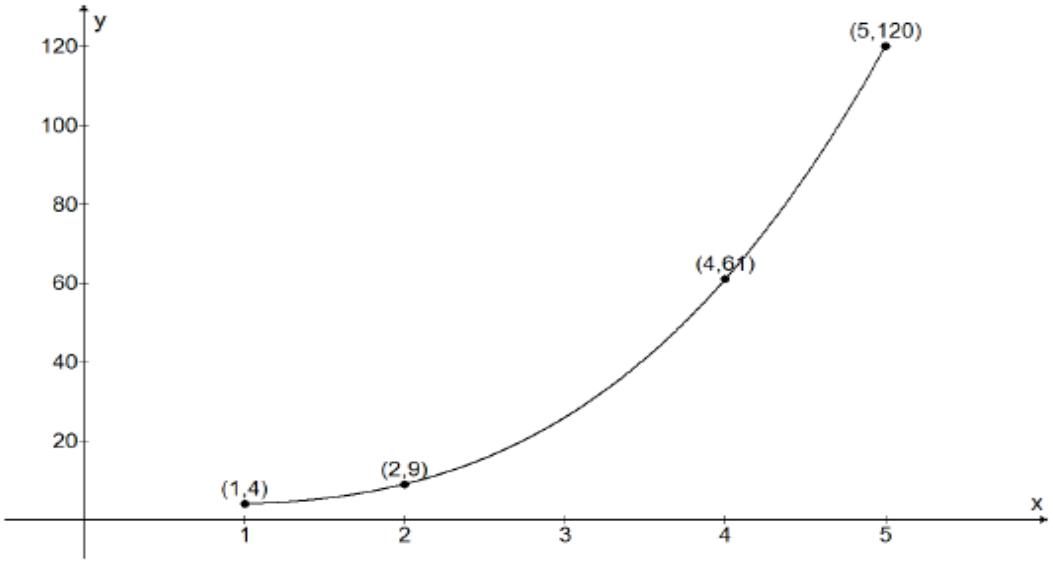
$$\begin{aligned}
a_0 &+ a_1(1) + a_2(1)^2 + a_3(1)^3 = 4 \\
a_0 &+ a_1(2) + a_2(2)^2 + a_3(2)^3 = 9 \\
a_0 &+ a_1(4) + a_2(4)^2 + a_3(4)^3 = 61 \\
a_0 &+ a_1(5) + a_2(5)^2 + a_3(5)^3 = 120
\end{aligned}$$

Solve this linear system of equations to get

$$a_0 = 5, \quad a_1 = -2, \quad a_2 = 0, \quad a_3 = 1$$

Substitute these values into the polynomial equation to get

$$y = 5 - 2x + x^3$$



$$y\big|_{(x\,=\,3.2)} = 5 - 2(3.2) + (3.2)^3 = 31.368$$

Lagrange Interpolation Polynomial

Here the polynomial passing through the given points is given directly (without solving simultaneous equations) and no. of L's = no. of known points

$$y = L_0 y_0 + L_1 y_1 + L_2 y_2 + \cdots + L_n y_n = \sum_{i=0}^{i=n} L_i y_i$$

where

$$L_0 = \frac{(x - x_1)(x - x_2)(x - x_3)\cdots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\cdots(x_0 - x_n)}$$

$$L_1 = \frac{(x - x_0)(x - x_2)(x - x_3)\cdots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\cdots(x_1 - x_n)}$$

$$\vdots$$

$$L_n = \frac{(x - x_0)(x - x_1)(x - x_2)\cdots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\cdots(x_n - x_{n-1})}$$

Example 2: For the following data, use **Lagrange Interpolation Polynomial** to find y at x = 3.2

x	1	2	4	5
y	4	9	61	120

Solution

Here there are 4 points. Then $y = L_0 y_0 + L_1 y_1 + L_2 y_2 + L_3 y_3$

$$y = \frac{(x-2)(x-4)(x-5)}{(1-2)(1-4)(1-5)} * 4 + \frac{(x-1)(x-4)(x-5)}{(2-1)(2-4)(2-5)} * 9 + \frac{(x-1)(x-2)(x-5)}{(4-1)(4-2)(4-5)} * 61 + \frac{(x-1)(x-2)(x-4)}{(5-1)(5-2)(5-4)} * 120$$

Substitute x = 3.2 to get y = 31.368

Notes:

1. To check the solution, ΣL = 1

   $L_0 + L_1 + L_2 + L_3 = -0.144 + 0.528 + 0.792 - 0.176 = 1$

2. If y = f(x) is required, then simplify the above equation to get

   $y = 5 - 2x + x^3$.