Lecture 3

Non dimensional Groups:

As you proceed with the study of chemical engineering, you will find that groups of symbols may be put together, either by theory or based on experiment, that have no net units. Such collections of variables or parameters are called dimensionless or nondimensional groups.

One example is the Reynolds number (group) arising in fluid mechanics.

$$\text{Reynolds number} = \frac{Dv\rho}{\mu} = N_{RE}$$

where D is the pipe diameter, say in cm; ν is the fluid velocity, say in cm/s; ρ is the fluid density, say in g/cm^3; and μ is the viscosity, say in centipoise, units that can be converted to g/(cm)(s). Introducing the consistent set of units for D, ν, ρ, and μ into Dνρ/μ, you will find that all the units cancel out so that the numerical value of 1 is the result of the cancellation of the units.

$$\frac{\cancel{cm}}{} \left| \frac{\cancel{cm}}{\cancel{s}} \right| \frac{\cancel{g}}{\cancel{cm^3}} \left| \frac{(\cancel{cm})\,(\cancel{s})}{\cancel{g}} \right.$$

Example 7

Explain without differentiating why the following differentiation cannot be correct:

$$\frac{d}{dx}\sqrt{1 + (x^2/a^2)} = \frac{2ax}{\sqrt{1 + (x^2/a^2)}}$$

where **x** is length and **a** is a constant.

Solution

- Observe that **x** and **a** must have the same units because the ratio x^2/a^2 must be dimensionless (because 1 is dimensionless).

- Thus, the left-hand side of the equation has units of **1/x** (from d/dx). However, the right-hand side of the equation has units of $\mathbf{x^2}$ (the product of **a*x**).

- Consequently, something is wrong as the equation is not dimensionally consistent.

Example 8

The following equation is proposed to calculate the pressure drop (Δp) across a length of pipe (L) due to flow through the pipe. Determine the dimensional consistency of this equation:

$$\Delta p = {}^1\!/_2 v^2 \left(\frac{L}{D}\right) f$$

where v is the average velocity of the fluid flowing through the pipe, D is the diameter of the pipe, and f is a dimensionless coefficient called the friction factor, which is a function of the Reynolds number.

Solution

Let's substitute SI units appropriate for each term into the proposed equation, recognizing that pressure is force per unit area (see Table 2.1). What are the units of Δp? They are

$$\frac{\text{N}}{\text{m}^2} = \frac{(\text{kg})(\text{m})}{\text{s}^2}\bigg|\frac{}{\text{m}^2} \rightarrow \frac{\text{kg}}{(\text{s}^2)(\text{m})}$$

What are the net units of the right-hand side of the proposed equation?

$$\left(\frac{\text{m}}{\text{s}}\right)^2 \Bigg| \frac{\text{m}}{\text{m}} \rightarrow \frac{\text{m}^2}{\text{s}^2}$$

Therefore, because the units of the left-hand side of the equation do not match the units of the right-hand side, the proposed equation is not dimensionally consistent. By some research or inspection, it was determined that the proposed equation was missing a density term on the right-hand side of the equation; that is, the equation should be

$$\Delta p = {}^1\!/_2\, \nu^2 \rho \left(\frac{L}{D}\right) f$$

With this modification, the units on the right-hand side of the equation become

$$\left(\frac{\text{m}}{\text{s}}\right)^2 \Bigg| \frac{\text{kg}}{\text{m}^3} \Bigg| \frac{\text{m}}{\text{m}} \rightarrow \frac{\text{kg}}{(\text{s}^2)(\text{m})}$$

Therefore, if the density is included, this equation is shown to be dimensionally consistent.

Questions

1. Which of the following best represents the force needed to lift a heavy suitcase?

a. 25 N b. 25 kN c. 250 N d. 250 kN

2. Pick the correct answer(s); a watt is

a. one joule per second b. equal to 1 (kg)(m^2)/s^2

c. the unit for all types of power

d. all of the above

e. none of the above

3. Is kg/s a basic or derived unit in SI?

4. Answer the following questions yes or no. Can you

a. divide ft by s? b. divide m by cm? c. multiply ft by s?

d. divide ft by cm? e. divide m by (deg) K? f. add ft and s?

g. subtract m and (deg) K h. add cm and ft? i. add cm and m^2?

j. add 1 and 2 cm?

5. Why is it not possible to add 1 ft and 1 ft^2?

7. Is the ratio of the numerator and denominator in a conversion factor equal to unity?

8. What is the difference, if any, between pound force and pound mass in the AE system?

9. Could a unit of force in the SI system be kilogram force?

10. What is the weight of a one pound mass at sea level? Would the mass be the same at the center of Earth? Would the weight be the same at the center of Earth?

11. What is the mass of an object that weighs 9.80 kN at sea level?

12. Explain what dimensional consistency means in an equation.

13. Explain why the so-called dimensionless group has no net dimensions.

14. If you divide all of a series of terms in an equation by one of the terms, will the resulting series of terms be dimensionless?

15. How might you make the following variables dimensionless:

a. Length (of a pipe). b. Time (to empty a tank full of water).

Problems

1. Classify the following units as correct or incorrect units in the SI system:

a. nm b. K c. sec d. N/mm e. kJ/(s)(m^3)

2. Add 1 cm and 1 m.

3. Subtract 3 ft from 4 yards.

4. Divide 3 $m^{1.5}$ by 2 $m^{0.5}$.

5. Multiply 2 ft by 4 lb.

7. Electronic communication via radio travels at approximately the speed of light (186,000 miles/second).The edge of the solar system is roughly at Pluto, which is 3.6×10^9 miles from Earth at its closest approach. How many hours does it take for a radio signal from Earth to reach Pluto?

8. Determine the kinetic energy of one pound of fluid moving in a pipe at the speed of 3 feet per second.

9. Convert the following from AE to SI units:

a. 4 lb_m/ft to kg/m b. 1.00 lb_m/(ft^3)(s) to kg/(m^3)(s)

10. Convert the following 1.57×10^{-2}g/(cm)(s) to lb_m/(ft)(s)

11. Convert 1.1 gal to ft^3.

12. Convert 1.1 gal to m^3.

13. An orifice meter is used to measure the rate of flow of a fluid in pipes. The flow rate is related to the pressure drop by the following equation:

$$u = c\sqrt{\frac{\Delta P}{\rho}}$$

Where u = fluid velocity

Δp = pressure drop 1force per unit area^2

ρ= density of the flowing fluid

c = constant

What are the units of c in the SI system of units?

14. The thermal conductivity k of a liquid metal is predicted via the empirical equation:

$$k = A \exp\,(B/T)$$

where k is in J/(s)(m)(K) and A and B are constants. What are the units of A and B?