Lecture two

Conversion of Units and Conversion Factors

The procedure for converting one set of units to another is simply to multiply any number and its associated units by ratios termed conversion factors to arrive at the desired answer and its associated units.

If a plane travels at twice the speed of sound (assume that the speed of sound is 1100 ft/s), how fastis it going in miles per hour?

We formulate the conversion as follows:

$$\frac{2 \times 1100\ \text{ft}}{\text{s}}\left|\frac{1\ \text{mi}}{5280\ \text{ft}}\right|\frac{60\ \text{s}}{1\ \text{min}}\left|\frac{60\ \text{min}}{1\ \text{hr}}\right.$$

$$\frac{\text{ft}}{\text{s}} \qquad \frac{\text{mi}}{\text{s}} \qquad \frac{\text{mi}}{\text{min}}$$

= 1500 mi/hr

Example 2

(a) Convert 2 km to miles. (b) Convert 400 in.^{3}/day to cm^3/min.

Solution

(a) One way to carry out the conversion is to look up a direct conversion factor, namely 1.61 km = 1 mile:

$$\frac{2\ \text{km}}{}\left|\frac{1\ \text{mile}}{1.61\ \text{km}}\right. = 1.24\ \text{mile}$$

Another way is to use conversion factors you know

$$\frac{2\ \cancel{\text{km}}}{}\left|\frac{10^5\ \cancel{\text{cm}}}{1\ \cancel{\text{km}}}\right|\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\left|\frac{1\ \cancel{\text{ft}}}{12\ \cancel{\text{in.}}}\right|\frac{1\ \text{mile}}{5280\ \cancel{\text{ft.}}} = 1.24\ \text{mile}$$

$$\text{(b)}\quad \frac{400\ \text{in.}^3}{\text{day}}\left|\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)^3\right|\frac{1\ \text{day}}{24\ \text{hr}}\left|\frac{1\ \text{hr}}{60\ \text{min}}\right. = 4.55\,\frac{\text{cm}^3}{\text{min}}$$

In part (b) note that not only are the numbers in the conversion of inches to centimeters raised to a power, but the units also are raised to the same power.

Example 3

An example of a semiconductor is ZnS with a particle diameter of 1.8 nanometers. Convert this value to(a) dm (decimeters) and (b) inches.

Solution

$$\text{(a)}\quad \frac{1.8\ \text{nm}}{}\left|\frac{10^{-9}\ \text{m}}{1\ \text{nm}}\right|\frac{10\ \text{dm}}{1\ \text{m}} = 1.8 \times 10^{-8}\ \text{dm}$$

$$\text{(b)}\quad \frac{1.8\ \text{nm}}{}\left|\frac{10^{-9}\ \text{m}}{1\ \text{nm}}\right|\frac{39.37\ \text{in.}}{1\ \text{m}} = 7.09 \times 10^{-8}\ \text{in.}$$

FORCE AND WEIGHT

According to Newton's second law of motion, force is proportional to the product of mass and acceleration (length/time^2). Units are, therefore, kg. m/s^2 (SI) and lb$_m$. ft/s^2 (AE). To avoid having to carry around these complex units in all calculations involving forces, derived force units have been defined in each system.

In the SI system, the derived force units is: **1 newton (N) ≡ 1 kg. m/s^2**

In the AE system, the derived force unit—called a pound-force (lb$_f$)—is defined as the product of a unit mass (1 lb$_m$) and the acceleration of gravity **at sea level and 45° latitude**, which is 32.174 ft/s^2 :

1 lb$_f$ ≡ 32.174 lb$_m$. ft/s^2

For example, the force in newtons required to accelerate a mass of 4.00 kg at a rate of 9.00 m/s^2 is

$$F = \left.\frac{4.00\ \text{kg}}{}\right|\left.\frac{9.00\ \text{m}}{\text{s}^2}\right|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2} = 36.0\ \text{N}$$

The force in lb_f required to accelerate a mass of 4.00 lb_m at a rate of 9.00 ft/s^2 is

$$F = \left.\frac{4.00\ \text{lb}_\text{m}}{}\right|\left.\frac{9.00\ \text{ft}}{\text{s}^2}\right|\frac{1\ \text{lb}_\text{f}}{32.174\ \text{lb}_\text{m}\cdot\text{ft/s}^2} = 1.12\ \text{lb}_\text{f}$$

Example 4

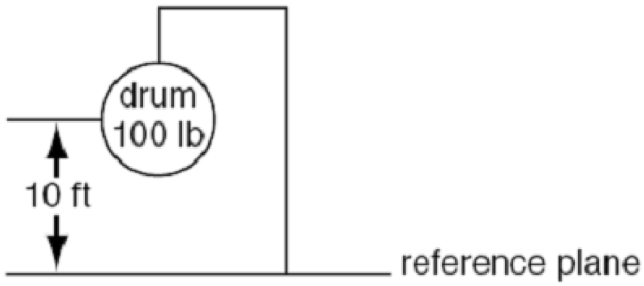
What is the potential energy in (ft)($1b_f$) of a 100 lb drum hanging 10 ft above the surface of the earth with reference to the surface of the earth?

Solution

Potential energy = P = m g h

Assume that the 100 lb means 100 lb mass; g = acceleration of gravity = 32.2 ft/s^2. Figure below is a sketch of the system.

$$P = \left.\frac{100\ \text{lb}_\text{m}}{}\right|\left.\frac{32.2\ \text{ft}}{\text{s}^2}\right|\left.\frac{10\ \text{ft}}{}\right|\frac{(\text{s}^2)(\text{lb}_\text{f})}{32.174(\text{ft})(\text{lb}_\text{m})} = 1000\ (\text{ft})(\text{lb}_\text{f})$$



Example 5

In biological systems, production rate of glucose is 0.6 μg mol/(mL)(min). Determine the production rate of glucose for this system in the units of lb mol/(ft^3)(day).

Solution

Basis: 1 min

$$\frac{0.6\ \mu\text{g mol}}{(\text{mL})(\text{min})}\left|\frac{1\ \text{g mol}}{10^6\ \mu\text{g mol}}\right|\frac{1\ \text{lb mol}}{454\ \text{g mol}}\left|\frac{1000\ \text{mL}}{1\ \text{L}}\right|\frac{1\ \text{L}}{3.531\times10^{-2}\ \text{ft}^3}\left|\frac{60\ \text{min}}{\text{hr}}\right|\frac{24\ \text{hr}}{\text{day}}$$

$$= 0.0539\frac{\text{lb mol}}{(\text{ft}^3)(\text{day})}$$

Dimensional Consistency (Homogeneity)

The concept of dimensional consistency can be illustrated by an equation that represents the pressure/volume/temperature behavior of a gas, and is known as van der Waals's equation.

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

Inspection of the equation shows that the constant **a** must have the units of [(pressure)(volume)^2] for the expression in the first set of parentheses to be consistent throughout. If the units of pressure are atm and those of volume are cm^3, **a** will have the units of [(atm)(cm)^6]. Similarly, **b** must have the same units as V, or in this particular case the units of cm^3.

Example 6

Your handbook shows that microchip etching roughly follows the relation

$$d = 16.2 - 16.2e^{-0.021t} \quad t < 200$$

where d is the depth of the etch in microns (micrometers, µm) and t is the time of the etch in seconds. What are the units associated with the numbers 16.2 and 0.021? Convert the relation so that d becomes expressed in inches and t can be used in minutes.

Solution

Both values of 16.2 must have the associated units of microns (µm). The exponential must be dimensionless so that 0.021 must have the associated units of s^{-1}.

$$d_{in} = \frac{16.2\ \mu\text{m}}{} \left| \frac{1\ \text{m}}{10^6\ \mu\text{m}} \right| \frac{39.27\ \text{in.}}{1\ \text{m}} \left[1 - \exp \frac{-0.021}{s} \left| \frac{60s}{1\ \text{min}} \right| \frac{t_{min}}{} \right]$$

$$= 6.38 \times 10^{-4}(1 - e^{-1.26t_{min}})\ \text{inches}$$