Metaheuristics for Multiobjective Optimization (MOP)

The success of metaheuristics on hard single-objective optimization problems is well recognized today. **However**, many real-life problems require taking into account several conflicting points of view corresponding to multiple objectives.

Indeed, optimization problems encountered in practice are seldom monoobjective. In general, there are many conflicting objectives to handle.

minimize the cost of product.

maximize the quality of product.

maximize the collected profit of product.

minimize the environmental impact of product.

Gandibleux, et. al. "Metaheuristics for Multiobjective Optimisation", Springer Science & Business Media, 2004

Talbi, E.-G. Metaheuristics: from design to implementation (Vol. 74): John Wiley & Sons, 2009.

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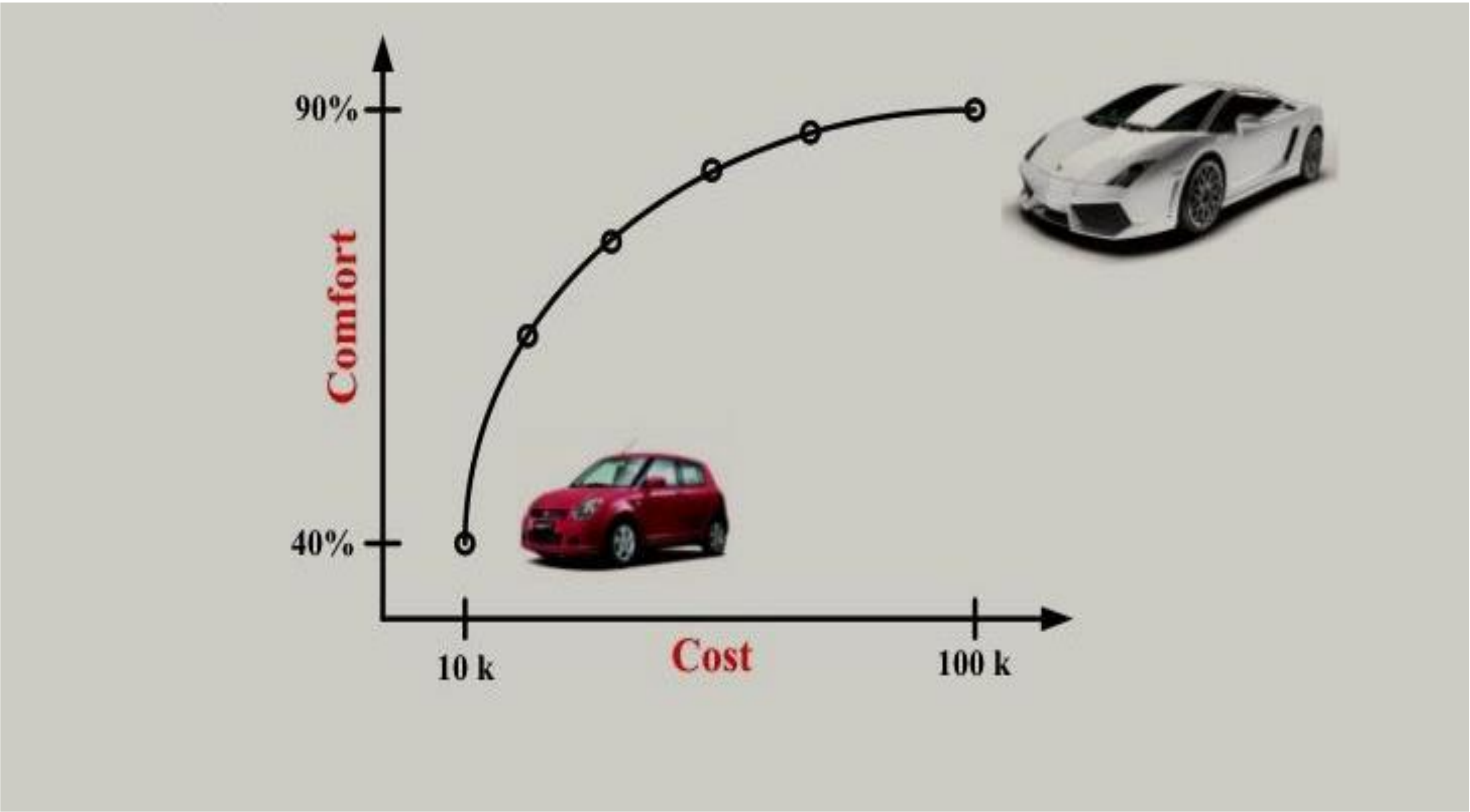

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The complexity of MOPs becomes more and more significant in terms of the size of the problem to be solved (e.g., number of objectives, size of the search space).

The optimal solution for MOPs is not a single solution as for monoobjective optimization problems, but a set of solutions defined as Pareto optimal solutions.

A solution is Pareto optimal if it is not possible to improve a given objective without deteriorating at least another objective.
This set of solutions represents the compromise solutions between the different conflicting objectives.

When metaheuristics are applied, the goal becomes to obtain an approximation of the Pareto optimal set having two properties:	convergence to the Pareto optimal	ensures the generation of near-optimal Pareto solutions.
	uniform diversity	indicates a good distribution of the obtained solutions around the Pareto optimal front.

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Compared to monoobjective optimization, the difficulty in solving MOPs lies in the following general facts

no commonly used definitions on the global optimality of a solution. The final choice depends on the decision maker.

The number of Pareto optimal solutions increases according to the size of the problem and mainly with the number of objectives being considered.

$$MOP = \begin{cases} \min \quad F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ \qquad\qquad s.c. \quad x \in S \end{cases}$$

- where n (n ≥ 2) is the number of objectives.
- S represents the set of feasible solutions .
- F(x) is the vector of objectives to be optimized.

The evaluation of a solution on each criterion based on: **the decision maker**.

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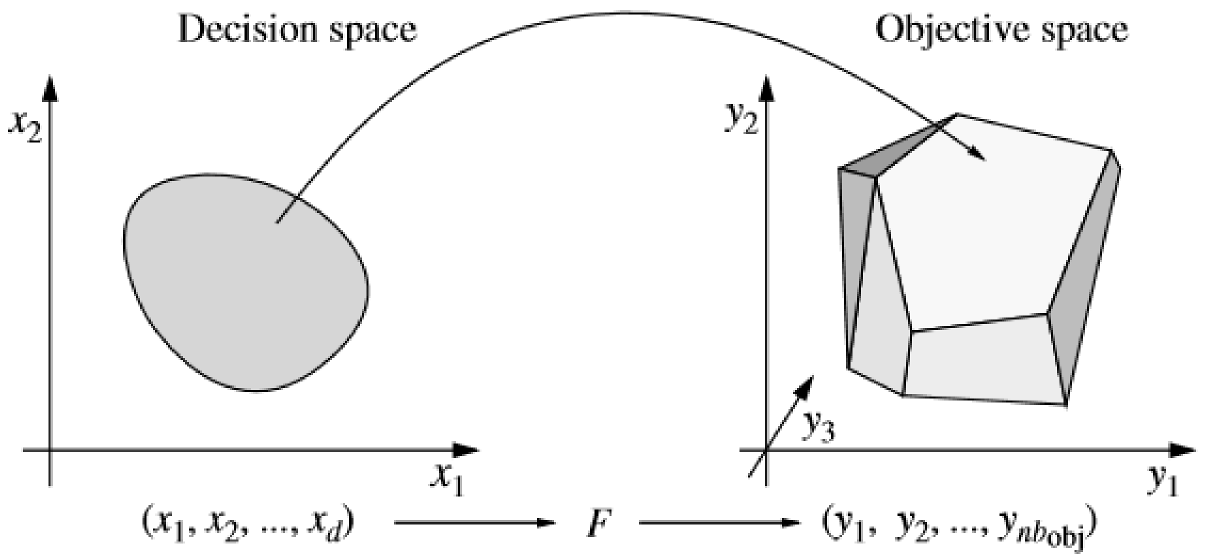

FIGURE 4.1 Decision space and objective space in a MOP.

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It is not usual to have a solution x', associated with a decision variable vector, where x' is optimal for all the objectives.

$$\forall x \in S, \quad f_i(x') \leq f_i(x), \qquad\qquad i = 1,2,\ldots,n$$

Given that this situation is not usual in real-life MOPs where the criteria are in conflict, other concepts were established to consider optimality:

Pareto dominance

An objective vector $u = (u_1,\ldots,u_n)$ is said to **dominate** $v = (v_1,\ldots,v_n)$ (denoted by $u \prec v$) if and only if no component of $\boldsymbol{v}$ is smaller than the corresponding component of $\boldsymbol{u}$ and at least one component of $\boldsymbol{u}$ is strictly smaller, that is:

$$\forall i \in \{1,\ldots,n\} : u_i \leq v_i \wedge \exists i \in \{1,\ldots,n\} : u_i < v_i$$

A **Pareto optimal solution** denotes that it is impossible to find a solution that improves the performances on a criterion without decreasing the quality of at least another criterion.

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Pareto optimality

*A solution $x^* \in S$ is **Pareto optimal** if for every $x \in S$, $F(x)$ does not dominate $F(x^*)$, that is, $F(x) \nprec F(x^*)$.*

In general, searching in a monoobjective problem leads to find a unique global optimal solution. A MOP may have a set of solutions known as the Pareto optimal set. The image of this set in the **objective space** is denoted as the Pareto front.

Pareto optimal set

*For a given MOP (F,S), the **Pareto optimal set** is defined as $P^* = \{x \in S / \nexists x' \in S, F(x') \prec F(x)\}$.*

Pareto front

*For a given MOP (F,S) and its Pareto optimal set P^*, the **Pareto front** is defined as $PF^* = \{F(x),\ x \in P^*\}$.*

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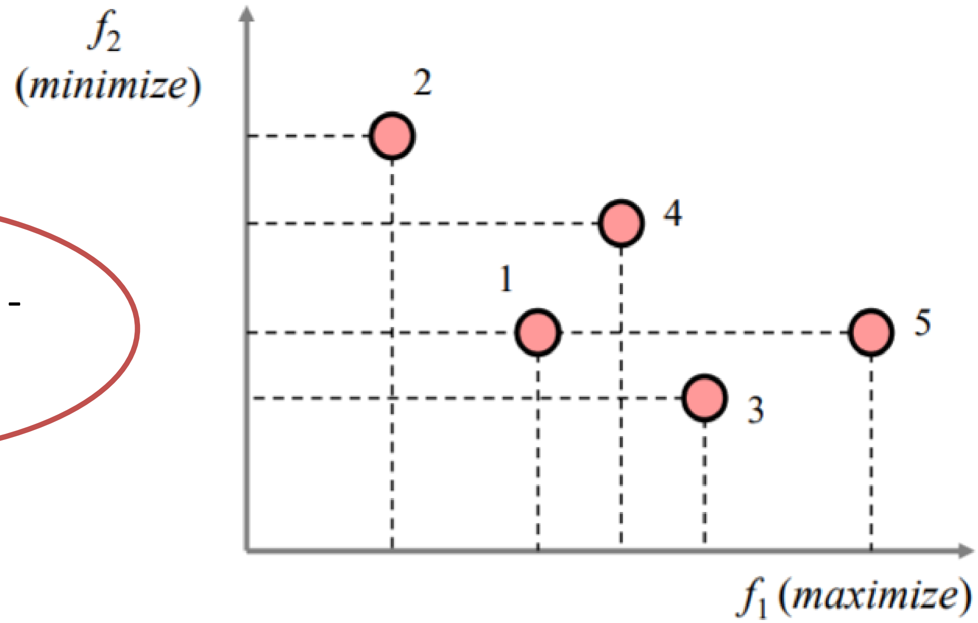
Dominance Test - Example



- 1 Vs 2: 1 dominates 2
- 1 Vs 5: 5 dominates 1
- 1 Vs 4: Neither solution dominates

https://engineering.purdue.edu/~sudhoff/ee630/Lecture09.pdf

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Multicriteria Decision Making

The aim of solving MOPs is to help a decision maker to find a Pareto solution. One of the fundamental questions in MOPs resolution is related to the **interaction** between **the problem solver** (e.g., metaheuristic) and **the decision maker**.

The role of the decision maker is to specify some extra information to select his favorite solution. This interaction can take one of the three following approaches.
Each approach has its weaknesses and strengths. The choice of a method depends on the problem properties and the abilities of the decision maker.

A priori

Here, the decision maker provides his preferences before the optimization process. For example, the decision maker is supposed to evaluate a priori the ***weight*** of each objective to define the utility function. The decision maker must have a minimum **knowledge** on his problem.

A posteriori

the search process determines a set of Pareto solutions. Then, the decision maker chooses one solution from the set of solutions provided by the solver. This approach is practical when the number of objectives is **small**.

Interactive

there is a progressive interaction between the decision maker and the solver. From the knowledge extracted during the problem resolution, the decision maker defines his preferences. If the decision maker is rational and the problem is well formulated, the final solution is always Pareto optimal

THANK YOU

$$e^{\frac{-\Delta f}{T}}$$