## Lecture 13

### 2.4 The Chemical Reaction Equation and Stoichiometry

The stoichiometric equation of a chemical reaction is a statement of the relative number of molecules or moles of reactants and products that participate in the reaction. For example, the stoichiometric equation
$2 \mathrm{SO}_{2}+\mathrm{O}_{2} \rightarrow 2 \mathrm{SO}_{3}$
indicates that for every two molecules ( g -moles, lb-moles) of $\mathrm{SO}_{2}$ that react, one molecule ( g -mole, lb -mole) of $\mathrm{O}_{2}$ reacts to produce two molecules (g-moles, lb-moles) of $\mathrm{SO}_{3}$.

The numbers that precede the formulas for each species are the stoichiometric coefficients of the reaction components.

The stoichiometric ratio of two molecular species participating in a reaction is the ratio of their stoichiometric coefficients in the balanced reaction equation. For the reaction: $\quad 2 \mathrm{SO}_{2}+\mathrm{O}_{2} \rightarrow 2 \mathrm{SO}_{3}$
you can write the stoichiometric ratios:

$$
\frac{2 \text { moles } \mathrm{SO}_{3} \text { generated }}{1 \mathrm{~mol} \mathrm{O} \mathrm{O}_{2} \text { consumed }}, \quad \frac{2 \mathrm{lb} \text { moles } \mathrm{SO}_{2} \text { consumed }}{2 \mathrm{lb} \text { moles } \mathrm{SO}_{3} \text { generated }}
$$

For example, if $1600 \mathrm{~kg} / \mathrm{h}$ of $\mathrm{SO}_{3}$ is to be produced, you can calculate the amount of oxygen required as:
$\frac{1600 \mathrm{SO}_{3} \text { generated }}{h}\left|\frac{1 \mathrm{kmol} \mathrm{SO}_{3}}{80 \mathrm{~kg} \mathrm{SO}_{3}}\right| \frac{1 \mathrm{kmol}_{2} \text { consumed }}{2 \mathrm{kmol} \mathrm{SO}}{ }_{3}$ generated $=10 \frac{\mathrm{kmol} \mathrm{O}_{2}}{h}$
$\longrightarrow 10 \frac{\mathrm{kmol}_{2}}{\mathrm{~h}} \left\lvert\, \frac{32 \mathrm{~kg} \mathrm{o}}{1 \mathrm{kmol}_{2}}=320 \mathrm{~kg} \mathrm{o} / \mathrm{h}\right.$

## Example 1

In the combustion of heptane, $\mathrm{CO}_{2}$ is produced. Assume that you want to produce 500 kg of dry ice per hour, and that $50 \%$ of the $\mathrm{CO}_{2}$ can be converted into dry ice, as shown in Figure E9.2. How many kilograms of heptane must be burned per hour?


## Solution

$\mathrm{C}_{7} \mathrm{H}_{16}+11 \mathrm{O}_{2} \rightarrow 7 \mathrm{CO}_{2}+8 \mathrm{H}_{2} \mathrm{O}$

Basis: 500 kg of dry ice (equivalent to 1 hr )
$\left.\underline{500 \mathrm{~kg} \text { dry ice }}\left|\frac{1 \mathrm{~kg} \mathrm{CO}_{2}}{0.5 \mathrm{~kg} \text { dry ice }}\right| \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{CO}}{2} \right\rvert\, \frac{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{7}$ H $\mathrm{H}_{16}$
$\left\lvert\, \frac{100.1 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}}{1 \mathrm{~kg} \mathrm{~mol} \mathrm{C}}{ }_{7} \mathrm{H}_{16} \quad=325 \mathrm{~kg} \mathrm{C}_{7} \mathrm{H}_{16}\right.$

## Extent of Reaction

The extent of reaction, $\xi$, denotes how much reaction occurs.

The extent of reaction is defined as follows:
$\xi=\frac{n_{i}-n_{i o}}{v_{i}}$
$\mathrm{n}_{\mathrm{i}}=$ moles of species i present in the system after the reaction occurs,
$\mathrm{n}_{\mathrm{io}}=$ moles of species i present in the system when the reaction starts,
$v_{i}=$ coefficient for species $i$ in the particular chemical reaction equation.
$\xi=$ extent of reaction (moles reacting)

- The coefficients of the products in a chemical reaction are assigned positive values and the reactants assigned negative values. Note that ( $\mathrm{n}_{\mathrm{i}}{ }^{-}$ $\mathrm{n}_{\mathrm{io}}$ ) is equal to the generation or consumption of component i by reaction.

Equation (1) can be rearranged to calculate the number of moles of component $i$ from the value of the extent of reaction:

$$
\begin{equation*}
n_{i}=n_{i 0}+\xi v_{i} \tag{2}
\end{equation*}
$$

## Example 2

Determine the extent of reaction for the following chemical reaction
$\mathrm{N}_{2}+3 \mathrm{H}_{2} \longrightarrow 2 \mathrm{NH}_{3}$
given the following analysis of feed and product:

|  | $\mathbf{N}_{\mathbf{2}}(\mathbf{g})$ | $\mathbf{H}_{2}(\mathbf{g})$ | $\mathbf{N H}_{\mathbf{3}}(\mathbf{g})$ |
| :--- | :--- | :--- | :--- |
| Feed | 100 | 50 | 5 |
| Product | --- | --- | 90 |

Also, determine the g and g mol of $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$ in the product.

## Solution

The extent of reaction can be calculated by applying Equation 1 based on $\mathrm{NH}_{3}:$
$n_{i}=90 \mathrm{~g} \mathrm{NH}_{3} \left\lvert\, \frac{1 \mathrm{~mol} \mathrm{NH}_{3}}{17 \mathrm{~g} \mathrm{NH}_{3}}=5.294 \mathrm{~mol} \mathrm{NH} 3\right.$
$n_{i o}=5 g \mathrm{NH}_{3} \left\lvert\, \frac{1 \mathrm{~mol} \mathrm{NH}_{3}}{17 g \mathrm{NH}_{3}}=0.294 \mathrm{~mol} \mathrm{NH}_{3}\right.$

$$
\xi=\frac{n_{i}-n_{i o}}{v_{i}}=\frac{(5.294-0.294) \mathrm{mol} \mathrm{NH}_{3}}{2}=2.5 \text { moles reacting }
$$

Equation 2 can be used to determine the mol of $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$ in the products of the reaction:
$\mathrm{N}_{2}$ :
$n_{i 0}=100 g N_{2} \left\lvert\, \frac{1 \mathrm{~mol} \mathrm{~N}_{2}}{28 g \mathrm{~N}_{2}}=3.57 \mathrm{~mol} \mathrm{~N}_{2}\right.$
$n_{N 2}=3.57+(-1)(2.5)=1.07 \mathrm{~mol} \mathrm{~N}_{2}$
$m_{N 2}=1.07 \mathrm{~mol} N_{2} \left\lvert\, \frac{28 g N_{2}}{1 \mathrm{~mol} \mathrm{~N}_{2}}=30 \mathrm{~g} \mathrm{~N}\right.$
$\mathrm{H}_{2}$ :
$n_{i 0}=50 \mathrm{~g} \mathrm{H}_{2} \left\lvert\, \frac{1 \mathrm{~mol} \mathrm{H}_{2}}{2 g \mathrm{H}_{2}}=25 \mathrm{~mol} \mathrm{H}_{2}\right.$
$n_{H 2}=25+(-3)(2.5)=17.5 \mathrm{~mol} \mathrm{H}_{2}$
$m_{N 2}=17.5 \mathrm{~mol} \mathrm{H}_{2} \left\lvert\, \frac{2 g \mathrm{H}_{2}}{1 \mathrm{~mol} \mathrm{H}_{2}}=35 \mathrm{~g} \mathrm{H}_{2}\right.$

Note: If several independent reactions occur in the reactor, say $k$ of them, $\quad \xi$ can be defined for each reaction, with $v_{\mathrm{ki}}$ being the stoichiometric coefficient of species $i$ in the kth reaction, the total number of moles of species i is:
$n_{i}=n_{i 0}+\sum_{k=1}^{R} v_{k i} \xi_{k}$

Where R is the total number of independent reactions.

