Lecture-Five

Viscous Turbulent Flow in Pipes

1- *Friction Factor Calculations.*

Experimentation shows the following to be true in turbulent flow.

1- The head loss varies directly as the length of the pipe.
2- The head loss varies almost as the square of the velocity.
3- The head loss varies almost inversely as the diameter.
4- The head loss depends upon the surface roughness of the interior pipe wall.
5- The head loss depends upon the fluid properties of density and viscosity.
6- The head loss is independent of the pressure.

$h_f = f\frac{L}{d}.\frac{V^2}{2g}$

$f = f(V, d, \rho, \mu, \epsilon, \acute{\epsilon}, m)$

$\in$ is a measure of the size of the roughness projection and has the dimension of a length.

$\acute{\in}$ is a measure of the arrangement or spacing of the roughness elements.

m is a form factor.

For smooth $\in = \in' = m = 0 \rightarrow f = f(V, D, \rho, \mu)$ averaged into non-dimensionless group namely $\frac{\rho dV}{\mu} = Re$

For rough pipes the terms $\in, \in'$ may be made dimensionless by dividing by d

$\therefore f = F\left(\frac{\rho dV}{\mu}, \frac{\in}{d}, \frac{\in'}{d}, m\right)$ Proved by experimental plot of friction factor aganst the R_e on a log-log chart.

Blasius presented his results by an empirical formula is valid up to about Re =100000

$f = \frac{0.316}{Re^{\frac{1}{4}}}$

In rough pipe $\in$/d is called relative roughness.

$f = F\left(Re, \frac{\in}{d}\right)$ is limited and not permit variation of $\in'$/d or m.

Moody has constructed one of the most convenient charts for determining friction factors. In laminar flow, the straight line masked "laminar flow" and the Hangen-Poiseuille equation is applied and from which $f = 64/Re$

$$h_f = f\frac{L}{d}\frac{V^2}{2g} \; ; \; V_{av} = \frac{\Delta p R^2}{8\mu L}$$

The Colebrook formula provides the shape of $\in$/d = constant curves in the transient region

$$\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{\frac{\in}{d}}{3.7} + \frac{2.51}{Re\ \sqrt{f}}\right) \qquad (1)$$

2- *Simple Pipe Problem.*

Six variables enter into the problem for incompressible fluid, which are Q, L, d, h_f, V, $\in$. Three of them are given (L, V, $\in$) and three will be find.

Now, the problems type can be solved as follows,

Problem	Given	To find (unknown)
I	$Q, L, d, V, \in$	h_F
II	$h_f, L, d, V, \in$	Q
III	$h_f, Q, L, V, \in$	d

In each of the above problem the following are used to find the unknown quantity
(i) The Darcy – Weisbach Equation.
(ii) The Continuity Equation.
(iii) The Moody diagram.

In place of the Moody diagram Fig. (1), the following explicit formula for(f)may be utilized with the restrictions placed on it

$$f = 0.0055\left[1 + \left(2000.\frac{\in}{d} + \frac{10^6}{Re}\right)^{\frac{1}{3}}\right] \text{Moody equation}$$

$$4 * 10^3 \leq Re \quad \leq 10^7 \& \frac{\in}{D} \leq 0.01$$

$$f = \frac{1.325}{\left[\ln\left(\frac{\in}{3.7d} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad 10^{-6} \leq \frac{\in}{D} \leq 10^{-12}\ ,\ 5000 \leq R_e \leq 10^8 \qquad (2)$$

1% yield diff-with Darcy equation

The following formula can be used without Moody chart is

$$\frac{1}{f^{1/2}} \approx -1.8\log\left[\frac{6.9}{Re_d} + \left(\frac{\in/d}{3.7}\right)^{1.11}\right] \qquad (3)$$

Eq. 3 is given by Haaland which varies less than 2% from Moody chart.

3- ***Solution Procedures.***

I- Solution for h_f.

With ***Q,∈,*** and ***d*** are known

$$Re = \frac{Vd}{\upsilon} = \frac{4Q}{\pi d \upsilon}$$

And ***f*** may be looked up in Fig.(1) or calculated from Eq. 2. Substitution of (***f***) in Darcy equation (L-4,Eq-19) yields h_f the energy loss due to flow through the pipe per unit weight of fluid.

Ex.1

Determine the head (energy) loss for flow of 140 l/s of oil υ=0.00001 m^2/s through 400 m pipe length of 200 mm- diameter cost–iron pipe

Sol.

$$Re = \frac{4Q}{\pi D \upsilon} = \frac{4(0.14)}{\pi(0.2)(0.00001)} = \mathbf{89127}$$

The relation roughness is ∈/D= 0.25/200= 0.00125 from a given diagram by interpolation ***f***= 0.023 by solution of Eq. 2, ***f***=0.0234; hence

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.023\frac{400}{0.2}\left[\frac{0.14}{\frac{\pi}{4}(0.2)^2}\right]^2\frac{1}{2(9.81)}$$

$$h_f = 46.58m.$$

II- Solution for Discharge Q.

V & f Are unknown then Darcy – Weisbach equation and moody diagram must be used simultaneously to find their values.

1. Givens $\left\{ \begin{array}{l} \boldsymbol{\epsilon/d} \\ \boldsymbol{f} \text{ value is assumed by inspection of the Moody diagram} \end{array} \right.$
2. Substitution of this trail ***f*** into the Darcy – equation produce a trial value of ***V.***
3. From ***V*** a trial ***Re*** is computed.
4. An improved value of ***f*** is found from moody diagram with help of ***Re***
5. When ***f*** has been found correct the corresponding ***V*** and ***Q*** is determined by multiplying by the area.

Ex.2

Water at 15 C° flow through a 300 mm diameter riveted steel pipe, ∈=3 mm with a head loss of 6 m in 300 m. Determine the flow rate in pipe.

Sol.

The relative roughness is ***∈/d*** = 0.003/0.3=0.01, and from diagram a trial ***f*** is taken as (0.038). By substituting into Darcy equation

$$6 = 0.038 \frac{300}{0.3} \frac{V^2}{2(9.81)}$$

$$\therefore\ V = 1.76 \frac{m}{s}$$

At T=15C° → $\upsilon = 1.13 * 10^{-6} \frac{m^2}{s}$

$$\therefore\ Re = \frac{Vd}{\upsilon} = \frac{1.715*0.3}{1.13*10^{-6}} = 467278$$

From the Moody diagram ***f*** = 0.038 at $\left(Re \quad \& \ \frac{\epsilon}{D}\right)$

And from Darcy → $V_{av} = \sqrt{\frac{h_f.d.2.g}{f.L}} = \sqrt{\frac{6*0.3*2*9.81}{0.038*300}} = 1.76 \frac{m}{S}$

$$\therefore Q = AV = \pi\ (0.15)^2 \sqrt{\frac{(6*0.3)(2)(9.81)}{(0.038)(300)}} = 0.1245 \frac{m^3}{s}$$

III- Solution for Diameter d.

Three unknown in Darcy-equation ***f***, ***V***, ***d***, two in the continuity equation ***V, d*** and three in the ***Re*** number equation

To element the velocity in Darcy equation & in the expression for ***Re***, simplifies the problem as follows.

$$h_f = f \frac{L}{d} \frac{Q^2}{2g\left(\frac{d^2\pi}{4}\right)^2}$$

$$Or \quad d^5 = \frac{8LQ^2}{h_f g\pi^2} f = C_1 f \qquad (4)$$

In which C_1 is the known quantity $\frac{8LQ^2}{h_f g\pi^2}$

From continuity $\quad Vd^2 = \frac{4Q}{\pi}$

$$Re \quad = \frac{Vd}{\upsilon} = \frac{4Q}{\pi\upsilon}\frac{1}{d} = \frac{C_2}{d} \qquad (5)$$

C_2 is the known quantity $\frac{4Q}{\pi\upsilon}$ the solution is now effected by the following procedure

1. Assume the value of ***f***.
2. Solve Eq. 4 for ***d***.

3- Solve Eq. 5 for ***Re.***
4- Find the relative roughness ***∈/d***.
5- With R_e and ***∈/d***, Look up new ***f*** from a diagram.
6- Use the new ***f***, and repeat the procedure.
7- When the value of ***f*** does not change in the two significant steps, all equations are satisfied and the problem is solved.

Ex.3

Determine the size of clean wrought-iron pipe required to convey 4000 gpm oil, υ=0.0001 $\frac{ft^2}{s}$, 10000 ft pipe length with a head loss of 75 ft .lb/lb.

Sol.

The discharge is $Q = \frac{4000}{448.4} = 8.93\ cfs$

From Eq. 4, $d^5 = \frac{8LQ^2}{h_f g \pi^2}\ f = \frac{8*10000*8.93^2}{75*32.2*\pi^2}\ f = \mathbf{267.65}\ \boldsymbol{f}$

And from Eq. 5,

$Re \;\; = \frac{4Q}{\pi \upsilon}\frac{1}{d} = \frac{4*8.93}{\pi*0.0001}\frac{1}{d} = \frac{113700}{d}$

And from Table 1 ∈= 0.00015 ft

If *f*=0.02 (assumed value) , ∴ d = 1.35 ft

$Re \;\; = 81400$ }

∈/d= 0.00011 } from Moody chart $f = 0.0191$

In repeating the procedure, d = 1.37 ft → $Re \;\; = 82991 - -\rightarrow f = 0.019$ Therefore

$d = 1.382 * 12 = 16.6\ in$

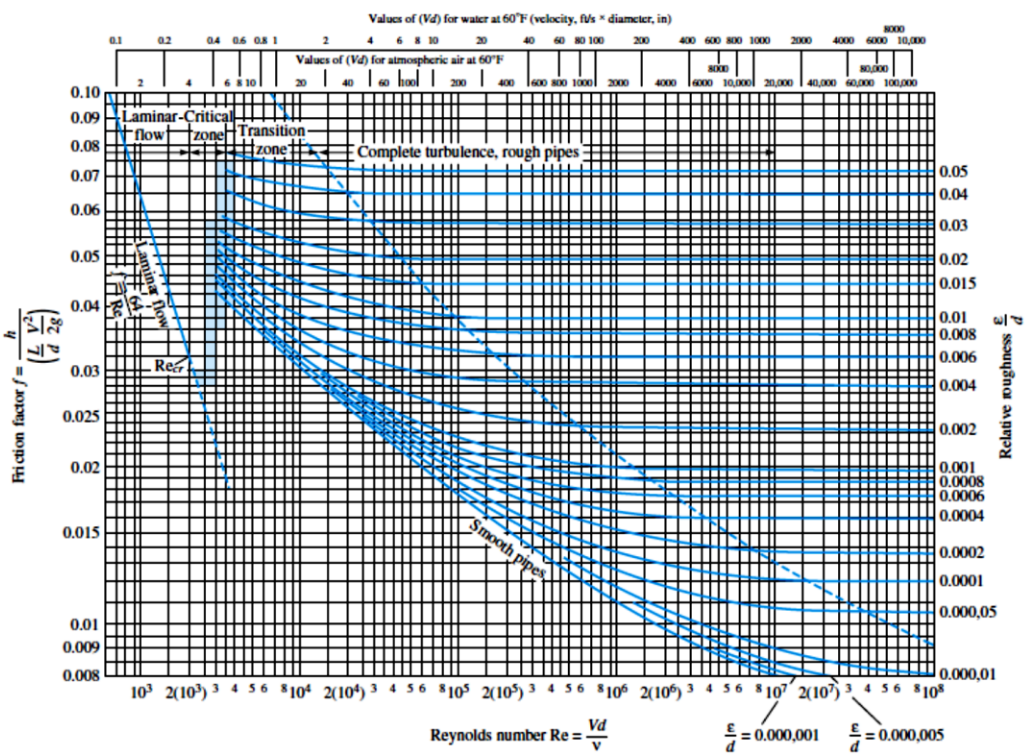

Figure (1): The Moody chart for pipe friction with smooth and rough walls.

Table 1: Recommended roughness values.

Material	Condition	ϵ ft	ϵ mm	Uncertainty, %
Steel	Sheet metal, new	0.00016	0.05	± 60
	Stainless, new	0.000007	0.002	± 50
	Commercial, new	0.00015	0.046	± 30
	Riveted	0.01	3.0	± 70
	Rusted	0.007	2.0	± 50
Iron	Cast, new	0.00085	0.26	± 50
	Wrought, new	0.00015	0.046	± 20
	Galvanized, new	0.0005	0.15	± 40
	Asphalted cast	0.0004	0.12	± 50
Brass	Drawn, new	0.000007	0.002	± 50
Plastic	Drawn tubing	0.000005	0.0015	± 60
Glass	—	Smooth	Smooth	
Concrete	Smoothed	0.00013	0.04	± 60
	Rough	0.007	2.0	± 50
Rubber	Smoothed	0.000033	0.01	± 60
Wood	Stave	0.0016	0.5	± 40