Lecture-Eight

Losses Analysis in Different Pipe Systems

1- *Pipe in Series.*

In this typical series–pipe system as in Fig.(1), the H (head) is required for a given Q or the Q wanted for a given H. Applying the energy equation from A to B including all losses gives

$$H + 0 + 0 = 0 + 0 + 0 + K_e \frac{V_1^2}{2g} + f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g} + \frac{V_2^2}{2g} \qquad (1)$$

From continuity eqn.

$V_1 d_1^2 = V_2 d_2^2$

V_2 is eliminated from eqn (1) , so that

$$H = \frac{V_1^2}{2g}\left\{K_e + \frac{f_1 L_1}{d_1} + \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 + \frac{f_2 L_2}{d_2}\left(\frac{d_1}{d_2}\right)^4 + \left(\frac{d_1}{d_2}\right)^4\right\} \qquad (2)$$

For known lengths and sizes of pipes this reduces to

$$H = \frac{V_1^2}{2g}\,(C_1 + C_2 f_1 + C_3 f_2) \qquad (3)$$

C_1, C_2 & C_3 *are Known*

- Q is given then R_e is computed and *f's* may be looked up in Moody diagram then H is found by direct substitution.
- For a given H, the values of V_1, f_1, f_2 are unknown in Eq. (3).

By Assuming values of f_1 & f_2 (may be equaled) then V_1 *is found* 1^{st} *trial*
and from $V_1 \dashrightarrow$ ***Re's*** are determined and values of f_1 & f_2 look up from Moody diagram.
And at these value, a better V_1 is computed from Eq. (2) since ***f*** varies so slightly with Re the trial solution converges very rapidly. The same procedures apply for more than two pieces in series.

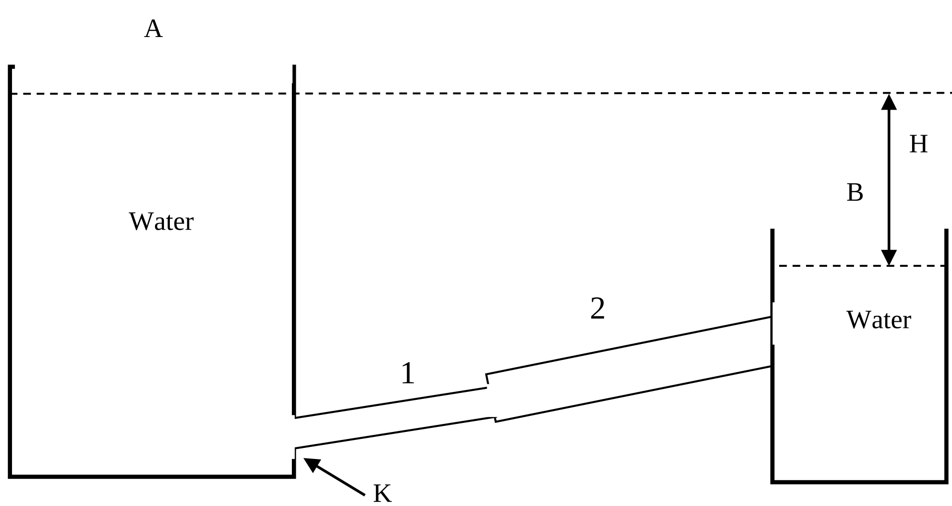

Figure 1: Series pipe system

2- Equivalent pipes.

Two pipe system (in series) are said to be equivalent when the same head loss produces the same discharge in both system. From Darcy equation.

$$h_{f1} = f_1 \frac{L_1}{d_1} \frac{Q_1^2}{\left(d_1^2 \frac{\pi}{4}\right)^2 2g} = f_1 \frac{L_1}{d_1^5} \frac{8Q_1^2}{\pi^2 g}$$

And for a second pipe

$$h_{f2} = f_2 \frac{L_2}{d_2^5} \frac{8Q_2^2}{\pi^2 g}$$

For the two pipes to be equivalent

$$h_{f1} = h_{f2} \qquad Q_1 = Q_2$$

After equating $h_{f1} = h_{f2}$ and simplifying

$$\frac{f_1 L_1}{d_1^5} = \frac{f_2 L_2}{d_2^5}$$

Solving for L_2 gives

$$L_2 = L_1 \frac{f_1}{f_2} \left(\frac{d_2}{d_1}\right)^5 \qquad (4)$$

Ex.1

From Fig.(1), $K_e = 0.5, L_1 = 300m\ , d_1 = 600mm, \epsilon_1 = 2mm, L_2 = 240m, d_2 = 1m, \epsilon_2 = 0.3mm\ , \nu = 3 * 10^{-6} \frac{m^2}{s}\ and\ H = 6m.$ Determine the discharge through the system.

Sol.

From the energy Eq. (2)

$$6 = \frac{V_1^2}{2g} \left[0.5 + f_1 \frac{300}{0.6} + (1 - 0.6^2)^2 + f_2 \frac{240}{1.0}\ 0.6^4 + 0.6^4 \right]$$

After simplifying

$$6 = \frac{V_1^2}{2g} (1.0392 + 500 f_1 + 31.104 f_2)$$

From $\frac{\epsilon_1}{d_1} = 0.0033, \frac{\epsilon_2}{d_2} = 0.0003,$ and Moody diagram values of f 's are assumed for the fully turbulent range.

$$f_1 = 0.026 \quad f_2 = 0.015$$

By solving for V_1 with these value, $V_1 = 2.848 \frac{m}{s}, V_2 = 1.025 \frac{m}{s}$

$$Re_1 = \frac{2.848 * 0.6}{3 * 10^{-6}} = 569600$$

$$Re_2 = \frac{1.025 * 1.0}{3 * 10^{-6}} = 341667$$

At these ***Re's*** and from Moody diagram, $f_1 = 0.0265, f_2 = 0.0168,$ by solving again for $V_1, V_1 = 2.819 \frac{m}{s}$ and $Q = 0.797 \frac{m^3}{s}$

Ex.2

Solve ***Ex.1*** by means of equivalent pipes.

Sol.

First by expressing the minor losses in terms of equivalent length, for pipe 1, since $K=K_e+(1-(d1/d2)^2)^2$

$$K_1 = 0.5 + (1 - 0.6^2)^2 = 0.91 \longrightarrow L_{e1} = \frac{K_1 d_1}{f_1} = \frac{0.91 * 0.6}{0.026} = 21m$$

For pipe-2 $K_2 = 1 \longrightarrow L_{e2} = \frac{K_2 d_2}{f_2} = \frac{1 * 1}{0.015} = 66.7\ m$

After selecting f_1 & f_2 . The problem is reduced to 321m of 600-mm diam. & 306.7 m of 1-m pipe diam. By expressing the 1-m pipe terms of an equivalent length of 600-mm pipe, by Eq. (4)

$$L_e = \frac{f_2}{f_1} L_2 \left(\frac{d_1}{d_2}\right)^5 = 306.7 \frac{0.015}{0.026} \left(\frac{0.6}{1}\right)^5 = 13.76\, m$$

Now, by adding to the 600-mm pipe, the problem is reduced to 334.76m of 600-mm for finding the discharge through it, $\in_1 = 2mm,\ H = 6\, m$.

$$6 = f \frac{334.76}{0.6} \frac{V^2}{2g}$$

$$f = 0.026, ---\rightarrow V = 2.848 --\rightarrow R_e = .848 * \frac{0.6}{(2 * 10^{-6})} = 569600$$

From Re and $\frac{\in}{d} = 0.0033$ from Moody diagram *f=0.0265*

From above equation

$$V = 2.821 \ \frac{m}{s} \ \rightarrow Q = \pi\, (0.3)^2\, (2.821) = 0.798 \ \frac{m^3}{s}$$

3- *Pipes in Parallel.*

The second type of pipe- system is the parallel flow type, in this case as shown in Fig.(2) the head losses are same in any of the lines and the total flow is the sum of flow rate in each pipe.

The minor losses are added into the lengths of each pipe as equivalent lengths. From Fig.(2) the conditions to be satisfied are

$$\left.\begin{aligned} & h_{f1} = h_{f2} = h_{f3} = \frac{p_A}{\gamma} + z_A - \left(\frac{p_B}{\gamma} + z_B\right) \\ & Q = Q_1 + Q_2 + Q_3 \end{aligned}\right\} \qquad (5)$$

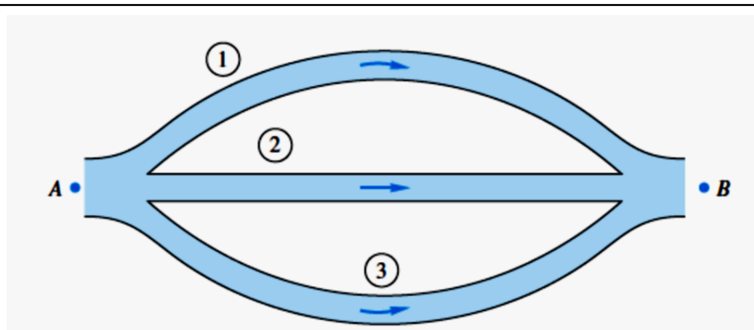

Figure 2: Parallel pipes system.

z_A & z_B are the elevations of point A&B.
Q is the discharge through the approach pipes.
Two types of problems occur

1) The elevations of HGL (hydraulic grid line) at A&B are known, to find the discharge.
2) Q is known, to find the distribution of flow and the head loss, size of pipe, fluid properties and roughness's are assumed to be known.

Case-1. as the simple pipe problem. Since, head loss is the drop in HGL. These discharges are added to determine the total discharge.

Case-2. The recommended procedure is as follows.

1- Assume a discharge Q', through pipe 1.
2- Solve for h'_{f1}, using assumed discharge.
3- Using h'_{f1}, find Q'_2 , Q'_3 .
4- With the three discharges for a common head loss, now assume that the given Q is split up among the pipes in the same proportion as Q'_1, Q'_2, Q'_3 ; thus

$$Q_1 = \frac{Q'_1}{\sum Q'} Q; \quad Q_2 = \frac{Q'_2}{\sum Q'} Q; \quad Q_3 = \frac{Q'_3}{\sum Q'} Q \qquad (6)$$

5- Check the correctness of these discharges by computing h_{f1}, h_{f2}, h_{f3} for the computing Q_1, Q_2, Q_3 .

Ex.3

In Fig. (2), $L_1 = 3000\ ft$, $d_1 = 1\ ft$, $\in_1 = 0.001\ ft$
$L_2 = 2000\ ft$, $d_2 = 8 in$, $\in_2 = 0.0001\ ft$
$L_3 = 4000\ ft$, $d_3 = 16\ in$, $\in_3 = 0.0008\ ft$
$\rho = 2.00 \frac{Slugs}{ft^3}$, $\boldsymbol{\nu} = 0.00003 \frac{\boldsymbol{ft^2}}{\boldsymbol{s}}$
$p_A = 80\ psi, z_A = 100\ ft$, $z_B = 80\ ft$.
For a total flow of 12 cfs, determine the flow through each pipe and the pressure at B.

Sol.

For pipe-1-

Assume $Q'_1 = 3\ cfs$; $then\ V'_1 = 3.82 \frac{ft}{s}$

$$\left.\begin{aligned} \therefore\ Re'_1 &= \frac{3.82*1}{0.00003} = 127000 \\ \frac{\in_1}{d_1} &= 0.001 \end{aligned}\right\} \quad \text{From Moody chart} \quad f'_1 = 0.022$$

$$\therefore h'_{f1} = 0.022 \frac{3000}{1.0} \frac{3.82^2}{64.4} = 14.97\ ft$$

For pipe-2-

$$14.97 = f'_2 \frac{2000}{0.667} \frac{V'^2_2}{2g} ------ (a)$$

Assume $f'_2 = 0.020$ (Recomended fully turbulent flow)

$$then\ V'_2 = 4.01 \frac{ft}{s} --\rightarrow\ Re'_2 = \frac{4.01*\frac{2}{3}*1}{0.00003} = 89000$$

$$\frac{\in_2}{d_2} = 0.00015$$

From Moody chart → $f'_2 = 0.019$

Then from Eq. (a) $V'_2 = 4.11 \frac{ft}{s} --\rightarrow\ Q'_2 = 1.44\ cfs$

For pipe -3-

$$14.97 = f'_3 \frac{4000}{1.333} \frac{V'^2_3}{2g} ------ (b)$$

Assume $f'_3 = 0.019\ then\ V'_3 = 4.01 \frac{ft}{s} --\rightarrow\ Re'_3 = \frac{4.01*1.333}{0.00003} = 178000$

$$\frac{\in_3}{d_3} = 0.0006$$

From Moody chart. At $\left(Re'_3\ \&\frac{\in_3}{d_3}\right) --\rightarrow\ f'_3 = 0.02$ from Eq.(b). $V'_3 = 4.0 \frac{ft}{s}$ & $Q'_3 = 5.6\ cfs$

The total discharge for the assumed conditions is

$$\sum Q' = 3.0 + 1.44 + 5.6 = 10.04\ cfs$$

From Eq. (6)

$Q_1 = \frac{Q_1'}{\sum Q'} . Q = \frac{3.00}{10.04}\ 12 = 3.58\ cfs$
$Q_2 = \frac{1.44}{10.04}\ 12 = 1.72\ cfs$
$Q_3 = \frac{5.6}{10.04}\ 12 = 6.7\ cfs$
Check the values of h_{f1}, h_{f2}, h_{f3}

$$V_1 = \frac{3.58}{\pi * \frac{1}{4}} = 4.56\frac{ft}{s} --\rightarrow\ Re_1, \frac{\in_1}{d_1} = 152000\} -\rightarrow\ \ f_1 = 0.021$$

$h_{f1}=20.4ft$

$$V_2 = \frac{1.72}{\frac{\pi}{9}} = 4.93\frac{ft}{s} --\rightarrow \frac{\in_2}{d_2}\ Re_2 = 109200\} -\rightarrow\ f_2 = 0.019$$

$h_{f2}=21.6\ ft$

$V_3 = \frac{6.7}{\frac{4\pi}{9}} = 4.8\frac{ft}{s} -\rightarrow \frac{\in_3}{d_3}\ Re_3 = 213000\} -\rightarrow\ f_3 = 0.019 f_2$
$h_{f3}=20.4ft$

is about midway between 0.018 & 0.019. To satisfy the condition $h_{f1} = h_{f2} = h_{f3}$
$\therefore if\ f_2 = 0.018\ then\ h_{f2} = 20.4\ ft \quad satisfying$.
To find p_B
$\frac{p_A}{\gamma} + z_A = \frac{p_B}{\gamma} + z_B + h_f$
$or\ \frac{p_B}{\gamma} = 80 * \frac{144}{64.4} + 100 - 80 - 20.4 = 178.1 ft$
In which the average head loss was taken. Then
$p_B = \frac{178.1*64.4}{144} = 79.6\ psi$