Lecture-Seven

Minor losses Analysis in Pipes

1- *Minor Losses.*

The losses which occur in pipelines because of bend, elbows, joints, valves, etc, are called minor losses $\boldsymbol{h_m}$. the total losses in pipeline are

$$h_L = h_f + h_m \qquad (1)$$

Also, others minor losses can be explain as follows,

A. *Losses due to sudden expansion in pipe.*

From momentum equation

$$\sum Fx = \rho Q\,(V_2 - V_1)$$

$$Q = A_2 V_2$$

$$p_1 A_1 - p_2 A_2 = \rho A_2 (V_2^2 - V_1 V_2)$$

Divided by γA_2 since $A_1 = A_2$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1 V_2}{g} \qquad ---(a)$$

From Bermonlli's equation between section 1&2 as in Fig.1

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_m$$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} + h_m \qquad ---(b)$$

Equating Eq. (a & b)

$$\frac{V_2^2 - V_1 V_2}{g} = \frac{V_2^2 - V_1^2}{2g} + h_m$$

$$h_m = \frac{V_2^2 - V_1 V_2}{g} - \frac{V_2^2 - V_1^2}{2g} = \frac{2V_2^2 - 2V_1\,V_2 - V_2^2 + V_1^2}{2g}$$

$$h_m = \frac{V_1^2 - 2V_1 V_2 + V_2^2}{2g} = \frac{(V_1 - V_2)^2}{2g} \qquad ---(c)$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1}{A_2} V_1 \qquad substitute\ in\ Eq.(c)$$

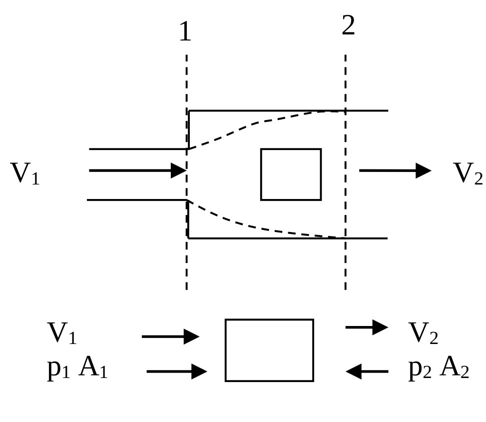


Figure 1: Sudden expansion.

$$\therefore\ h_m = \frac{V_1^2 - 2V_1\frac{A_1}{A_2}V_1 + \left(\frac{A_1}{A_2}V_1\right)^2}{2g}$$

$$h_m = \frac{V_1^2\left(1 - 2\frac{A_1}{A_2} + \left(\frac{A_1}{A_2}\right)^2\right)}{2g}$$

$$h_m = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 = K\frac{V_1^2}{2g} \qquad (2)$$

$$K = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(1 - \frac{d_1^2}{d_2^2}\right)^2$$

If sudden expansion from pipe to a reservoir $\frac{d_1}{d_2} = 0$

$$\therefore\ h_m = \frac{V_1^2}{2g}$$

B. <u>Head loss due to a sudden contraction in the pipe cross section</u>.

$Q = A_2V_2 = A_cV_c$

$C_c = A_c/A_2$

$p_cA_2 - p_2A_2 = \rho Q(V_2 - V_c)$

$p_cA_2 - p_2A_2 = \rho A_2(V_2^2 - V_2V_c)$

Divided by $\gamma\ A_2$

$$\frac{p_c - p_2}{\gamma} = \frac{V_2^2 - V_cV_2}{g} \qquad ---(a)$$

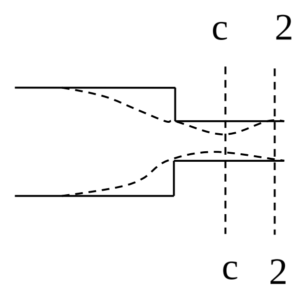


Figure 2: Sudden contraction.

Applying B.E between sections c & 2

$$\frac{p_c}{\gamma} + \frac{V_c^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_m$$

$$\frac{p_c - p_2}{\gamma} = \frac{V_2^2 - V_c^2}{2g} + h_m ----(b)$$

$V_c = \frac{A_2}{A_c}V_2$, Equating Eq (a& b)

$$\frac{V_2^2 - V_cV_2}{g} = \frac{V_2^2 - V_c^2}{2g} + h_m$$

$$h_m = \frac{V_2^2 - V_cV_2}{g} - \left(\frac{V_2^2 - V_c^2}{2g}\right) = \frac{2V_2^2 - 2V_cV_2 - V_2^2 + V_c^2}{2g}$$

$$h_m = \frac{V_c^2 - 2V_cV_2 + V_2^2}{2g} = \frac{V_2^2}{2g}\left(\left(\frac{A_2}{A_c}\right)^2 - \frac{2A_2}{A_c} + 1\right)$$

$$h_m = \frac{V_2^2}{2g}\left[\left(\frac{1}{C_c}\right)^2 - 1\right]^2$$

$$h_m = K\frac{V_2^2}{2g} \qquad (3)$$

$K = \left(\frac{1}{C_c} - 1\right)^2$, From experimental, $K \approx 0.42\left(1 - \frac{d_2^2}{d_1^2}\right)$

A_c is the cross – sectional area of the vena-contracts

C_c is the coefficient of contraction is defined by $C_c = \frac{A_c}{A_2}$

C. The head loss at the entrance to a pipe line.

From a reservoir the head losses is usually taken as

$h_m = \frac{0.5V^2}{2g}$ is the opening is square-edged

$h_m = \frac{0.01V^2}{2g} \sim \frac{0.05V^2}{2g}$ if the rounded entrance

Table (1) lists the loss coefficient ***K*** for four types of valve, three angles of elbow fitting and two tee connections. Fitting may be connected by either internal screws or flanges, hence the two are listings.

Table 1: Head loss coefficients K for typical fittings.

	Nominal diameter, in								
	Screwed				Flanged				
	$\frac{1}{2}$	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

Entrance losses are highly dependent upon entrance geometry, but exit losses are not. Sharp edges or protrusions in the entrance cause large zones of flow separation and large losses as shown in Fig. 3. As in Fig. 4, a bend or curve in a pipe, always induces a loss larger than the simple Moody friction loss, due to flow separation at the walls and a swirling secondary flow arising from centripetal acceleration.

Table (1) gives the losses coefficients for the fully open condition. In case of partially open valve the losses can be much higher. Fig. (5) gives average losses for three valves as a function of percentage open. The opining distance ratio ***h***/D as the x-axis in Fig. (5) is shown by Fig.(6) of valve geometry.

Minor losses may be expressed in terms of the equivalent L_e of pipe that has the same head loss in m.N /N

$$f\frac{L_e}{d}\frac{V^2}{2g} = K\frac{V^2}{2g}$$

K is the sum of several losses, solving for L_e gives $L_e = \frac{Kd}{f}$ (4)

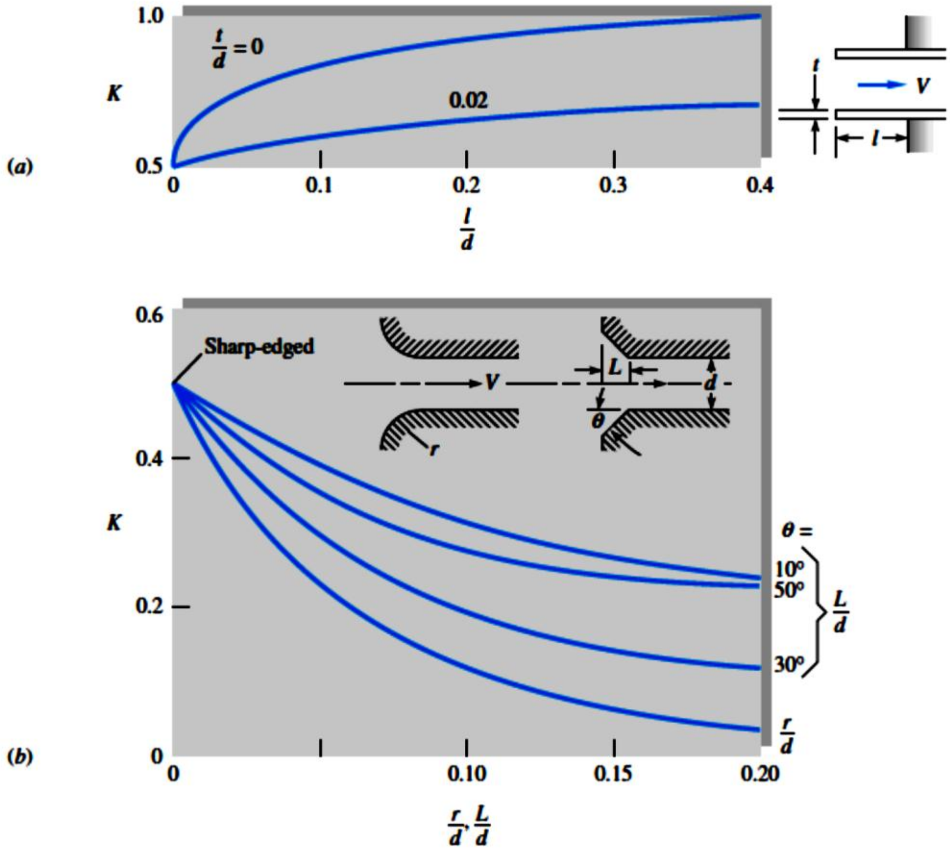


Figure 3: Entrance and exit loss coefficients,(a) reentrant inlets, (b) rounded and beveled inlets. Exit losses are K=1.

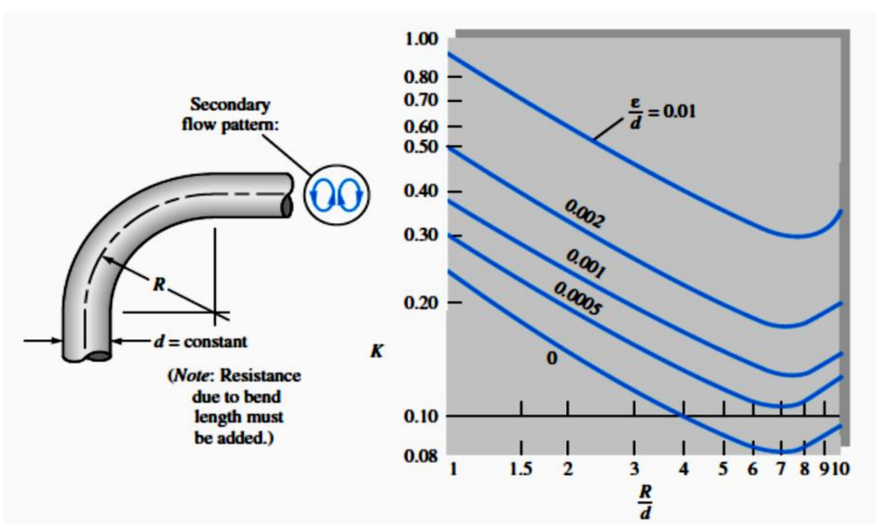


Figure 4: Resistance coefficients for 90° bends.

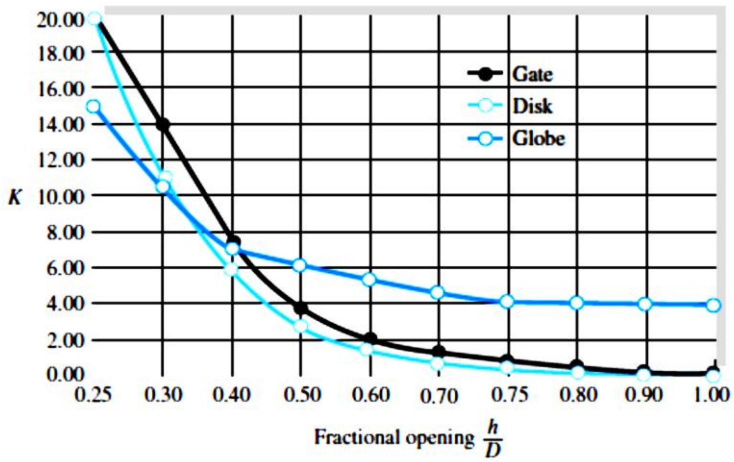


Figure 5: Average-loss coefficients for partially open valves.

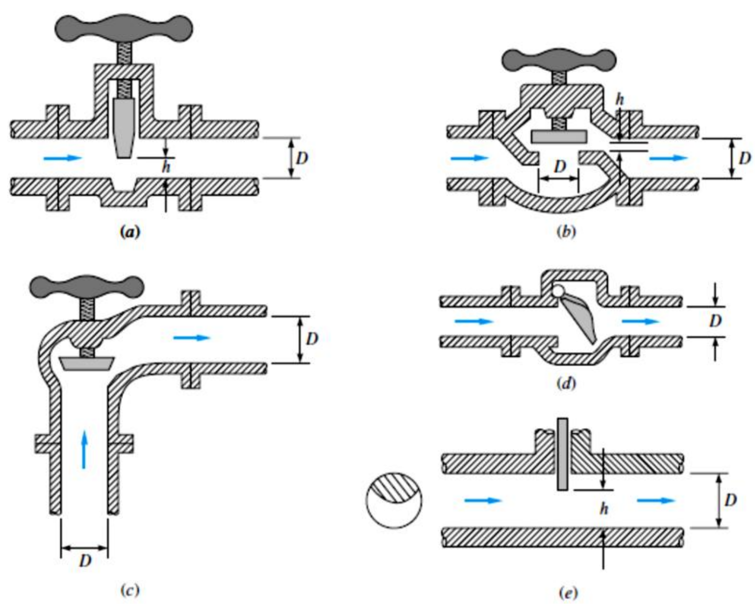


Figure 6: Typical commercial valve; (a) gate valve, (b) globe valve, (c) angle valve, (d) swing-check valve, (e) disk-type gate valve.

2- Application-1

Water, ρ = 1.94 slugs/ft^3, $and\ \nu = 1.1 * 10^{-5}\frac{ft^2}{s}$, is pumped between two reservoir at 0.2 $\frac{ft^3}{s}$ through 400 ft of 2 in diameter pipe and several minor losses, as shown in figure. The roughness ratio is $\frac{\epsilon}{d} = 0.001$. Compute the horse power required.

Sol.

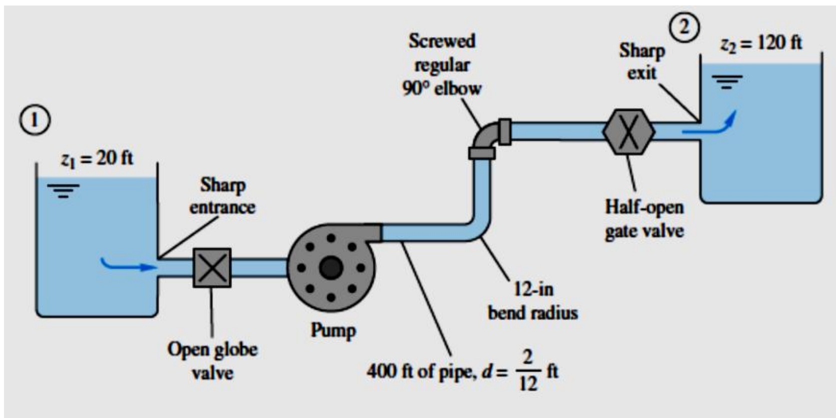
Write the steady- flow energy equation between section 1 &2 the two reservoir surface:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{2g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_m - h_p$$

Where h_p is the head increase across the pump $p_1 = p_2$, $V_1 = V_2 \approx 0$, solve for the pump head

$$h_p = z_2 - z_1 + h_f + \sum h_m = 120 - 20 + \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right)$$

$$V = \frac{Q}{A} = \frac{0.2}{\frac{1}{4}\pi\left(\frac{2}{12}\right)^2} = 9.17\frac{ft}{s}$$



Now list and sum the minor loss coefficients

Loss	K
Sharp entrance (fig, 7.9)	0.5
Open globe value (2 in Table 7.3)	6.9
12-in bend Fig. 7.10 $\frac{R}{d} = 6, \frac{\epsilon}{d} = 0.001$	0.15
Regular 90 elbow ($Table$ 7.3)	0.95
Half – closed gate value (Fig. 7.11)	3.8
Sharp exit (Fig. 7.9)	1.0
$\sum K$	13.3

Calculate the Re and pipe friction factor

$$Re = \frac{Vd}{\nu} = \frac{9.17\left(\frac{2}{12}\right)}{1.1*10^{-5}} = 139000$$

For $\frac{\epsilon}{d} = 0.001$, from the Moody chart read f = 0.0216

$$\therefore\ h_p = 100 + \frac{9.17^2}{2(32.2)}\left[\frac{0.0216(400)}{\left(\frac{2}{12}\right)} + 13.3\right]$$

$h_p = 100 + 84 = 184\ ft\ pump\ head$

$P = \rho g Q h_p = 1.94(32.3)(0.2)(184) = 2300\ \frac{ft\ lb_f}{s}$

$1\quad h.p. = 550\ \frac{ft\ lb_f}{s}\ \therefore P = \frac{2300}{550} = 4.2\ h.p.$

For an efficiency 70 to 80% , a pump is needed with an input power about 6 h.p.

3- *Application-2*

Water is to be supplied to the inhabitants of a college campus through a supply main. The following data is given,

Distance of the reservoir from the campus = 3000 m

Number of inhabitance = 4000

Consumption of water per day of each inhabitant= 180 liters

Loss of head due to friction = 18 m,

Co-efficient of friction for the pipe, *f* = 0.007. If the half of the daily supply is pumped in 8 hr, determine the size of the supply main (***d***) .

Sol.

Total supply per day $= 4000 * \frac{180}{1000} = 720\frac{m^3}{day}$

$Q = \frac{720}{2*8*3600} = 0.0125\frac{m^3}{s}$

$h_f = 18m$

Assume *f*= 0.03 $\quad \therefore d^5 = \frac{8*L\,Q^2}{h_f\,g\,\pi^2} * f = \frac{8*3000*(0.0125)^2}{18*9.81*\pi^2} * 0.03 = 6.455 * 10^{-5}$

$\therefore d = 0.1452m$

$R_e = \frac{4Q}{\pi v d} = \frac{4*0.0125}{\pi*1.007*10^{-6}*0.1452} = 108904 = 1.08 * 10^5$

$\frac{\epsilon}{d} = \frac{0.046*10^{-3}}{0.1452} = 3.168 * 10^{-4}$ $\qquad$ From $\left(R_e\ \&\ \frac{\epsilon}{d}\right) - -\rightarrow f = 0.0195$

$d^5 = 4.2 * 10^{-5} - -\rightarrow d = 0.13352\ m$.

4- *Application-3*

I) Find the discharge through the pipeline as in below figure for H=10m, II) determine the head loss $\boldsymbol{h_L}$ for Q=60 l/s. III) compare the result of discharge with equivalent length .

Sol.

The energy equation applied between points 1 & 2, including all the losses, may be written

I) $\quad H_1 + 0 + 0 = \frac{V_2^2}{2g} + 0 + 0 + \frac{1}{2}\frac{V_2^2}{2g} + f\,\frac{102}{0.2032}\frac{V_2^2}{2g} + 2 * 0.26\frac{V_2^2}{2g} + \frac{5.3\,V_2^2}{2g}$

Loss coefficients (K):-

Entrance = 0.5

Each elbow = 0.26

Globe valve (partially open h/d=0.6) = 5.3

$\therefore\ H_1 = \frac{V_2^2}{2g}\,(7.32 + 502\,f) \qquad\qquad - - - - - (A)$

When the head is given, this problem is solved as the second type of simple pipe problem. If $\frac{\epsilon}{d} = \frac{0.26}{203.2} = 1.28 * 10^{-3}$, *f* = 0.0205

$10 = \frac{V_2^2}{2g}\,(\,7.32 + 502 * 0.0205) - -\rightarrow\ V_2 = 3.337\ \frac{m}{s}$

$\nu = 1.01 * 10^{-6} \frac{m^2}{s}$

$\frac{\epsilon}{d} = 0.00128$; $R_e = \frac{(3.337*0.2032)}{(1.01*10^{-6})} = 6.7 * 10^5$

From Moody chart at $\left\{ Re \quad \& \frac{\epsilon}{d} \right\} - - \rightarrow f = 0.0208$

Repeating the procedure gives $V_2 = 3.32 \frac{m}{s}$, $Re = 6.6 * 10^5$, $and \frac{\epsilon}{d}$, $f = 0.0209$. from eq. A gives V_2=3.31 m/s. The discharge is

$Q = V_2 A_2 = (3.31)\left(\frac{\pi}{4}\right)(0.2032)^2 = 107.34\ l/s$

II) For the second part , with Q is known, the solution is straight forward;

$V_2 = \frac{Q}{A} = \frac{0.06}{\left(\frac{\pi}{4}\right)(0.2032)^2} = 1.85 \frac{m}{s} \quad Re = 3.7 * 10^5\ and\ \frac{\epsilon}{d}, f = 0.0212$

From Eq. A

$\therefore\ h_L = \frac{(1.85)^2}{2(9.806)}(6.32 + 502 * 0.0212) = 2.959\ m$

III) With equivalent lengths Eq.(4) the value of ***f*** is an approximated, say ***f*** = 0.0205. The sum of minor losses is K= 6.32

$L_e = \frac{Kd}{f} = \frac{6.32*0.2032}{0.0205} = 62.64\ m$

The total length of pipe is

$62.64 + 102 = 164.64\ m$

By Darcy equation. $10 = f\ \frac{L+L_e}{d}\frac{V_2^2}{2g} = f\ \frac{164.64}{0.2032}\frac{V_2^2}{2g}$

If $f = 0.0205, V_2 = 3.43 \frac{m}{s},\ Re = 6.9 * 10^5\ and \frac{\epsilon}{d}, f = 0.0203$

From eq.A , $V_2 = 3.347 \frac{m}{s} and\ Q = 108.5\ l/s$