## Lecture 11

## Example 6

Figure below illustrates a nanoporous membrane that is used in the separation of nitrogen and oxygen from air. What is the composition of the waste stream if the waste stream amounts to $80 \%$ of the input stream?


## Solution

Basis: 100 g mol of F

Tatal mole balance:

Input $=$ output
$\mathrm{F}=\mathrm{P}+\mathrm{W}$
$100=P+0.8(100)$
$\mathrm{P}=20 \mathrm{~mol}$
$\underline{\mathrm{O}}_{2}$ mol balance

Input $=$ output
$0.21(\mathrm{~F})=0.25(\mathrm{P})+\mathrm{y}_{\mathrm{O} 2}(\mathrm{~W})$
$0.21(100)=0.25(20)+\mathrm{y}_{\mathrm{O} 2}(80)$
$y_{02}=0.2$

$$
\mathrm{y}_{\mathrm{N} 2}=1-0.2
$$

$$
y_{\mathrm{N} 2}=0.8
$$

## Example 7

A cylinder containing $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}$, and $\mathrm{N}_{2}$ has to be prepared containing a $\mathrm{CH}_{4}$ to $\mathrm{C}_{2} \mathrm{H}_{6}$ mole ratio of 1.5 to 1 . Available to prepare the mixture are (1) a cylinder containing a mixture of $80 \% \mathrm{~N}_{2}$ and $20 \% \mathrm{CH}_{4}$, (2) a cylinder containing a mixture of $90 \% \mathrm{~N}_{2}$ and $10 \% \mathrm{C}_{2} \mathrm{H}_{6}$, and (3) a cylinder containing pure $\mathrm{N}_{2}$. What is the number of degrees of freedom, i.e., the number of independent specifications that must be made, so that you can determine the respective contributions from each cylinder to get the desired composition in the cylinder with the three components?

## Solution



Do you count seven unknowns-three values of $x_{\mathrm{i}}$ and four values of $F_{i}$ ? How many independent equations can be written?
Three material balances: $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}$, and $\mathrm{N}_{2}$ One specified ratio: moles of $\mathrm{CH}_{4}$ to $\mathrm{C}_{2} \mathrm{H}_{6}$ equal 1.5 or $\left(\mathrm{x}_{\mathrm{CH}_{4}} / \mathrm{X}_{\mathrm{C}_{2} \mathrm{H}_{6}}\right)=1.5$ One summation of mole fractions: $\Sigma x_{i}^{P_{4}}=1$

Thus, there are seven minus five equals two degrees of freedom. If you pick a basis, such as $F_{4}=1$, one other value has to be specified to solve the problem to calculate composition of $F_{4}$. Keep in mind that you must be careful in making any specifications to maintain only independent equations. Avoid transforming one or more independent equations in a set such that the resulting set contains redundant (dependent) equations. Did you notice in the problem formulation that the $\sum x_{i}=1$ equations for $\mathrm{F}_{1}, \mathrm{~F}_{2}$, and $\mathrm{F}_{3}$ were redundant because of the way the specification of the mole fractions was made?

## Example 8

Membranes represent a relatively new technology for the separation of gases. One use that has attracted attention is the separation of nitrogen and oxygen from air. Figure E8.2a illustrates a nanoporous membrane that is made by coating a very thin layer of polymer on a porous graphite supporting layer.

What is the composition of the waste stream if the waste stream amounts to $80 \%$ of the input stream?


## Solution

Basis: $100 \mathrm{~g} \mathrm{~mol}=F$

A degree of freedom analysis that includes all of the variables comes next.

Number of variables: 9

$$
F, P, W \text { and } 6 n_{i}
$$

Number of equations: 9

Specifications: $\quad n_{\mathrm{O}_{2}}^{F}=0.21(100)=21$

$$
n_{N_{2}}^{F}=0.79(100)=79
$$

$$
y_{\mathrm{O}_{2}}^{P}=n_{\mathrm{O}_{2}}^{P} / P=0.25 \quad n_{\mathrm{O}_{2}}^{P}=0.25 P
$$

$$
y_{N_{2}}^{P}=n_{N_{2}}^{P} / P=0.75 \quad n_{N_{2}}^{P}=0.75 P
$$

$$
W=0.80(100)=80
$$

Material balances: $\mathrm{O}_{2}$ and $\mathrm{N}_{2}$
Implicit equations: $\Sigma n_{i}^{W}=W$ or $\Sigma y_{i}^{W}=1$

The problem has zero degrees of freedom

|  | In | Out | In | Out |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{2}$ | 0.21 (100) | $=0.25 P+y^{W} W_{2}(80)$ | 0.21 (100) | $=0.25 P+n_{O_{2}}^{W}$ |
| $\mathrm{N}_{2}$ | 0.79 (100) | $=0.75 P+y_{N_{2}}^{W}(80)$ | 0.79 (100) | $=0.75 P+n_{N_{2}}^{W}$ |
|  | 1.00 | $=y_{\mathrm{O}_{2}}^{W}+y_{\mathrm{N}_{2}}^{W}$ | 80 | $=n_{\mathrm{O}_{2}}^{W}+n_{\mathrm{N}_{2}}^{W}$ |

The solution of these equations is $n_{\mathrm{O}_{2}}^{W}=16$ and $n_{\mathrm{N}_{2}}^{W}=64$, or $y_{\mathrm{O}_{2}}^{W}=0.20$ and $y_{\mathrm{N}_{2}}^{W}=0.80$, and $P=20 \mathrm{~g} \mathrm{~mol}$.

## Problems

1. Strawberries contain about $15 \mathrm{wt} \%$ solids and $85 \mathrm{wt} \%$ water. To make strawberry jam, crushed strawberries and sugar are mixed in a $45: 55$ mass ratio, and the mixture is heated to evaporate water until the residue contains one-third water by mass. Calculate how many pounds of strawberries are needed to make a pound of jam.
2. Three hundred gallons of a mixture containing $75.0 \mathrm{wt} \%$ ethanol (ethyl alcohol) and $25 \%$ water (mixture specific gravity 0.877 ) and a quantity of a $40.0 \mathrm{wt} \%$ ethanol- $60 \%$ water mixture (SG 0.952) are blended to produce a mixture containing $60.0 \mathrm{wt} \%$ ethanol. Calculate the required volume of the $40 \%$ mixture $\left(\mathrm{V}_{40}\right)$.
